$$t - x \in [t-1, t+1] \cap [0,1] =$$

$$= \begin{cases} \emptyset & t < -1 \\ [0, t+1] & t \in [-1, 0] \\ [0, 1] & t \in [0, 1) \\ [t-1, 1] & t \in [1, 2] \\ \emptyset & t > -2 \end{cases}$$

- $t - x \in [0, t+1]$

  $-x \in [-t, 1]$

  $x \in [1, t]$

- $t - x \in [0, 1]$

  $-x \in [-t, 1-t]$

  $x \in [t-1, t]$

- $t - x \in [t-1, 1]$

  $-x \in [-1, 1-t]$ $\qquad x \in [t-1, 1]$

$t \in [-1,0] \qquad x \in [1, t]$

$$\int_1^t f_1(x) f_2(t-x)\,dx =$$

$$= \int_1^t (1-x^2)(t-x)\,dx$$

$$= \int_1^t (x^3 - tx^2 - x + t)\,dx$$

$$= \left[\tfrac{1}{4}x^4\right]_1^t - t\left[\tfrac{1}{3}x^3\right]_1^t - \left[\tfrac{1}{2}x^2\right]_1^t + t\left[x\right]_1^t =$$

$$= \left(\tfrac{1}{4}t^4 - \tfrac{1}{4}\right) - t\left(\tfrac{1}{3}t^3 - \tfrac{1}{3}\right) - \tfrac{1}{2}\left(t^2 - 1\right) + t(t-1)$$

$$= -\frac{1}{12}t^4 + \frac{1}{2}t^2 - \frac{2}{3}t + \frac{1}{2}$$

$t \in [0,1] \qquad x \in [t-1, t]$

$$(f_1 * f_2)(t) = \int_{t-1}^t (x^3 - tx^2 - x + t)$$

$$= \left[\frac{1}{4}x^4\right]_{t-1}^{t} - t\left[\frac{1}{3}x^3\right]_{t-1}^{t}$$

$$- \left[\frac{1}{2}x^2\right]_{t-1}^{t} + t\left[x\right]_{t-1}^{t} \quad \text{atd.}$$

...

$t \in [1,2] \qquad x \in [t-1, 1]$

⋮

12.3. (ii)

$$f_1(x) = \frac{1}{x} \qquad x \neq 0$$

$$f_2(x) = \begin{cases} 1 & x \in [-1,1] \\ 0 & \text{jinak} \end{cases}$$

$$(f_1 * f_2)(t) = \int_{-\infty}^{\infty} f_1(x)\, f_2(t-x)\, dx$$

$$t - x \in [-1, 1]$$

$$-x \in [-1-t, 1-t]$$

$$x \in [t-1, t+1]$$

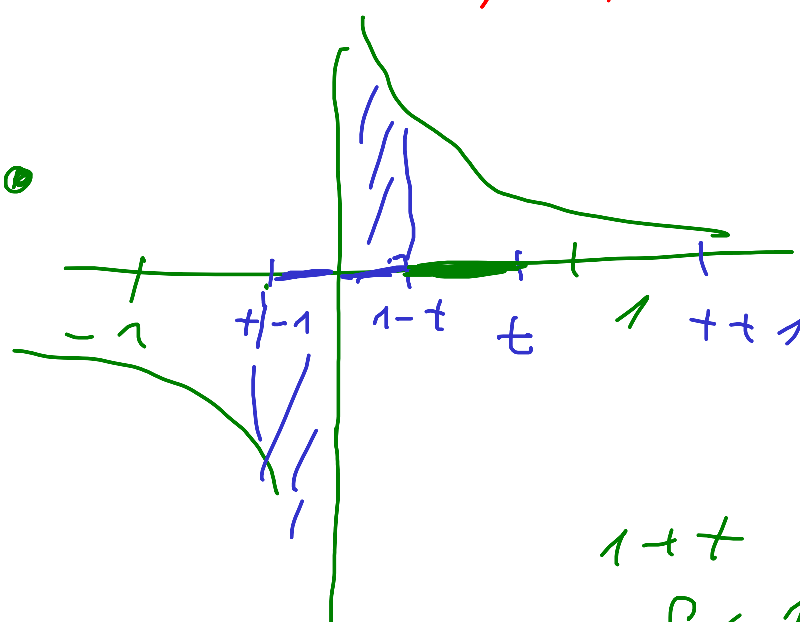
$$(f_1 * f_2)(t) = \int_{t-1}^{t+1} \underbrace{f_1(x)}_{\frac{1}{x}} \, dx$$

- $t < -1$ $\longrightarrow$ $[t-1, t+1] \not\ni 0$
- $t \in [-1, 1]$
- $t > -1$

$$(f_1 * f_2)(t) = \int_{t-1}^{t+1} \frac{1}{x} \, dx =$$

$$= [\ln x]_{t-1}^{t+1} = \underline{\underline{\ln \frac{t+1}{t-1}}}$$

$t \in [-1, 1]$

- 

$$(f_1 * f_2)(t) = \int_{1-t}^{1+t} \frac{1}{x} \, dx$$

$$= \left[\ln x \right]_{1-x}^{1+x} = \ln \frac{1+x}{1-x}$$

$$(f_1 * f_2)(t) = 0$$

$$t \in (-\infty, -1) \cup (1, \infty)$$