

Uvažme Taylorov rozvoj funkcie  $f$ :  $f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(a)(x-a)^n$ .

Upravme  $g$  na  $g(x) = \begin{cases} 1 & x \in (-\delta, 0) \\ -1 & x \in (0, \delta) \\ 0 & \text{inak} \end{cases}$  aby to vyšlo, inak vyjde rovnako.

$$\begin{aligned} (f * g)(a) &= \int_{-\infty}^{\infty} g(x) f(a-x) dx = \int_{-\delta}^0 g(x) f(a-x) dx + \int_0^{\delta} g(x) f(a-x) dx = \\ &= \int_{-\delta}^0 f(a) + f'(a) \cdot (-x) + f''(a) \cdot \frac{1}{2} (-x)^2 + \dots + o(x^3) dx + \int_0^{\delta} -f(a) - f'(a)(-x) - f''(a) \frac{1}{2} (-x)^2 - \dots - o(x^3) dx = \\ &= \left[ f(a)x - \frac{1}{2} f'(a)x^2 + f''(a) \frac{1}{6} x^3 + \dots \right]_{-\delta}^0 + \left[ -f(a)x + \frac{1}{2} f'(a)x^2 - \frac{1}{6} f''(a)x^3 + \dots \right]_0^{\delta} = \\ &= \cancel{\delta f(a)} + \frac{1}{2} f'(a) \cdot \delta^2 + \delta^3 \left( \frac{1}{6} f''(a) + \dots \right) - \cancel{\delta f(a)} + \frac{1}{2} f'(a) \cdot \delta^2 + \delta^3 \left( -\frac{1}{6} f''(a) + \dots \right) = \\ &= \frac{2}{2} \cdot f'(a) \cdot \delta^2 + \delta^3 (\dots) \end{aligned}$$

$$\lim_{\delta \rightarrow 0^+} \frac{1}{\delta^2} (f * g)(a) = \lim_{\delta \rightarrow 0^+} \left( \frac{\delta^2}{\delta^2} f'(a) + \delta(\dots) \right) = f'(a)$$