

Interpolacni polynom 5ti a 9ti bodu

```
> bla;
                                bla (1)
```

```
> p1 := add(cat(a, n)·x^n, n = 0..4); p2 := add(cat(a, n)·x^n, n = 0..8);
    p1 := a4 x^4 + a3 x^3 + a2 x^2 + a1 x + a0
    p2 := a8 x^8 + a7 x^7 + a6 x^6 + a5 x^5 + a4 x^4 + a3 x^3 + a2 x^2 + a1 x + a0 (2)
```

Body [x,y] pro interpolaci, prvnic 5 bodu je pouzito pro prvni polynom.

```
> Points := [[0, 0], seq([k, 1], k = 1..4), seq([kK 1/2, 1], k = 1..4)];
    Points := [[0, 0], [1, 1], [2, 1], [3, 1], [4, 1], [1/2, 1], [3/2, 1], [5/2, 1], [7/2, 1]] (3)
```

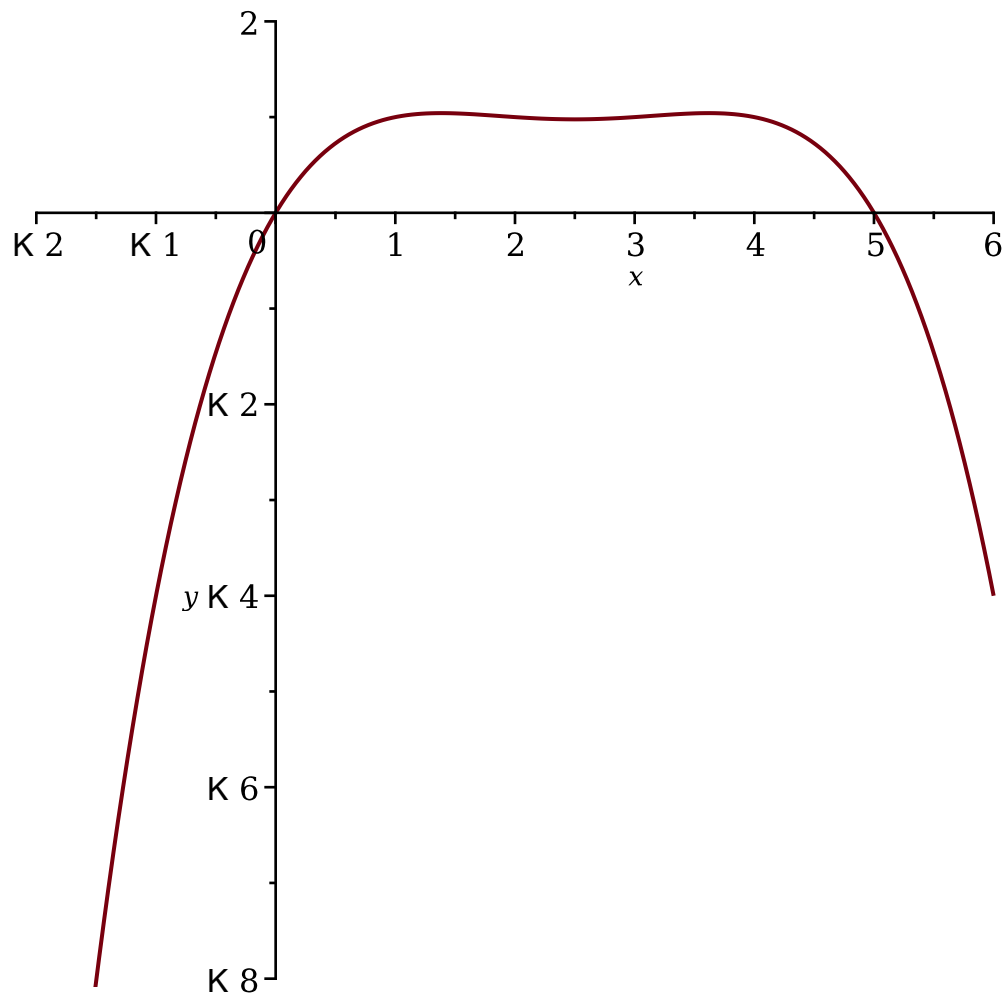
```
> system1 := [seq(eval(p1, x = Points[k, 1]) = Points[k, 2], k = 1..5)];
system1 := [a0 = 0, a0 + a1 + a2 + a3 + a4 = 1, a0 + 2 a1 + 4 a2 + 8 a3 + 16 a4 = 1, a0 + 3 a1 + 9 a2 + 27 a3 + 81 a4 = 1, a0 + 4 a1 + 16 a2 + 64 a3 + 256 a4 = 1] (4)
```

```
> seq(print(%[k]), k = 1..numelems(%));
    a0 = 0
    a0 + a1 + a2 + a3 + a4 = 1
    a0 + 2 a1 + 4 a2 + 8 a3 + 16 a4 = 1
    a0 + 3 a1 + 9 a2 + 27 a3 + 81 a4 = 1
    a0 + 4 a1 + 16 a2 + 64 a3 + 256 a4 = 1 (5)
```

```
> solve(system1, [seq(cat(a, n), n = 0..4)]);
    [[a0 = 0, a1 = 25/12, a2 = K 35/24, a3 = 5/12, a4 = K 1/24]] (6)
```

```
> p1_fin := eval(p1, %[1]);
    p1_fin := K 1/24 x^4 + 5/12 x^3 K 35/24 x^2 + 25/12 x (7)
```

```
> plot(p1_fin, x = K 2..6, y = K 8..2);
```



```
> system2 := [seq(eval(p2, x = Points[k, 1]) = Points[k, 2], k = 1
..numelems(Points))]:
```

```
> seq(print( %[k] ), k = 1 ..numelems(%));
a0 = 0
```

$$a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 = 1$$

$$a_0 + 2a_1 + 4a_2 + 8a_3 + 16a_4 + 32a_5 + 64a_6 + 128a_7 + 256a_8 = 1$$

$$a_0 + 3a_1 + 9a_2 + 27a_3 + 81a_4 + 243a_5 + 729a_6 + 2187a_7 + 6561a_8 = 1$$

$$a_0 + 4a_1 + 16a_2 + 64a_3 + 256a_4 + 1024a_5 + 4096a_6 + 16384a_7 + 65536a_8 = 1$$

$$a_0 + \frac{a_1}{2} + \frac{a_2}{4} + \frac{a_3}{8} + \frac{a_4}{16} + \frac{a_5}{32} + \frac{a_6}{64} + \frac{a_7}{128} + \frac{a_8}{256} = 1$$

$$a_0 + \frac{3a_1}{2} + \frac{9a_2}{4} + \frac{27a_3}{8} + \frac{81a_4}{16} + \frac{243a_5}{32} + \frac{729a_6}{64} + \frac{2187a_7}{128} + \frac{6561a_8}{256} = 1$$

$$a_0 + \frac{5a_1}{2} + \frac{25a_2}{4} + \frac{125a_3}{8} + \frac{625a_4}{16} + \frac{3125a_5}{32} + \frac{15625a_6}{64} + \frac{78125a_7}{128}$$

$$+ \frac{390625 a_8}{256} = 1$$

$$a_0 + \frac{7 a_1}{2} + \frac{49 a_2}{4} + \frac{343 a_3}{8} + \frac{2401 a_4}{16} + \frac{16807 a_5}{32} + \frac{117649 a_6}{64} + \frac{823543 a_7}{128} + \frac{5764801 a_8}{256} = 1 \quad (8)$$

> solve(system2, [seq(cat(a, n), n = 0..8)]);

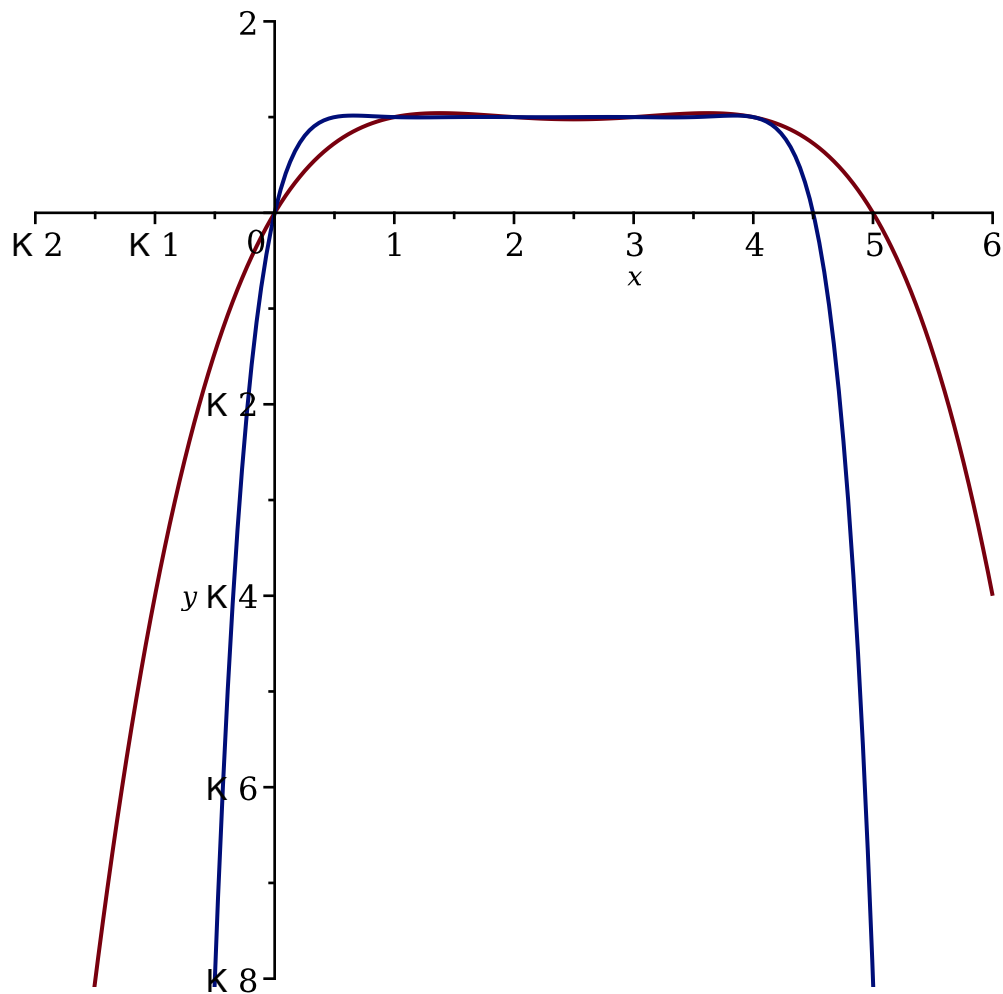
$$\left[\left[a_0 = 0, a_1 = \frac{761}{140}, a_2 = K \frac{29531}{2520}, a_3 = \frac{267}{20}, a_4 = K \frac{1069}{120}, a_5 = \frac{18}{5}, a_6 = K \frac{13}{15}, a_7 = \frac{4}{35}, a_8 = K \frac{2}{315} \right] \right] \quad (9)$$

> p2_fin := eval(p2, %[1]);

$$p2_fin := K \frac{2}{315} x^8 + \frac{4}{35} x^7 K \frac{13}{15} x^6 + \frac{18}{5} x^5 K \frac{1069}{120} x^4 + \frac{267}{20} x^3 + \frac{29531}{2520} x^2 + \frac{761}{140} x \quad (10)$$

Polynomy prochazejici pocatkem a majici hodnotu 1 pro x=1, 2, 3, 4, modry graf navic ma hodnotu 1 v x=1/2, 3/2, 5/2, 7/2

> plot([p1_fin, p2_fin], x=K 2..6, y=K 8..2);



Prirozeny splaj

Interpoluji se vzdy jen dva nasledujici body polynomem tretiho stupne, interpolacni polynomy maji na spojich prvni a druhou derivaci stejnou, na krajich je **druha** derivace nulova.

$$\begin{aligned} > \text{Points} := [[K\ 1, 0], [0, 1], [1, 0]] \\ & \qquad \qquad \qquad \text{Points} := [[K\ 1, 0], [0, 1], [1, 0]] \end{aligned} \quad (11)$$

$$\begin{aligned} > p1 := \text{add}(\text{cat}(a, n) \cdot x^n, n = 0..3); \\ & \qquad \qquad \qquad p1 := a3 x^3 + a2 x^2 + a1 x + a0 \end{aligned} \quad (12)$$

$$\begin{aligned} > p2 := \text{add}(\text{cat}(b, n) \cdot x^n, n = 0..3); \\ & \qquad \qquad \qquad p2 := b3 x^3 + b2 x^2 + b1 x + b0 \end{aligned} \quad (13)$$

$$\begin{aligned} > \text{System_values} := [\text{eval}(p1, x = \text{Points}[1, 1]) = \text{Points}[1, 2], \text{eval}(p1, x \\ = \text{Points}[2, 1]) = \text{Points}[2, 2], \text{eval}(p2, x = \text{Points}[2, 1]) = \text{Points}[2, 2], \\ \text{eval}(p2, x = \text{Points}[3, 1]) = \text{Points}[3, 2]]; \\ \text{System_values} := [a0K\ a1 + a2K\ a3 = 0, a0 = 1, b0 = 1, b0 + b1 + b2 + b3 \\ = 0] \end{aligned} \quad (14)$$

$$> \text{System_der} := \text{eval}([\text{diff}(p1, x) = \text{diff}(p2, x), \text{diff}(p1, x, x) = \text{diff}(p2, x, x)], x$$

$$= 0); \quad \text{System_der} := [a1 = b1, 2 a2 = 2 b2] \quad (15)$$

$$\text{> System_ends} := [\text{eval}(\text{diff}(p1, x, x), x = K 1) = 0, \text{eval}(\text{diff}(p2, x, x), x = 1) = 0];$$

$$\text{System_ends} := [2 a2 K 6 a3 = 0, 2 b2 + 6 b3 = 0] \quad (16)$$

$$\text{> solution} := \text{solve}([\text{seq}(\text{System_values}), \text{seq}(\text{System_der}), \text{seq}(\text{System_ends})], [\text{seq}(\text{cat}(a, n), n = 0..3), \text{seq}(\text{cat}(b, n), n = 0..3)]);$$

$$\text{solution} := \left[\left[a0 = 1, a1 = 0, a2 = K \frac{3}{2}, a3 = K \frac{1}{2}, b0 = 1, b1 = 0, b2 = K \frac{3}{2}, b3 = \frac{1}{2} \right] \right] \quad (17)$$

$$\text{> p1_fin} := \text{eval}(p1, \text{solution}[1]); \text{p2_fin} := \text{eval}(p2, \text{solution}[1]);$$

$$p1_fin := K \frac{1}{2} x^3 K \frac{3}{2} x^2 + 1$$

$$p2_fin := \frac{1}{2} x^3 K \frac{3}{2} x^2 + 1 \quad (18)$$

$$\text{> plot1} := \text{plot}(p1_fin, x = K 1..0) : \text{plot2} := \text{plot}(p2_fin, x = K 0..1) :$$

$$\text{> plots[display]}(\text{plot1}, \text{plot2}, \text{scaling} = \text{constrained});$$

