9.1 křivka daná parametricky $[x(t), y(t)]$, kde $x(t) = t^2$
$y(t) = t^3$

$t \in [0, \sqrt{5}]$

Pozn: $y(t) = (\sqrt{x})^3 = x^{3/2}$

$$\text{délka} = \int_0^{\sqrt{5}} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt =$$

$$= \int_0^{\sqrt{5}} \sqrt{(2t)^2 + (3t^2)^2}\, dt = \int_0^{\sqrt{5}} \sqrt{4t^2 + 9t^4}\, dt$$

$$= \int_0^{\sqrt{5}} t\sqrt{4 + 9t^2}\, dt = \left| \begin{array}{l} 4 + 9t^2 = u \\ 18t\, dt = du \end{array} \right|$$

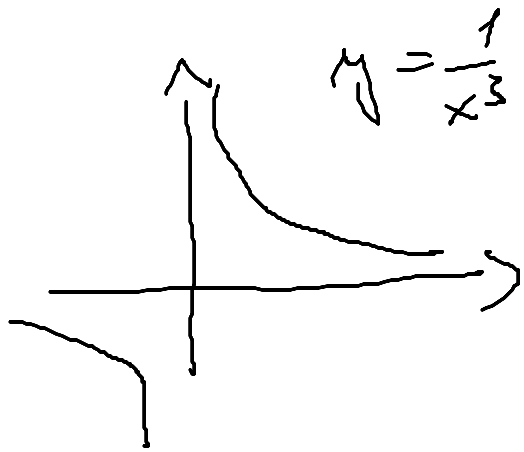
$$\left| \begin{array}{l} 4 + 9t^2 = u^2 \\ 18t\, dt = 2u\, du \end{array} \right|$$

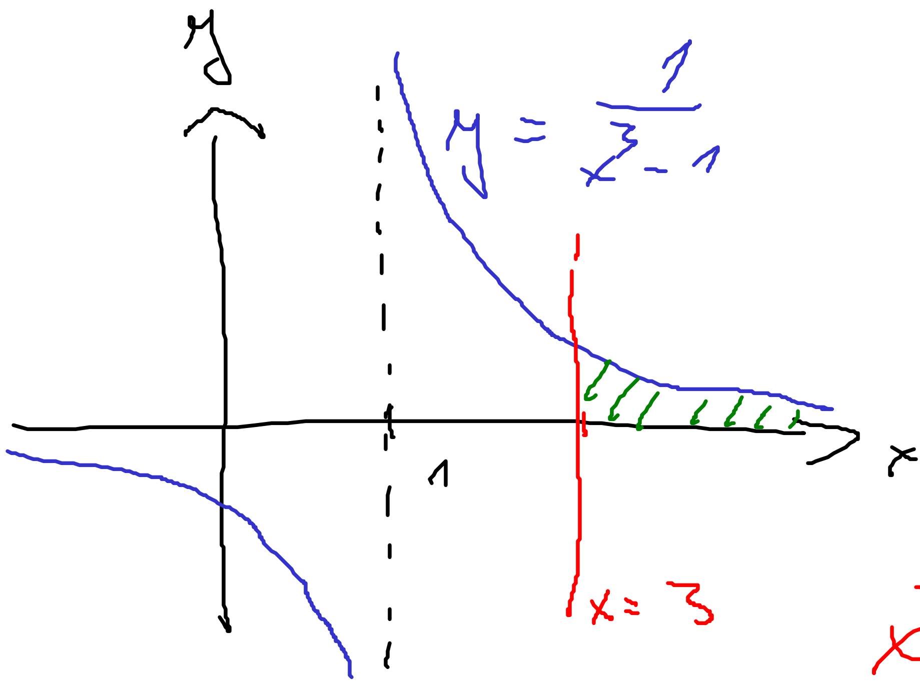
$$= \int_4^{49} \frac{1}{18}\sqrt{u}\;du =$$

$$= \frac{1}{18}\int_4^{49} u^{1/2}\,du = \frac{1}{18}\left[\frac{2}{3}u^{3/2}\right]_4^{49}$$

$$= \frac{1}{18}\cdot\frac{2}{3}\left[7^3 - 2^3\right] = \dots$$

9.2 (i)





$$x^3 - 1 = (x-1)(x^2+x+1)$$

$$\int_3^\infty \frac{1}{x^3-1}\,dx = \qquad \frac{1}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$\cdot (x-1)(x^2+x+1)$$

$$1 = A(x^2+x+1) + (Bx+C)(x-1)$$

$$1 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$$

$$1 = (A+B)x^2 + (A-B+C)x + (A-C)$$

$$\Rightarrow \begin{array}{l|l} A+B = 0 & A = -B \\ A-B+C = 0 & A = C+1 \\ A-C = 1 & C = A-1 = -B-1 \end{array}$$

$$-B - B + (-B-1) = 0 \Rightarrow \boxed{B = -\frac{1}{3}}$$

$$\boxed{A = \frac{1}{3}} \qquad \boxed{C = \frac{2}{3}}$$

$$= \int_3^\infty \left(\frac{1}{3(x-1)} + \frac{-x+2}{3(x^2+x+1)} \right) dx$$

$$= \frac{1}{3} \int_3^\infty \frac{1}{x-1}\,dx + \frac{1}{3} \int_3^\infty \frac{-x+2}{x^2+x+1}\,dx$$

$$= \frac{1}{3} \left[\ln(x-1) \right]_3^\infty$$

$$+\frac{1}{3}\int_3^\infty \frac{\frac{1}{2}(2x+1)+\frac{5}{2}}{x^2+x+1}\,dx$$

$$=\frac{1}{3}\Big[\ln(x-1)\Big]_3^\infty-\frac{1}{6}\int_3^\infty \frac{2x+1}{x^2+x+1}\,dx$$

$$+\frac{5}{6}\int_3^\infty \frac{1}{x^2+x+1}\,dx=$$

$$=\frac{1}{3}\Big[\ln(x-1)\Big]_3^\infty-\frac{1}{3}\cdot\frac{1}{2}\Big[\ln(x^2+x+1)\Big]_3^\infty$$

$$+\frac{5}{6}\int_3^\infty \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx= \left|\begin{array}{l} t=x+\frac{1}{2}\\ dt=dx\end{array}\right|$$

$$=\frac{1}{3}\left(\Big[\ln(x-1)\Big]_3^\infty-\Big[\ln\sqrt{x^2+x+1}\Big]_3^\infty\right)$$

$$+\frac{5}{6}\int_{7/2}^\infty \frac{1}{t^2+\frac{3}{4}}\,dt$$

$$=\frac{1}{3}\left[\ln\frac{x-1}{\sqrt{x^2+x+1}}\right]_3^\infty$$

$$+\frac{5}{6}\int_{7/2}^{\infty}\frac{1}{\frac{3}{4}\left(\frac{4}{3}t^{2}+1\right)}\,dt =$$

$$\lim_{x\to\infty}\ln\frac{x-1\;/\cdot\frac{1}{x}}{\sqrt{x^{2}+x+1}\;/\frac{1}{x}} =$$

$$= \lim_{x\to\infty}\ln\frac{1-\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^{2}}}}$$

$$= \ln 1 = 0$$

$$= \frac{1}{3}\left[0-\ln\frac{2}{\sqrt{13}}\right]$$

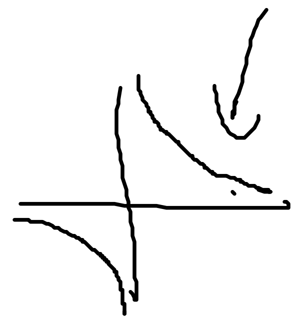
$$+\frac{5}{6}\cdot\frac{4}{3}\int_{7/2}^{\infty}\frac{1}{\left(\frac{2t}{\sqrt{3}}\right)^{2}+1}\,dt$$

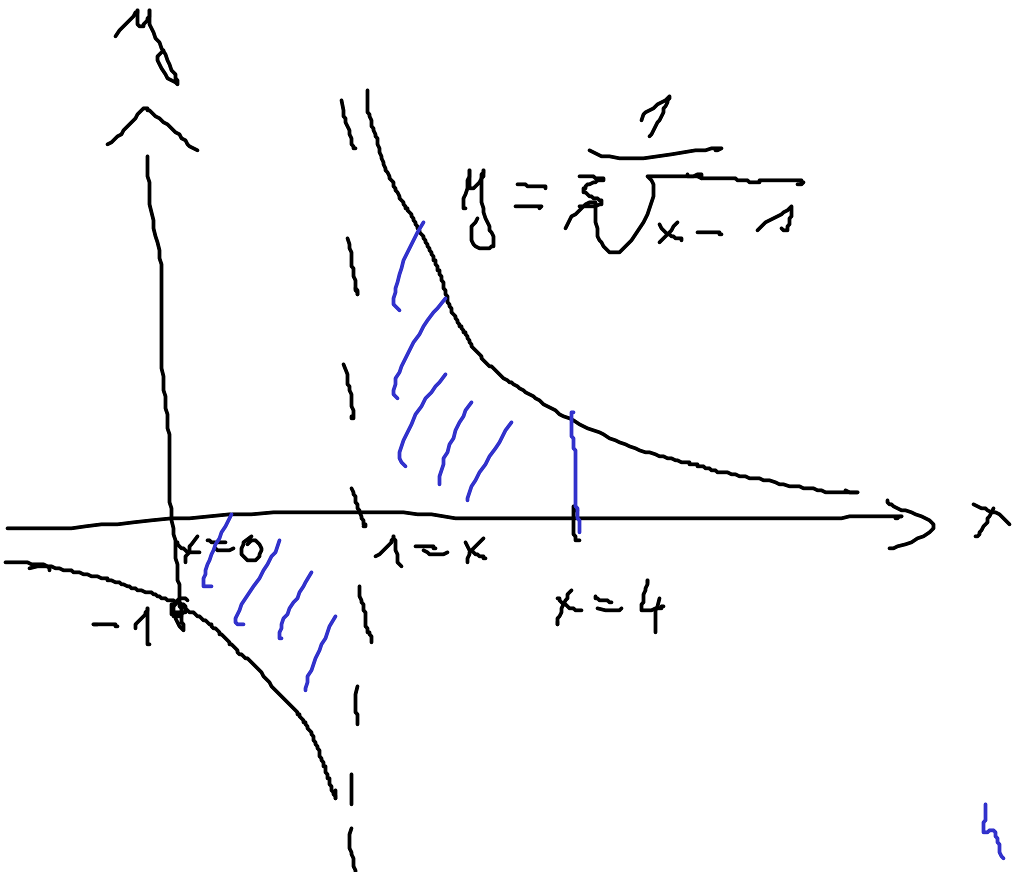
$$= -\frac{1}{3}\ln\frac{2}{\sqrt{13}}+\frac{10}{9}\left[\frac{\sqrt{3}}{2}\operatorname{arctg}\left(\frac{2t}{\sqrt{3}}\right)\right]_{7/2}^{\infty}$$

$$= \ldots$$

9.2.(ii) $y = \frac{1}{\sqrt[3]{x-1}}$

$y = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$





$$S = -\int_0^1 \frac{1}{\sqrt[3]{x-1}}\,dx + \int_1^4 \frac{1}{\sqrt[3]{x-1}}\,dx$$

$$= -\int_0^1 (x-1)^{-1/3}\,dx + \int_1^4 (x-1)^{-1/3}\,dx$$

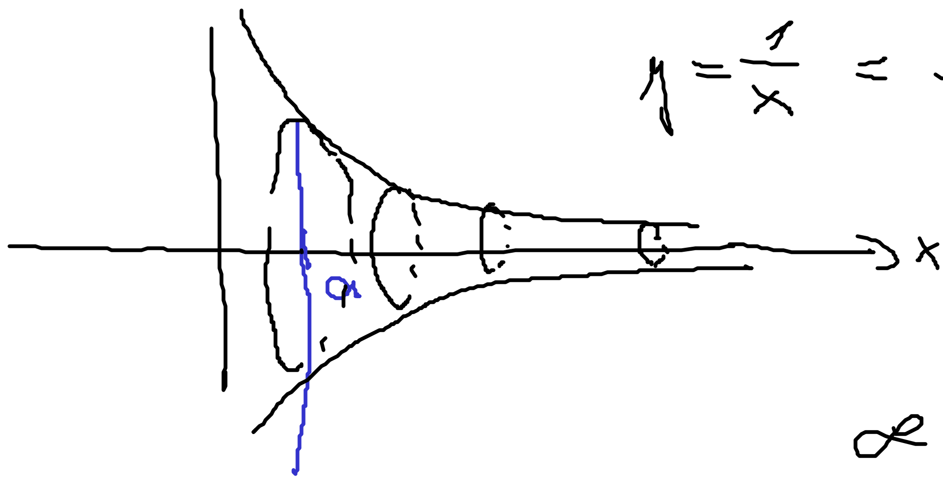
$$= -\left[\frac{3}{2}\cdot(x-1)^{2/3}\right]_0^1 + \left[\frac{3}{2}(x-1)^{2/3}\right]_1^4$$

$$= -\frac{3}{2}\left(0-(-1)^2\right)+\frac{3}{2}\left(\sqrt[3]{9}-0\right)$$

$$= \frac{3}{2}+\frac{3}{2}\cdot\sqrt[3]{9}$$

9.3

$y=\frac{1}{x}=f(x)$



$x \geq a$

$$V=\pi\int_a^\infty (f(x))^2\,dx = \pi\int_a^\infty \frac{1}{x^2}\,dx$$

$$=\pi\int_a^\infty x^{-2}\,dx = \pi\left[-x^{-1}\right]_a^\infty$$

$$=\pi\left[-0+\frac{1}{a}\right]=\frac{\pi}{a}$$ konečný objem

$$P = 2\pi \int_a^{\infty} f(x) \sqrt{1 + (f'(x))^2}\, dx$$

$$= 2\pi \int_a^{\infty} \frac{1}{x} \sqrt{1 + \left(-\frac{1}{x^2}\right)^2}\, dx$$

$$= 2\pi \int_a^{\infty} \frac{1}{x} \sqrt{1 + \frac{1}{x^4}}\, dx$$

$$= 2\pi \int_a^{\infty} \frac{1}{x} \cdot \frac{1}{x^2} \cdot \sqrt{x^4 + 1}\, dx$$

$$\geq 2\pi \int_a^{\infty} \frac{1}{x}\, dx =$$

$$= 2\pi \left[\ln x \right]_a^{\infty} = \infty$$

9.4 (i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ – podílov

odmocnin

⋮

X

$$\int_2^{\infty} \frac{1}{x \ln x}\, dx = \left| \begin{array}{l} y = \ln x \\ dy = \frac{1}{x} dx \end{array} \right|$$

$$= \int_{\ln 2}^{\infty} \frac{1}{y}\,dy = [\ln y]_{\ln 2}^{\infty} = \infty$$

$\Rightarrow$ řada diverguje

$\sum \frac{1}{n}$ div

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \int_1^{\infty} x^{-2}\,dx = \left[-x^{-1}\right]_1^{\infty}$$

$$= [-0+1] = 1$$

konverguje

(ii) $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$ konverguje? od mocninové krit.

Nápověda: použijte

$$\int_2^\infty \frac{dx}{x^{n+1}} = \frac{1}{n \cdot 2^n}$$

Rada $\sum_{n=1}^{\infty} \frac{1}{x^{n+1}}$ je stejně měrně konvergentní

Weierstrassovo kritérium

$[2, \infty]$ $\frac{1}{x^{n+1}} \leq \frac{1}{2^n}$

$\sum \frac{1}{2^n}$ konverg.

$\sum_{n=2}^{\infty} f_n(x)$ stejn. konverg.

pa $\int \sum_{n=2}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \int f(x)$

$$f(x) := \sum_{n=1}^{\infty} \frac{1}{x^{n+1}} = \frac{1}{x^2} \cdot \frac{1}{1 - \frac{1}{x}}$$

$$= \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

stejnoměrné konvergují?

$$\Rightarrow \sum_{n=1}^{\infty} \int_2^{\infty} \frac{dx}{x^{n+1}} = \int_2^{\infty} f(x) = \int_2^{\infty} \frac{dx}{x(x-1)}$$

na lib intervalu, kde řada stejnoměrně konverguje

interval := $[2,\infty)$

$$\sum_{n=1}^{\infty} \int_2^{\infty} x^{-(n+1)}\,dx = \sum_{n=1}^{\infty} \left[-\frac{1}{n}\cdot x^{-n}\right]_2^{\infty} =$$

$$= \sum_{n=1}^{\infty} \left(-0 + \frac{1}{n} 2^{-n}\right) = \sum_{n=1}^{\infty} \frac{1}{n\cdot 2^n}$$

$\parallel$

Prověstrama: $\int_2^{\infty} \frac{dx}{x(x-1)} = \dots = \ln 2$