Bonusový příklad Hledáme interpolační polynom

tg(x) pro hodnoty $x_i = -\frac{\pi}{3}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{3}$

$\boxed{f(x) = a_3x^3 + a_1x}$

$\boxed{\begin{array}{l} tg\left(\frac{\pi}{4}\right) = 1 \\ tg\left(\frac{\pi}{3}\right) = \sqrt{3} \end{array}}$

$x = \frac{\pi}{4} \to a_3\left(\frac{\pi}{4}\right)^3 + a_1\left(\frac{\pi}{4}\right) = 1$

$x = \frac{\pi}{3} \to a_3\left(\frac{\pi}{3}\right)^3 + a_1\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\Downarrow$

$$\begin{array}{cc} a_3 & a_1 \end{array}$$

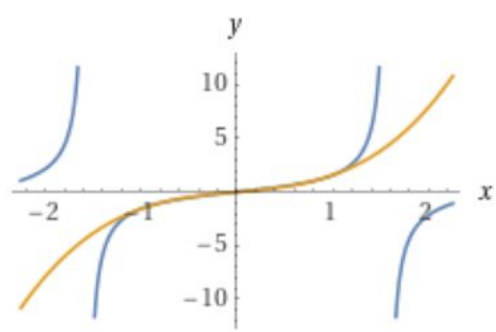
$$\left(\begin{array}{cc|c} \frac{\pi^3}{64} & \frac{\pi}{4} & 1 \\ \frac{\pi^3}{27} & \frac{\pi}{3} & \sqrt{3} \end{array}\right) \sim \left(\begin{array}{cc|c} \pi^3 & 16\pi & 64 \\ -\pi^3 & -9\pi & -27\sqrt{3} \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|c} \pi^3 & 16\pi & 64 \\ 0 & 7\pi & 64-27\sqrt{3} \end{array}\right) \sim \left(\begin{array}{cc|c} 7\pi^3 & 112\pi & 448 \\ 0 & -112\pi & -1024+432\sqrt{3} \end{array}\right) \sim$$

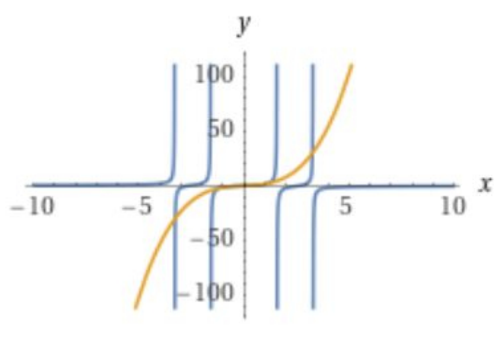
$$\sim \left(\begin{array}{cc|c} 7\pi^3 & 0 & -576+432\sqrt{3} \\ 0 & 7\pi & 64-27\sqrt{3} \end{array}\right) \Rightarrow \begin{array}{l} a_3 = \frac{432\sqrt{3}-576}{7\pi^3} \\ a_1 = \frac{64-27\sqrt{3}}{7\pi} \end{array}$$

$$f(x) = \frac{432\sqrt{3}-576}{7\pi^3}x^3 + \frac{64-27\sqrt{3}}{7\pi}x$$

Porovnání aproximací



$$\frac{2\left(4\sqrt{2}-1\right)\pi^3 x - 32\left(\sqrt{2}-1\right)\pi x^3}{\left(\pi^2-4x^2\right)\left(16\left(2\sqrt{2}-3\right)x^2+3\pi^2\right)}$$



Bonusový příklad Dokažte, že pro každé $c>0$ konverguje posloupnost a_n

$$a_{n+1}=\sqrt{c+a_n}\,;\quad a_1=\sqrt{c}$$

Posloupnost je rostoucí pokud platí $a_{n+1}>a_n\,;\ n\in\mathbb{N}$

$\Rightarrow \sqrt{c+a_n}>a_n\,;$ pro $a_1=\sqrt{c}\rightarrow \sqrt{c+\sqrt{c}}>\sqrt{c}\,;\ c>0$

indukční krok

$$\sqrt{c+a_{n+1}}>a_{n+1}$$

$$\sqrt{c+\sqrt{c+a_n}}>\sqrt{c+a_n}\ \Big|^2$$

$$\cancel{c}+\sqrt{c+a_n}>\cancel{c}+a_n\ \checkmark$$

$$\sqrt{c+\sqrt{c}}>\sqrt{c}\ \Big|^2$$

$$c+\sqrt{c}>c\ \Big|-c$$

$\sqrt{c}>0$ pro $c>0$ platí $\checkmark$

Posloupnost je shora ohraničená $a_n<d\,;\ n\in\mathbb{N}$

Jako d „náhodně volím“ $\dfrac{1+\sqrt{1+4c}}{2}\,;\ c>0.$

$$a_n<\frac{1+\sqrt{1+4c}}{2}$$

; pro $a_1=\sqrt{c}\rightarrow \sqrt{c}<\dfrac{1+\sqrt{1+4c}}{2}\ \Big|^2$

$$\cancel{c}<\frac{1+\sqrt{1+4c}}{2}+\cancel{c}$$

$$0<\frac{1+\sqrt{1+4c}}{2}$$

platí pro $c>0$ $\checkmark$

indukční krok

$$a_{n+1}<\frac{1+\sqrt{1+4c}}{2}$$

$$\sqrt{c+a_n}<\frac{1+\sqrt{1+4c}}{2}\ \Big|^2$$

$$c+a_n<\frac{(1+\sqrt{1+4c})(1+\sqrt{1+4c})}{4}=\frac{1+\sqrt{1+4c}+2c}{2}\ \Big|-c$$

$$a_n<\frac{1+\sqrt{1+4c}}{2}+\cancel{\frac{2c}{2}}-\cancel{c}\ \checkmark$$

Posloupnost konverguje pro každé $c>0$.