8.1 (i)

$$\frac{x}{(x-1)^2(x^2+2x+2)} = \frac{1}{25(x-1)} + \frac{1}{5(x-1)^2} - \frac{x+8}{25(x^2+2x+2)}$$

$$\int = \int \frac{1}{25(x-1)}\,dx + \int \frac{1}{5(x-1)^2}\,dx$$
$$- \int \frac{x+8}{25(x^2+2x+2)}\,dx$$

$$\int y^k\,dy = \frac{1}{k+1}\,y^{k+1} + C$$

$$= \frac{1}{25}\ln|x-1| - \frac{1}{5}\cdot\frac{1}{x-1}$$
$$- \frac{1}{25}\int \frac{\frac{1}{2}(2x+2)+7}{x^2+2x+2}\,dx$$

$$\int \frac{f'(y)}{f(y)}\,dy = \ln f(y)$$

$$= \frac{1}{25}\ln|x-1| - \frac{1}{5}\,\frac{1}{x-1}$$

$$-\frac{1}{50}\ln(x^2+2x+2)-\frac{7}{25}\int\frac{dx}{(x+1)^2+1}$$

$$=\frac{1}{25}\ln|x-1|-\frac{1}{5}\,\frac{1}{x-1}$$

$$-\frac{1}{50}\ln(x^2+2x+2)-\frac{7}{25}\operatorname{arctg}(x+1)+C$$

(ii) $$\int\frac{30x-77}{x^2-6x+13}\,dx=\int\frac{15(2x-6)+13}{x^2-6x+13}\,dx$$

$$D=36-4\cdot 13<0$$

$$=15\ln(x^2-6x+13)+13\int\frac{dx}{x^2-6x+13}$$

$$=15\ln(x^2-6x+13)+$$

$$+13\int\frac{dx}{(x-3)^2+4}=$$

$$=\;\dots\;+13\int\frac{\frac{1}{4}\,dx}{\left(\frac{x-3}{2}\right)^2+1}=$$

$$\left|\begin{array}{l} y=\frac{x-3}{2} \\ dy=\frac{1}{2}dx \end{array}\right|$$

$$= \dots + \frac{13}{4}\int \frac{2\,dy}{y^2+1} =$$

$$= \dots + \frac{13}{2}\,\mathrm{arctg}\, y + C$$

$$= 15\ln(x^2-6x+13) + \frac{13}{2}\,\mathrm{arctg}\,\frac{x-3}{2} + C$$

(iii) $$\int \frac{\overbrace{2x+1}^{=(2x+4)-3}}{(x^2+4x+13)^2}\,dx = \left| \begin{array}{l} y = x^2+4x+13 \\ dy = (2x+4)\,dx \end{array} \right|$$

$$= \int \frac{dy}{y^2} - 3\int \frac{1}{\left((x+2)^2+3^2\right)^2}\,dx$$

$$= -\frac{1}{y} - 3K_2(-2,3) + C,$$

kde $$\boxed{K_m(x_0,a) = \int \frac{dx}{\left((x-x_0)^2+a^2\right)^m}}$$

$$K_n(x_0,a) =$$

$$u = \frac{1}{\left((x-x_0)^2+a^2\right)^n} \qquad u' = \frac{-2n(x-x_0)}{\left((x-x_0)^2+a^2\right)^{n+1}}$$

$$v' = 1 \qquad v = x - x_0$$

$$= \frac{x-x_0}{\left((x-x_0)^2+a^2\right)^n} + 2n\int \frac{\left((x-x_0)^2+a^2\right)-a^2}{\left((x-x_0)^2+a^2\right)^{n+1}}\,dx$$

$$= \frac{x-x_0}{\left((x-x_0)^2+a^2\right)^n} + 2n\,K_n(x_0,a) - 2a^2 n\,K_{n+1}(x_0,a)$$

$$K_{n+1}(x_0,a) = \frac{2n-1}{2a^2 n}\,K_n(x_0,a) + \frac{1}{2a^2 n}\cdot\frac{x-x_0}{\left((x-x_0)^2+a^2\right)^n} + C$$

$$K_1(x_0,a) = \frac{1}{a}\operatorname{arctg}\frac{x-x_0}{a} + C$$

8.2 (i) $\int \frac{x}{1+x^4}\,dx = \left| \begin{matrix} y = x^2 \\ dy = 2x\,dx \end{matrix} \right|$

$= \int \frac{\frac{1}{2}dy}{1+y^2} = \frac{1}{2}\operatorname{arctg} y + C$

$= \frac{1}{2}\operatorname{arctg} x^2 + C$

(ii) $\int \frac{5\ln x}{x\ln^3 x + x\ln^2 x - 2x}\,dx$

$= \left| \begin{matrix} y = \ln x \\ dy = \frac{1}{x}dx \end{matrix} \right|$

$y^3 + y^2 - 2 = (y-1)(y^2+2y+2)$

	1	1	0	-2
1	1	2	2	0

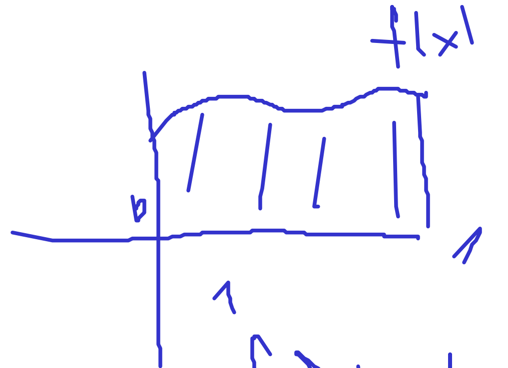
$= \int \frac{5y}{y^3+y^2-2}\,dy =$

$\frac{5y}{y^3+y^2-2} = \frac{A}{y-1} + \frac{Bx+C}{y^2+2y+2}$

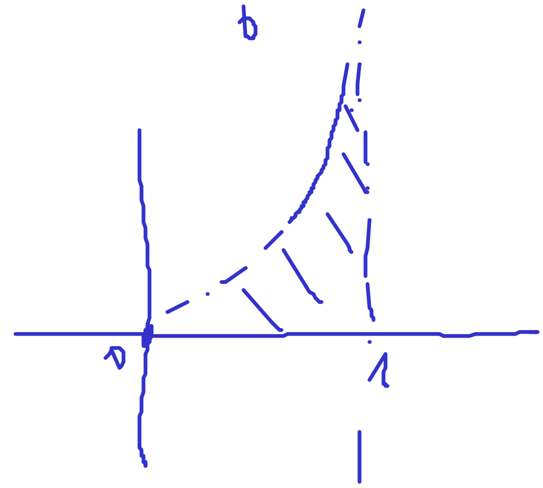
. - . . .

$$= \frac{1}{y-1} + \frac{-y+2}{y^2+2y+2}$$

$= \quad . \quad . \quad . \quad . \quad . \quad .$



$$\int_0^1 f(x)\,dx$$



8.3 $$\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \left| \begin{array}{l} y = 1 - x^2 \\ dy = -2x\,dx \end{array} \right|$$

$$= \int_1^0 -\frac{1}{2\sqrt{y}}\,dy$$

$$= +\frac{1}{2}\int_0^1 \frac{1}{\sqrt{y}}\,dy$$

$$= +\frac{1}{2}\int_0^1 y^{-1/2}\,dy = +\frac{1}{2}\left[\frac{y^{1/2}}{1/2}\right]_0^1 =$$

$$= +\frac{1}{2}\left[2\sqrt{y}\right]_0^1 = +(1-0) = \underline{\underline{1}}$$

(iii) $\int_0^1 \left(\frac{e^x}{e^{2x}+3} + \frac{1}{\cos^2 x}\right) dx =$

$= \left| \begin{matrix} y = e^x \\ dy = e^x dx \end{matrix} \right|$

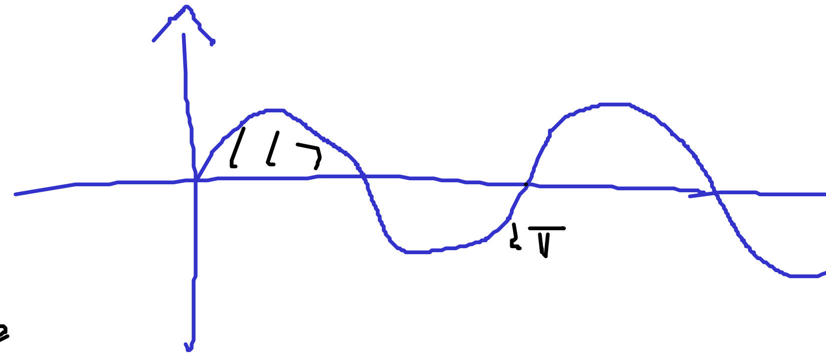
$= \int_1^e \frac{dy}{y^2+3} + \left[\operatorname{tg} x\right]_0^1$

$= \int_1^e \frac{dy}{3\left(\left(\frac{y}{\sqrt{3}}\right)^2+1\right)} + \left(\operatorname{tg} 1 - \operatorname{tg} 0\right)$

$= \frac{1}{3}\left[\sqrt{3}\operatorname{arctg}\left(\frac{y}{\sqrt{3}}\right)\right]_1^e + \operatorname{tg} 1$

$= \frac{\sqrt{3}}{3}\operatorname{arctg}\frac{e}{\sqrt{3}} - \frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right) + \operatorname{tg} 1$

8.4 (i) $\int_1^\infty \sin x\,dx =$

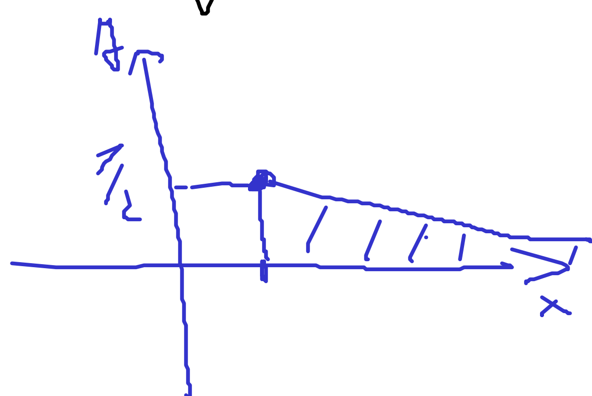


$$= \left[-\cos x\right]_1^\infty =$$

$$= -\left[\underbrace{\lim_{a\to\infty} \cos a}_{\text{neex.}} - \cos 1\right]$$

$\Rightarrow$ integral neex. div

(ii) $\int_1^\infty \frac{1}{x^2+x^4}\,dx$



$$\frac{1}{x^2+x^4} = \frac{1}{x^2(1+x^2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{1+x^2}$$

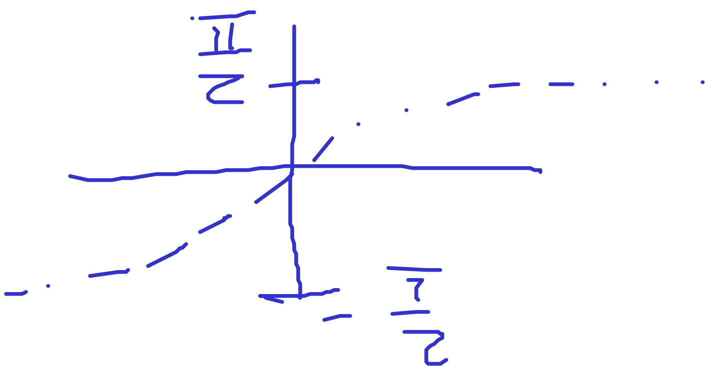
$$= \frac{1}{x^2} - \frac{1}{1+x^2}$$

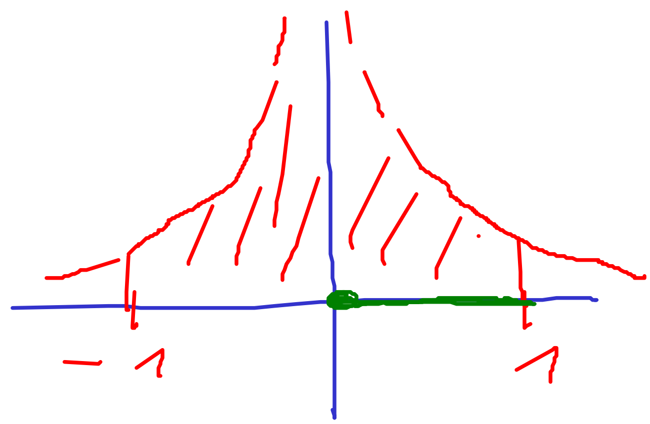
$$= \int_1^\infty \frac{1}{x^2}\,dx - \int_1^\infty \frac{1}{1+x^2}\,dx =$$

$$= \left[-\frac{1}{x}\right]_1^\infty - \left[\operatorname{arctg} x\right]_1^\infty =$$

$$= \left[-0+\frac{1}{1}\right] - \left[\frac{\pi}{2} - \operatorname{arctg} 1\right]$$

$$= 1 - \frac{\pi}{2} + \operatorname{arctg} 1$$





(iii) $$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

$$= 2\int_0^1 \frac{1}{x^2}\,dx =$$

$$= 2\left[-\frac{1}{x}\right]_0^1 = 2\left[-1+\infty\right]$$

$$= \infty$$