cv. 9. Lap. rovnice

$$\dot{x} = \alpha x - 2y + x^2 + xy + y^2$$
$$\dot{y} = x + y$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha & -2 \\ 1 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{\begin{pmatrix} x^2 + xy + y^2 \\ 0 \end{pmatrix}}_{f(x,y)}$$

$\operatorname{tr} A = \alpha + 1$ Hopf: $\alpha = -1$

$\det A = \alpha + 2 \big|_{\alpha=-1} = 1 > 0$

$$A(\alpha) \Rightarrow A(-1) = \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix}$$

vl. čísla: $\begin{vmatrix} -1-\lambda & -2 \\ 1 & 1-\lambda \end{vmatrix} = \lambda^2 + 1 = 0$

$\lambda_{1,2} = \pm i$

vl. vektory příslušný $-i$:

$$\begin{pmatrix} -1+i & -2 \\ 1 & 1+i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \; v = \begin{pmatrix} 1+i \\ -1 \end{pmatrix}$$

$$\Rightarrow T = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

$$T^{-1} A T = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \checkmark$$

$$\dot{x} = Ax + f(x)$$

$$x = Tu \Rightarrow T\dot{u} = ATu + f(Tu)$$

$$\dot{u} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} u + \underline{\underline{T^{-1} f(Tu)}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 + u_2 \\ -u_1 \end{pmatrix}$$

$$f(u_1 + u_2, -u_1) = \begin{pmatrix} (u_1+u_2)^2 - (u_1+u_2)u_1 + u_1^2 \\ 0 \end{pmatrix}$$

$$T^{-1} \cdot f(u_1 + u_2, -u_1) =$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} u_1^2 + u_1 u_2 + u_2^2 \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ u_1^2 + u_1 u_2 + u_2^2 \end{pmatrix}$$

$$Q_{11} = 2, \quad Q_{12} = 1, \quad Q_{22} = 2 \quad \Rightarrow \quad l_1 = \frac{1}{8} \cdot \left(-1 \cdot (2+2)\right) = -\frac{1}{2} < 0 \text{ stab.}$$