El-mag 2

Elektrické pole od nabité přímky.

Elektrické pole na ose nabitého kroužku.

Elektrické pole nabité sféry (výpočtem, fyz. úvahou).

Gaussova věta elektrostatiky.

intenzita elektrického pole v okolí nabité přímky

$$\vec{E} = \frac{\tau}{4\pi\varepsilon_0} \int_S$$

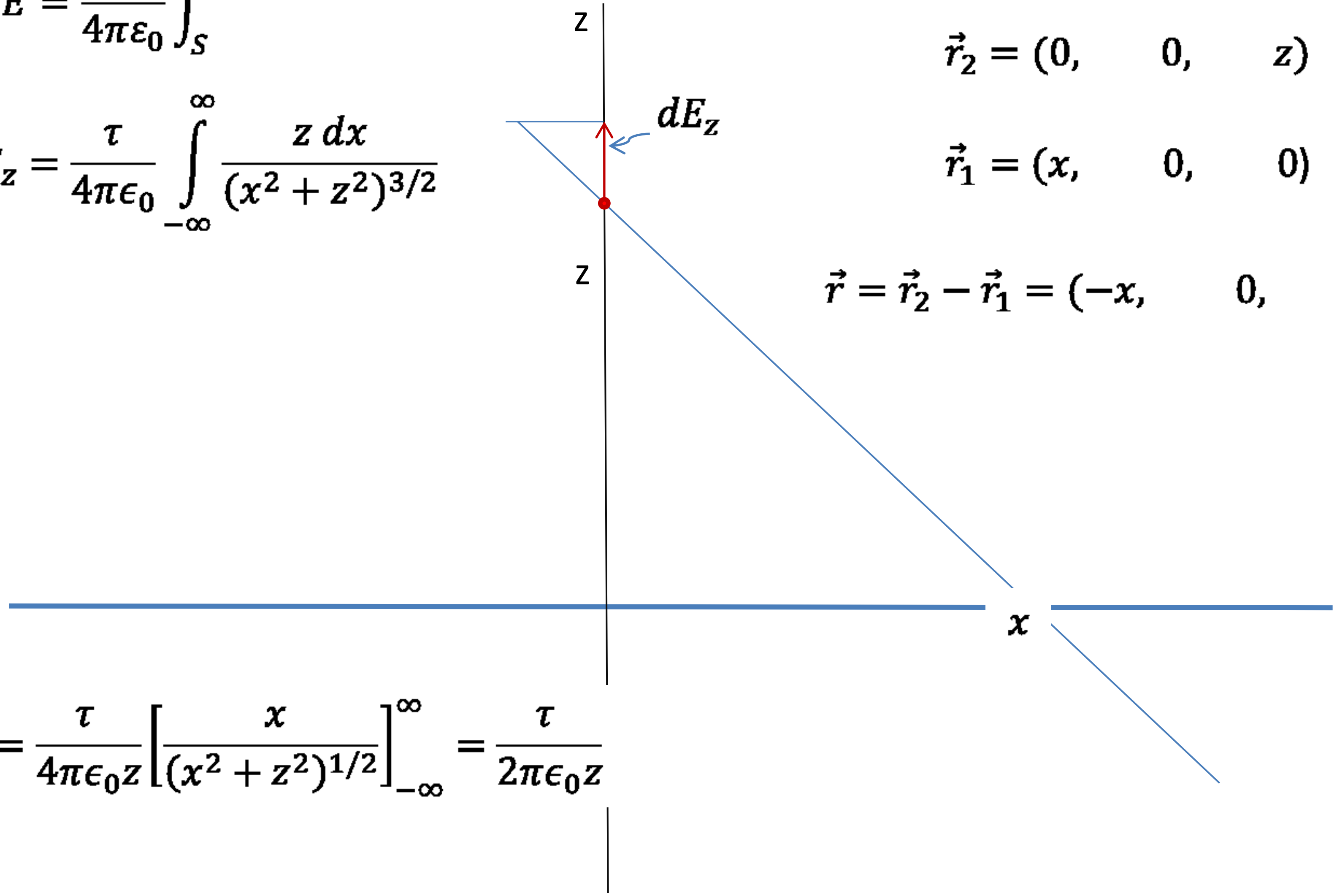
$$E_z = \frac{\tau}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{z\,dx}{(x^2+z^2)^{3/2}}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_2 = (0, \quad 0, \quad z)$$

$$\vec{r}_1 = (x, \quad 0, \quad 0)$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (-x, \quad 0, \quad z)$$



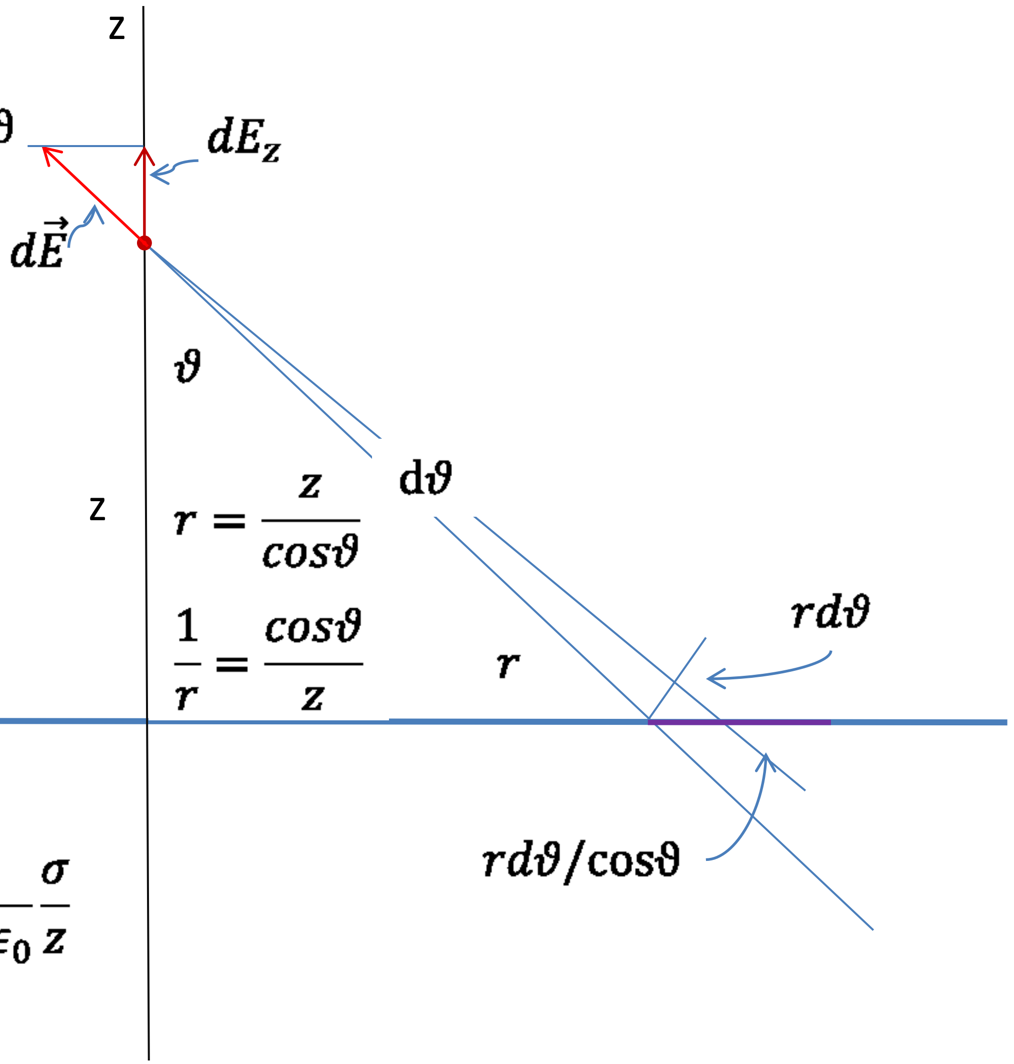
$$E_z = \frac{\tau}{4\pi\epsilon_0 z}\left[\frac{x}{(x^2+z^2)^{1/2}}\right]_{-\infty}^{\infty} = \frac{\tau}{2\pi\epsilon_0 z}$$

intenzita elektrického pole v okolí nabité přímky

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma r d\vartheta/\cos\vartheta}{r^2}\cos\vartheta$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma d\vartheta}{r}$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma d\vartheta}{z}\cos\vartheta$$



$$r = \frac{z}{\cos\vartheta}$$

$$\frac{1}{r} = \frac{\cos\vartheta}{z}$$

$$E_z = 2\int_0^{\pi/2}\frac{1}{4\pi\epsilon_0}\frac{\sigma}{z}\cos\vartheta d\vartheta = \frac{1}{2\pi\epsilon_0}\frac{\sigma}{z}$$

intenzita elektrického pole v okolí nabité roviny

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{\vec{r}}{|\vec{r}|^3}\, dS$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

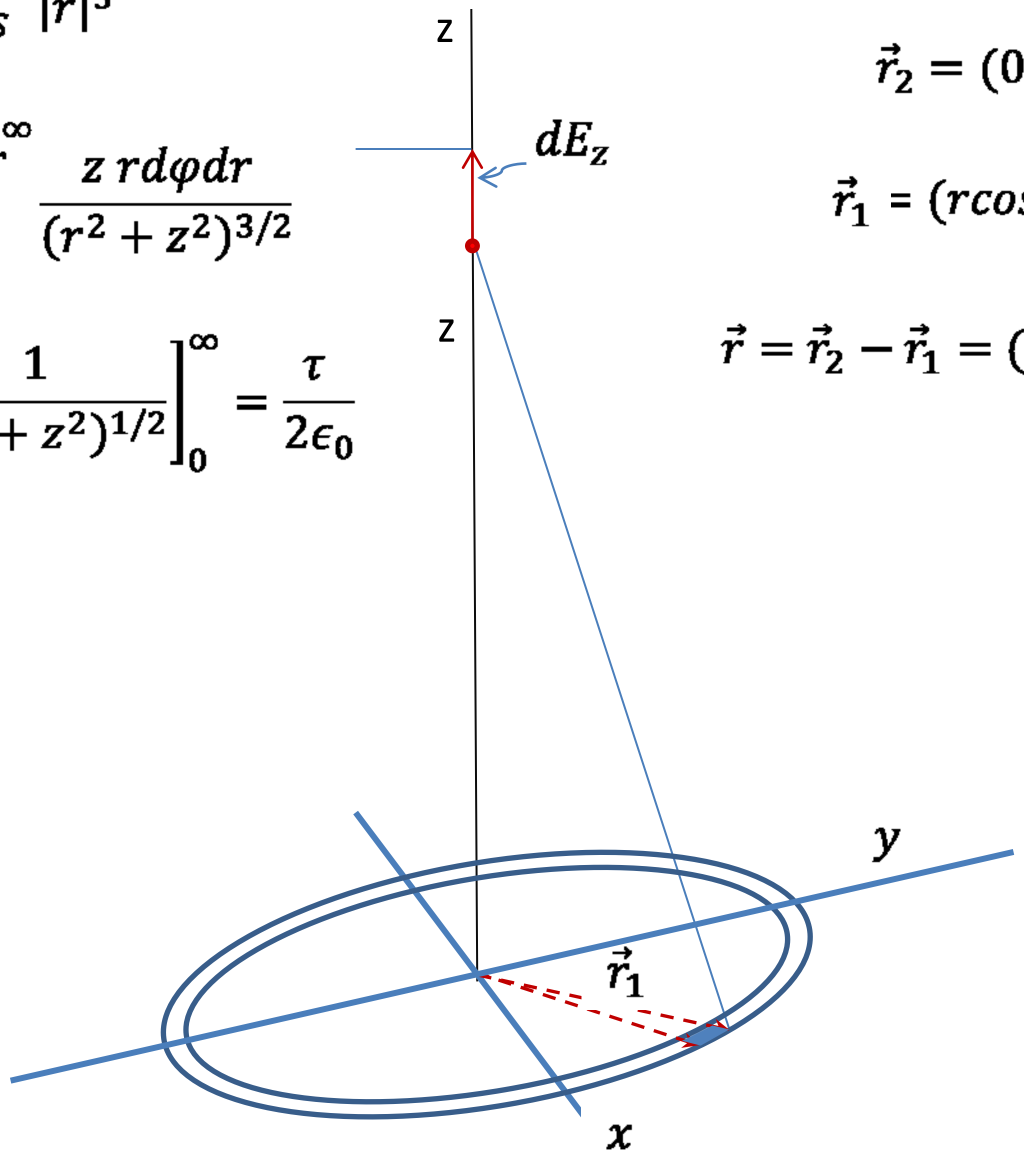
$$\vec{r}_2 = (0, \quad 0, \quad z)$$

$$E_z = \frac{\sigma}{4\pi\epsilon_0}\iint_{0,0}^{2\pi,\infty} \frac{z\, r d\varphi dr}{(r^2+z^2)^{3/2}}$$

$$\vec{r}_1 = (r cos\varphi, \ r sin\varphi, \quad 0)$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (-r cos\varphi\, , -r sin\varphi, \ z)$$

$$E_z = \frac{2\pi\ \sigma z}{4\pi\epsilon_0}\left[-\frac{1}{(r^2+z^2)^{1/2}}\right]_0^\infty = \frac{\tau}{2\epsilon_0}$$

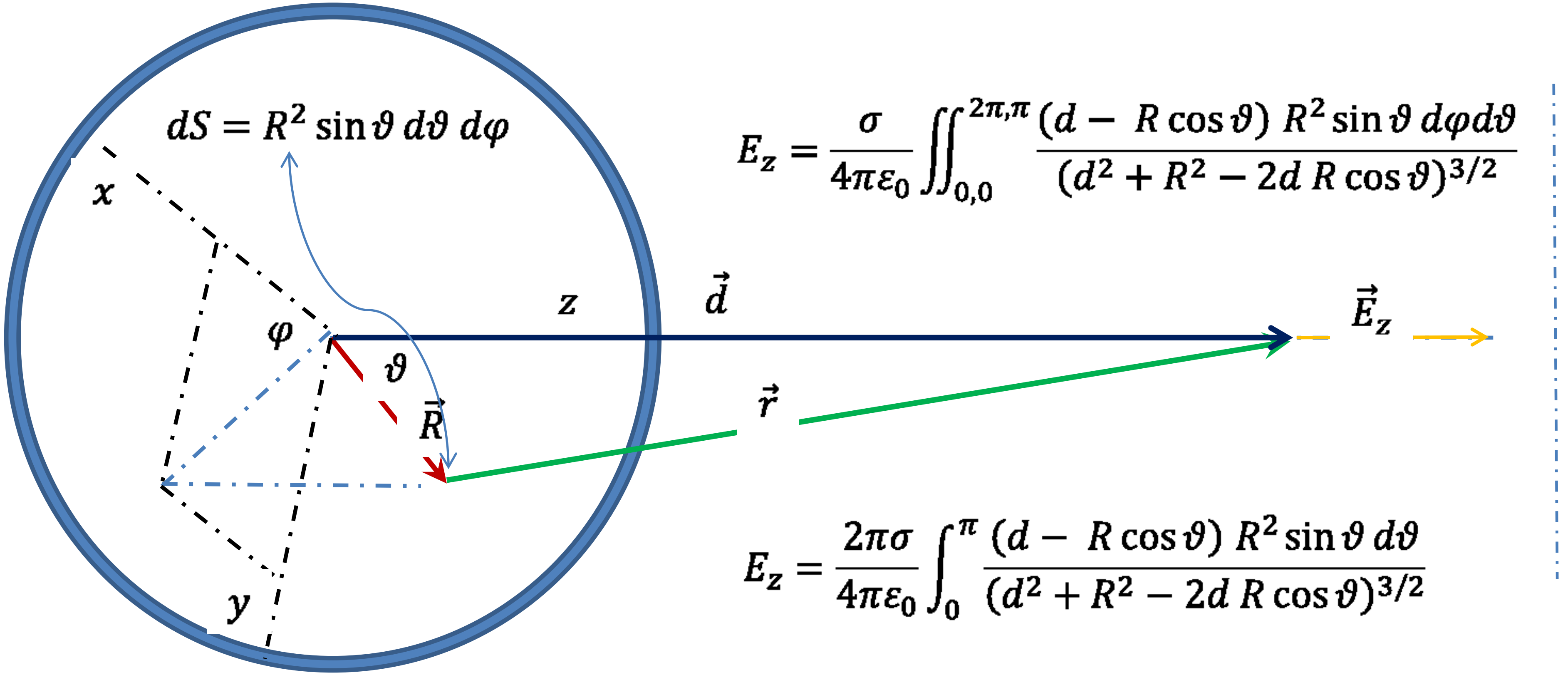


Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{\vec{r}}{|\vec{r}|^3}\, dS \qquad \vec{d} = (0, \quad 0, \quad d) \qquad \vec{R} = (\mathrm{R}\sin\vartheta\cos\varphi, \mathrm{R}\sin\vartheta\sin\varphi, \mathrm{R}\cos\vartheta)$$

$$\vec{r} = \vec{d} - \vec{R} \qquad r^2 = d^2 + R^2 - 2d\,R\cos\vartheta \qquad r^3 = \left|\vec{d} - \vec{R}\right|^3 = (d^2 + R^2 - 2d\,R\cos\vartheta\ )^{3/2}$$

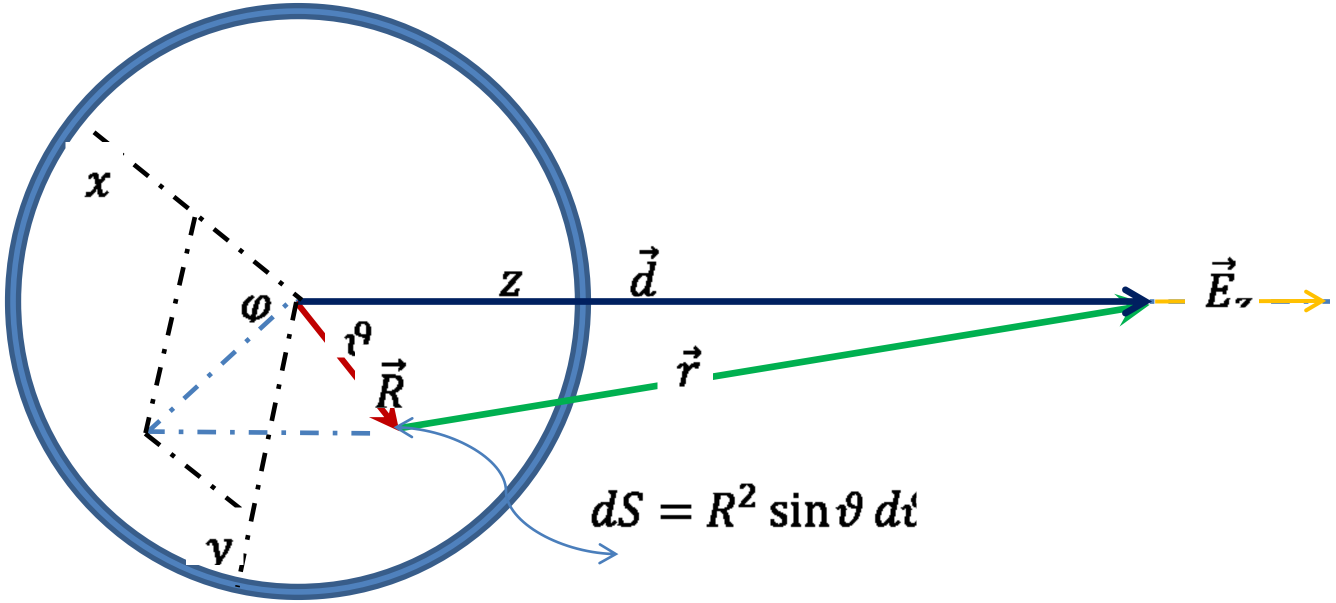
$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{\left(\vec{d} - \vec{R}\right)}{\left|\vec{d} - \vec{R}\right|^3}\, dS \qquad r^2 = \left(\vec{d} - \vec{R}\right)\left(\vec{d} - \vec{R}\right) \qquad \vec{r} = (-\mathrm{R}\sin\vartheta\cos\varphi, -\mathrm{R}\sin\vartheta\sin\varphi, d - \mathrm{R}\cos\vartheta)$$



$$E_z = \frac{\sigma}{4\pi\varepsilon_0}\iint_{0,0}^{2\pi,\pi} \frac{(d - \ R\cos\vartheta)\ R^2\sin\vartheta\, d\varphi d\vartheta}{(d^2 + R^2 - 2d\,R\cos\vartheta)^{3/2}}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{(d - \ R\cos\vartheta)\ R^2\sin\vartheta\, d\vartheta}{(d^2 + R^2 - 2d\,R\cos\vartheta)^{3/2}}$$

Intenzita elektrického pole v okolí homogenně nabité koulové sféry



$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{R^3\left(\frac{d}{R} - \cos\vartheta\right)\sin\vartheta\, d\vartheta}{R^3\left(\left(\frac{d}{R}\right)^2 + 1 - 2\frac{d}{R}\cos\vartheta\right)^{3/2}}$$

$$\frac{d}{R} = \xi$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{(d - R\cos\vartheta)\, R^2 \sin\vartheta\, d\vartheta}{(d^2 + R^2 - 2d\, R\cos\vartheta)^{3/2}}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{(\xi - \cos\vartheta)\sin\vartheta\, d\vartheta}{\left((\xi)^2 + 1 - 2\xi\cos\vartheta\right)^{3/2}}$$

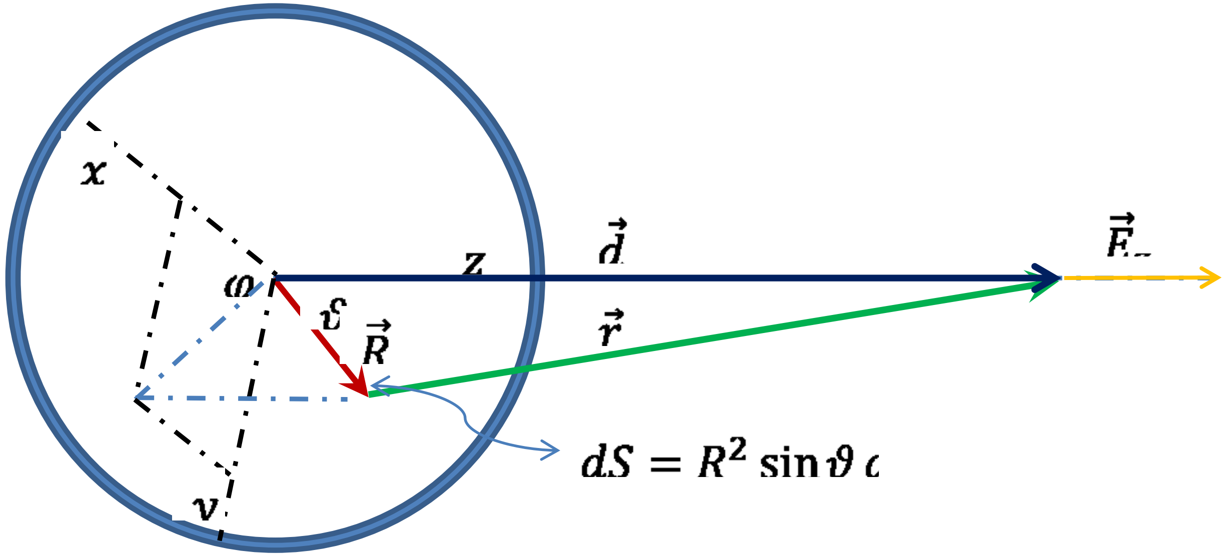
Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi}\frac{(\xi - \cos\vartheta)\sin\vartheta\, d\vartheta}{(\xi^2+1-2\xi\cos\vartheta)^{3/2}}$$

$$\frac{d}{R} = \xi$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{\left[\xi+1-(\xi-1)-\left(|\xi-1| - \frac{\xi^2-1}{|\xi-1|}\right)\right]}{2\xi^2}$$



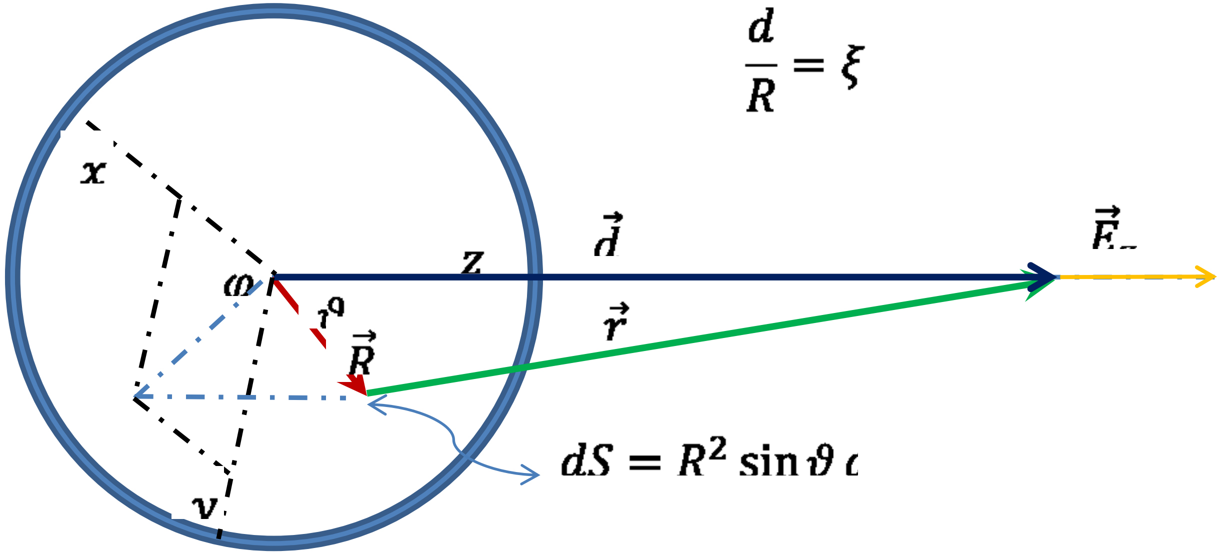
Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0} \frac{\left[\xi + 1 - (\xi - 1) - \left(|\xi - 1| - \frac{\xi^2 - 1}{|\xi - 1|}\right)\right]}{2\xi^2}$$

$$\xi > 1$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0} \frac{[\xi + 1 - \xi + 1 - \xi + 1 + \xi + 1]}{2\xi^2} = \frac{4\pi\sigma}{4\pi\varepsilon_0 \xi^2} = \frac{4\pi\sigma R^2}{4\pi\varepsilon_0 d^2} = \frac{q}{4\pi\varepsilon_0 d^2}$$

$$\frac{d}{R} = \xi$$

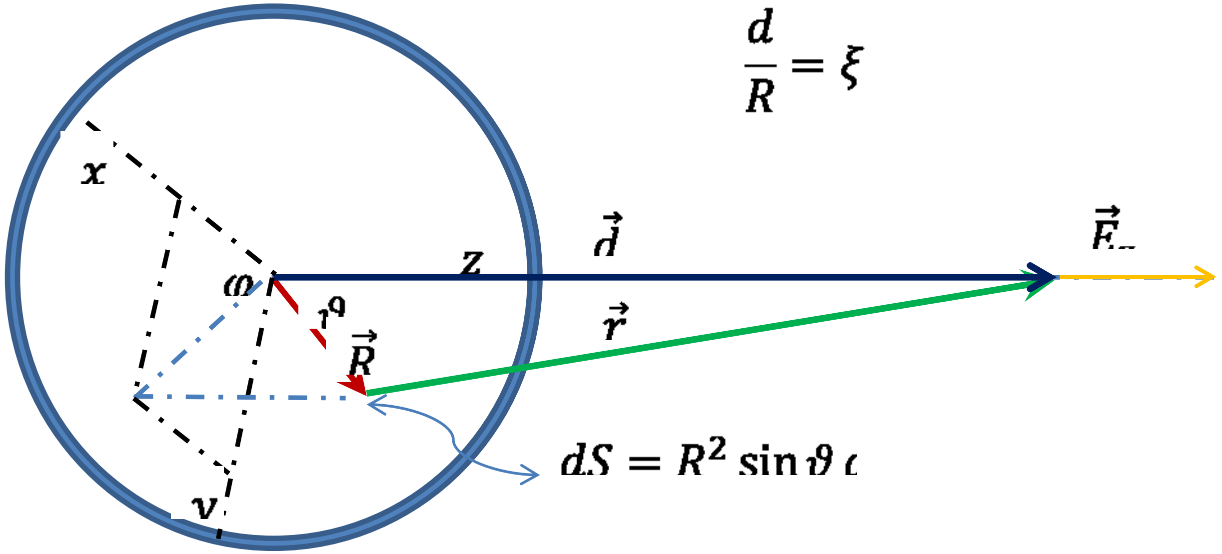


Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{\left[\xi + 1 - (\xi - 1) - \left(|\xi - 1| - \frac{\xi^2 - 1}{|\xi - 1|}\right)\right]}{2\xi^2}$$

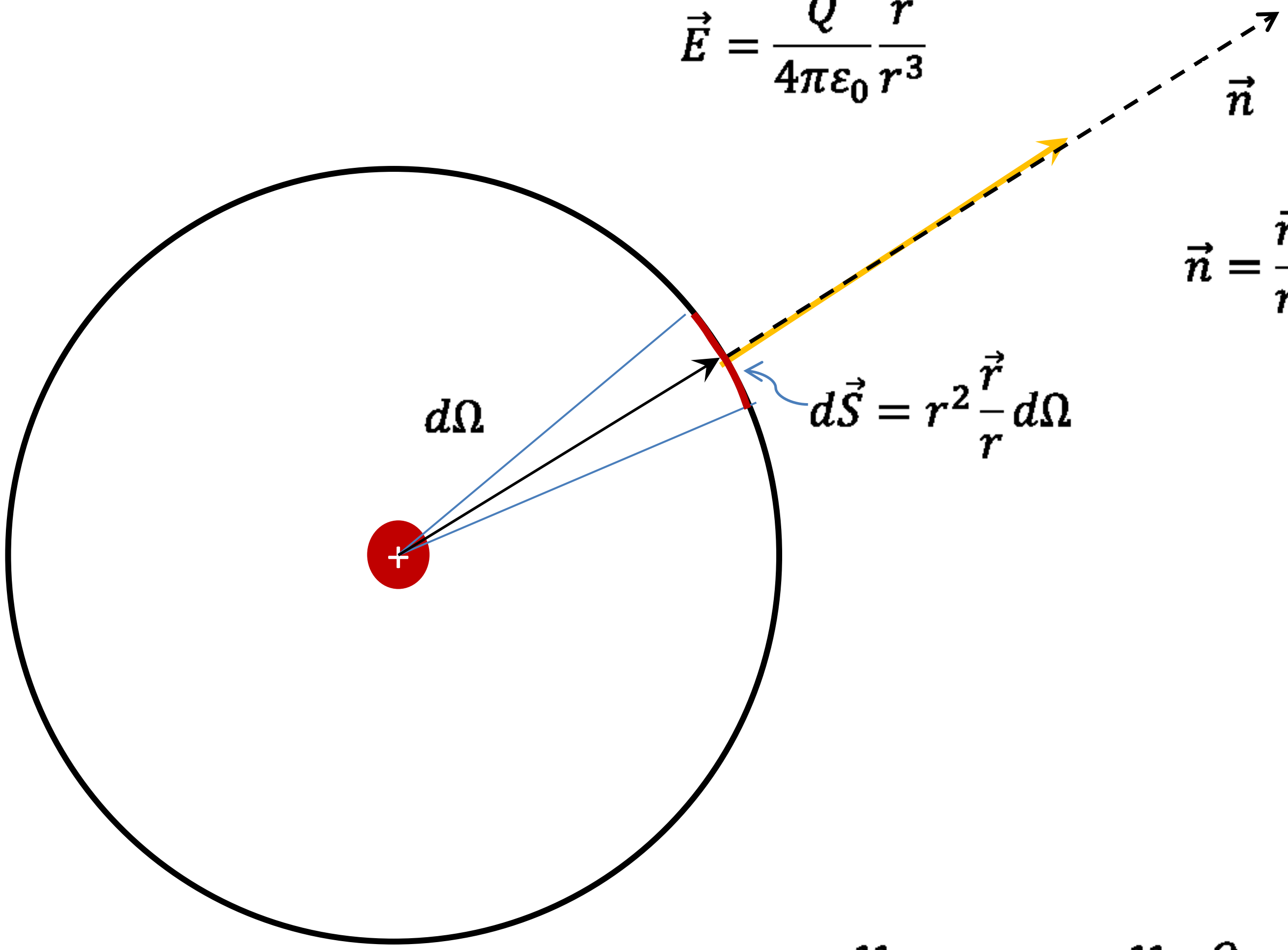
$$\xi < 1$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{[\xi + 1 - \xi + 1 + \xi - 1 - \xi - 1]}{2\xi^2} = 0$$



Tok vektoru $\vec{E}$ od bodového náboje kulovou sférou se středem v náboji

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

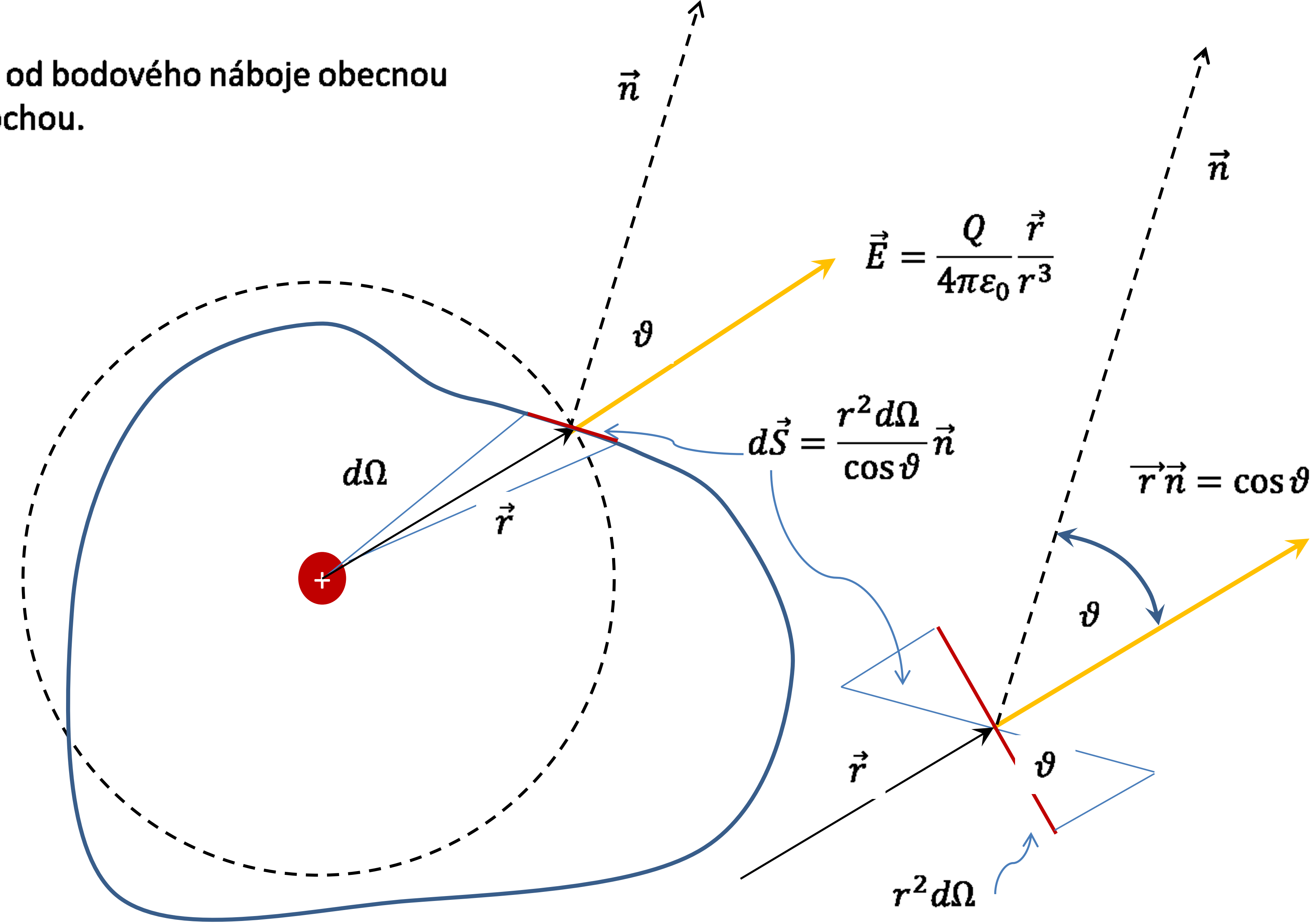


$$\vec{n} = \frac{\vec{r}}{r}$$

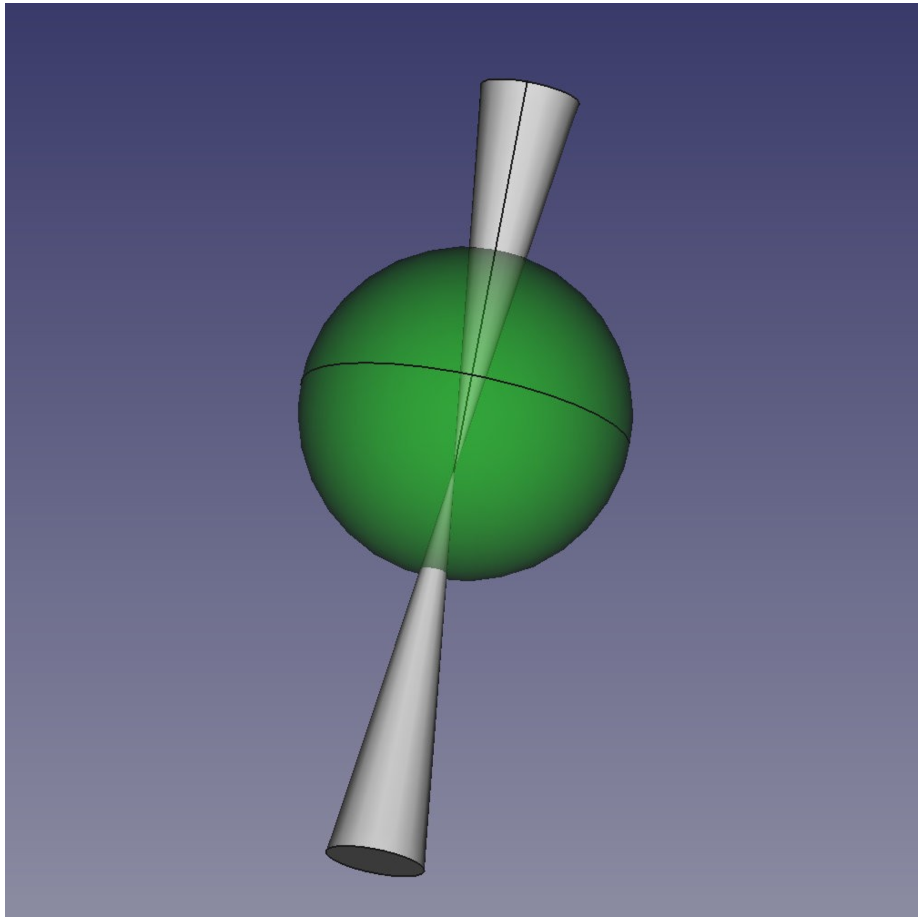
$$d\vec{S} = r^2\frac{\vec{r}}{r}d\Omega$$

$$\oiint \vec{E}(\vec{r})d\vec{S} = \oiint \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}d\vec{S} = \int_0^{4\pi} \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} * r^2\frac{\vec{r}}{r}d\Omega = \frac{Q}{\varepsilon_0}$$

Tok vektoru $\vec{E}$ od bodového náboje obecnou uzavřenou plochou.



$$\oiint \vec{E}(\vec{r})d\vec{S} = \oiint \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}\frac{r^2 d\Omega}{\cos\vartheta}\vec{n} = \int_0^{4\pi}\frac{Q}{4\pi\varepsilon_0 r}\frac{1}{\cos\vartheta}\vec{r}\vec{n}\,d\Omega = \int_0^{4\pi}\frac{Q}{4\pi\varepsilon_0}\frac{1}{\cos\vartheta}\cos\vartheta\;d\Omega = \frac{Q}{\varepsilon_0}$$

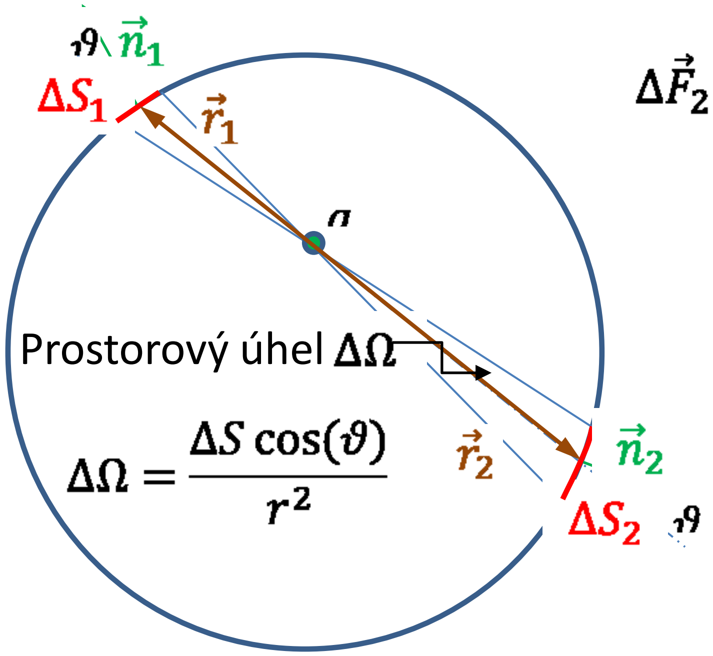


Síla působící na náboj uvnitř rovnoměrně nabité kulové sféry.

$\Delta\Omega$ je prostorový úhel, pod kterým je vidět ploška ΔS_1, ΔS_2

$$\Delta\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\Delta S_1}{r_1^2}\vec{e} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\Delta\Omega\, r_1^2}{\cos(\vartheta)\, r_1^2}\vec{e}$$

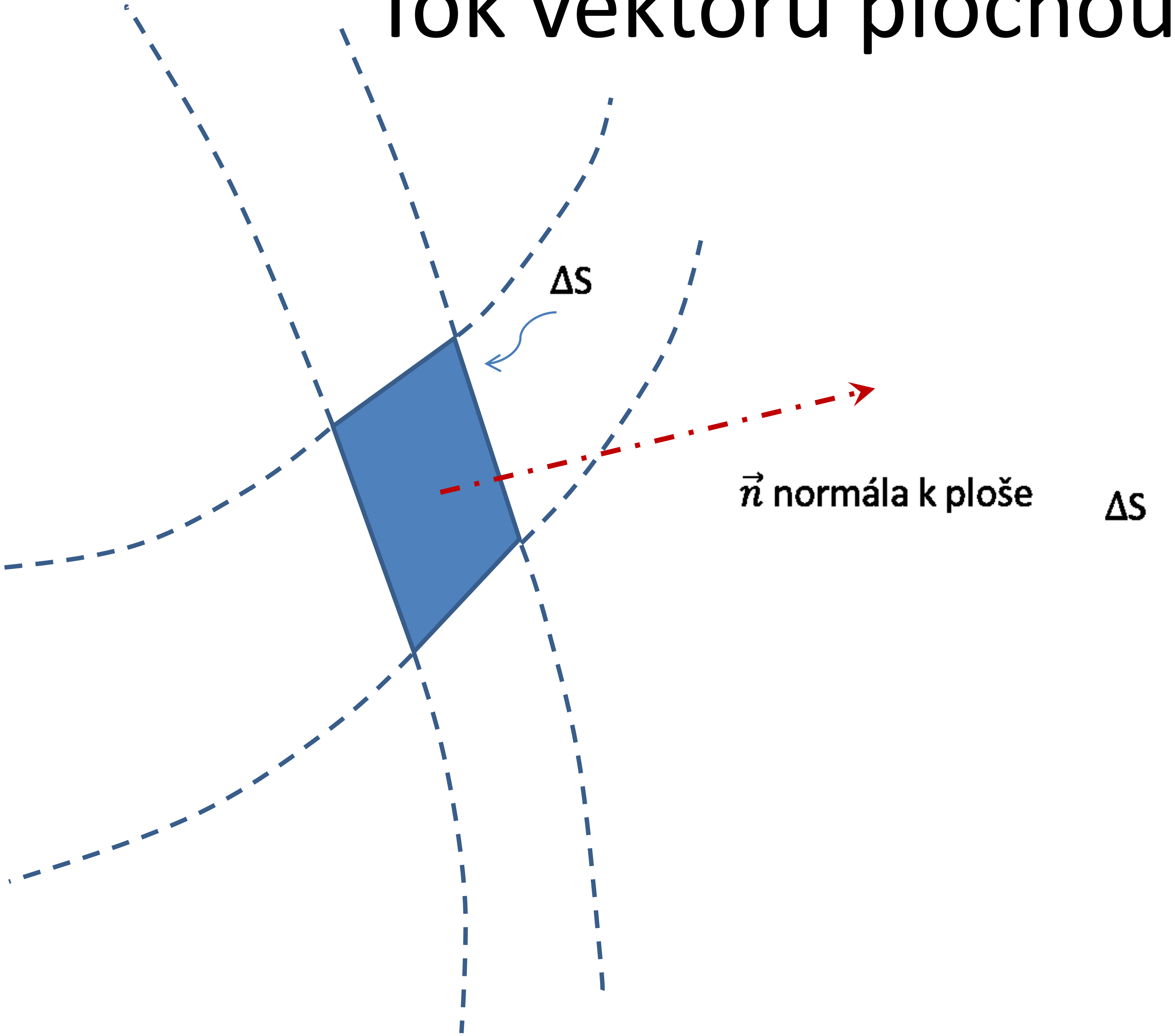
$$\Delta\vec{F}_2 = -\frac{1}{4\pi\varepsilon_0}\frac{\sigma\,\Delta S_2}{r_2^2}\vec{e} = -\frac{1}{4\pi\varepsilon_0}\frac{\sigma\,\Delta\Omega\, r_2^2}{\cos(\vartheta)\, r_2^2}\vec{e}$$



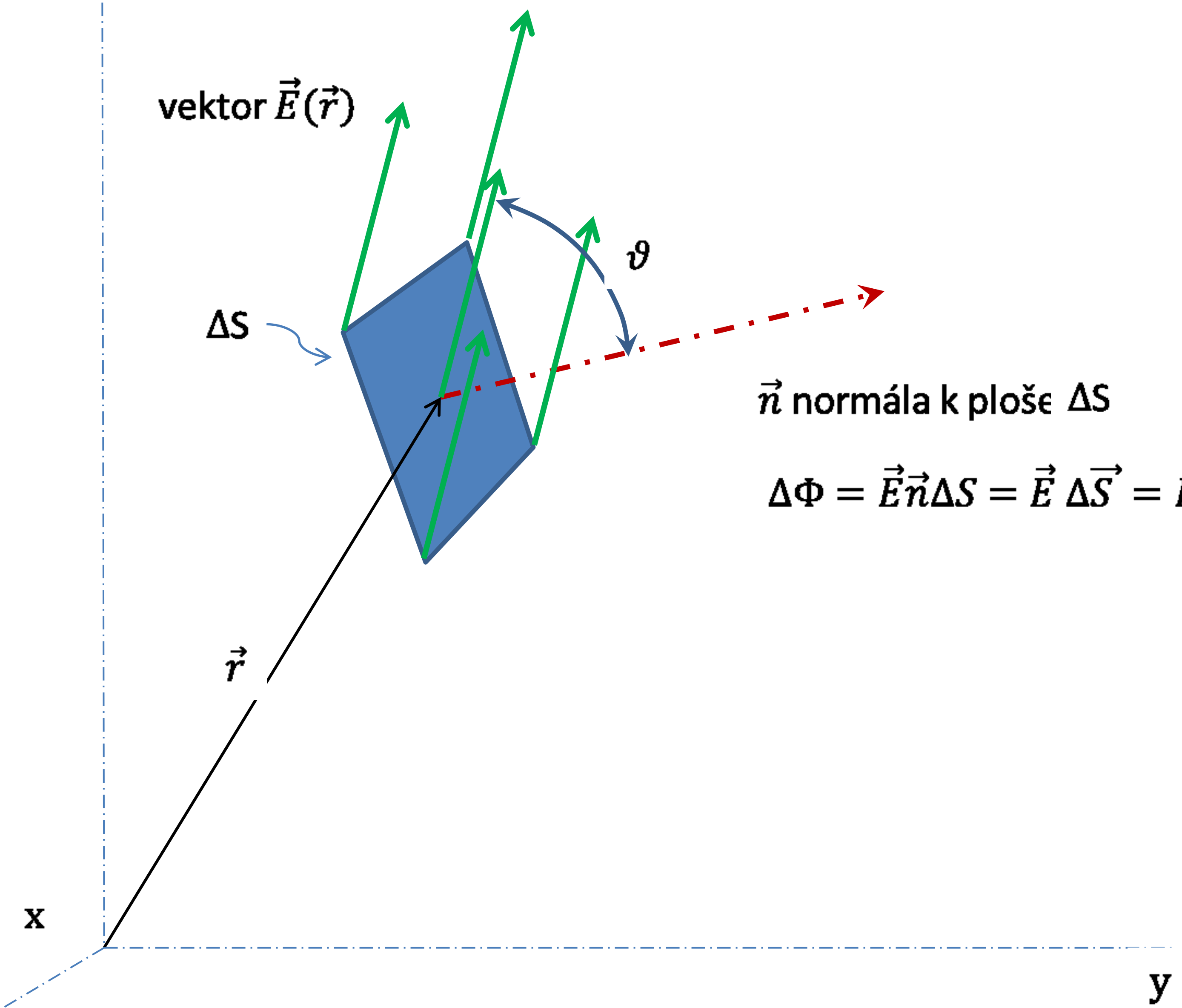
$$\Delta F = \frac{\sigma\Delta\Omega}{4\pi\varepsilon_0\cos(\vartheta)}$$

$\Delta\vec{F}_1 + \Delta\vec{F}_2 = 0$

Tok vektoru plochou

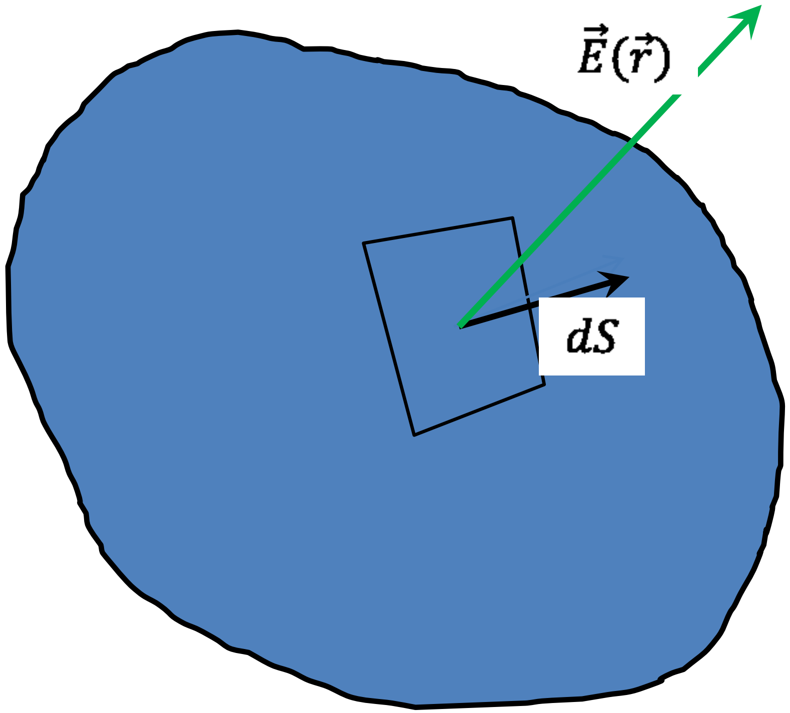


Tok Φ vektoru plochou



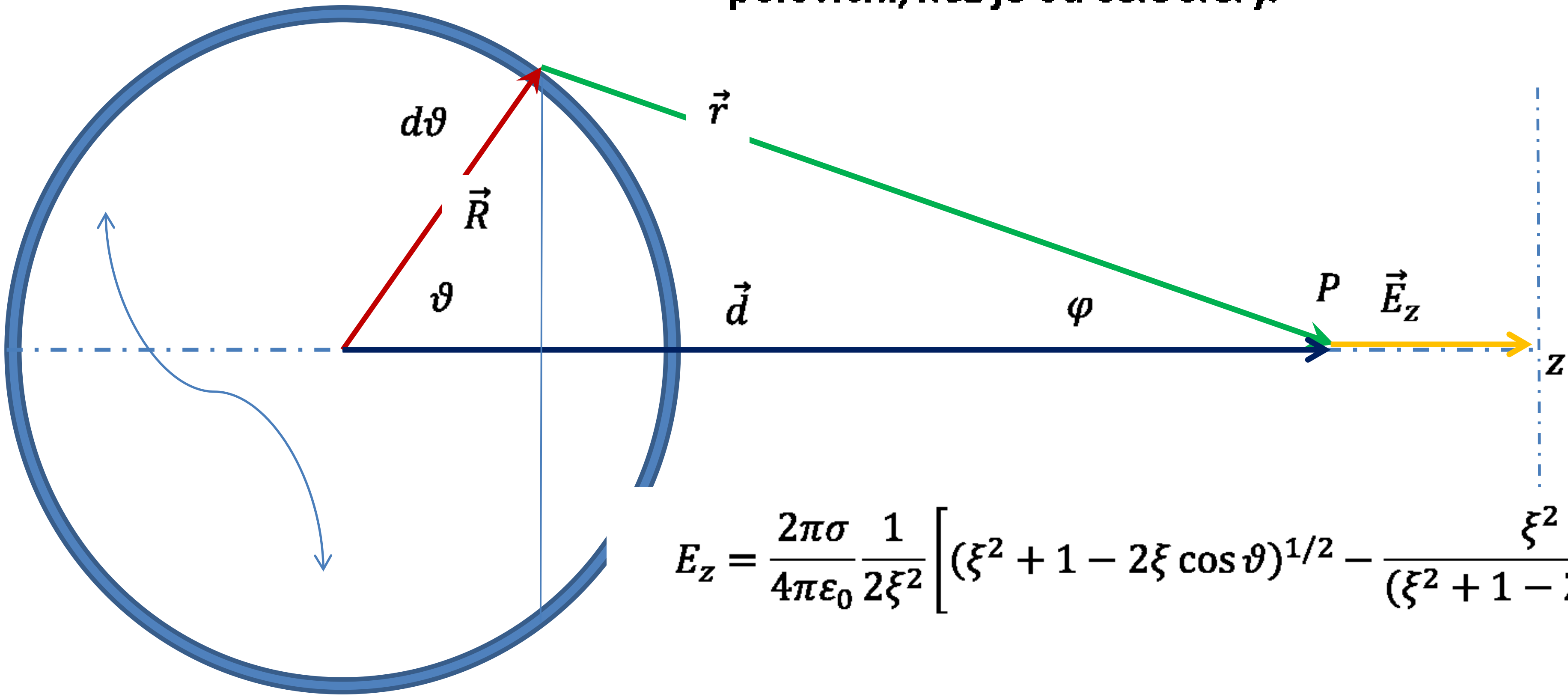
$$\Delta\Phi = \vec{E}\vec{n}\Delta S = \vec{E}\,\Delta\vec{S} = E\Delta S\cos\vartheta$$

Gaussova věta elektrostatiky



$$\oiint \vec{E}(\vec{r})dS = \frac{Q}{\varepsilon_0}$$

Příklad 1: určete pro jaký úhel ϑ_0 bude el. pole v bodě P od každé z obou částí homogenně nabité sféry vymezené kružnicí, která je určena úhlem ϑ_0 poloviční, než je od celé sféry.

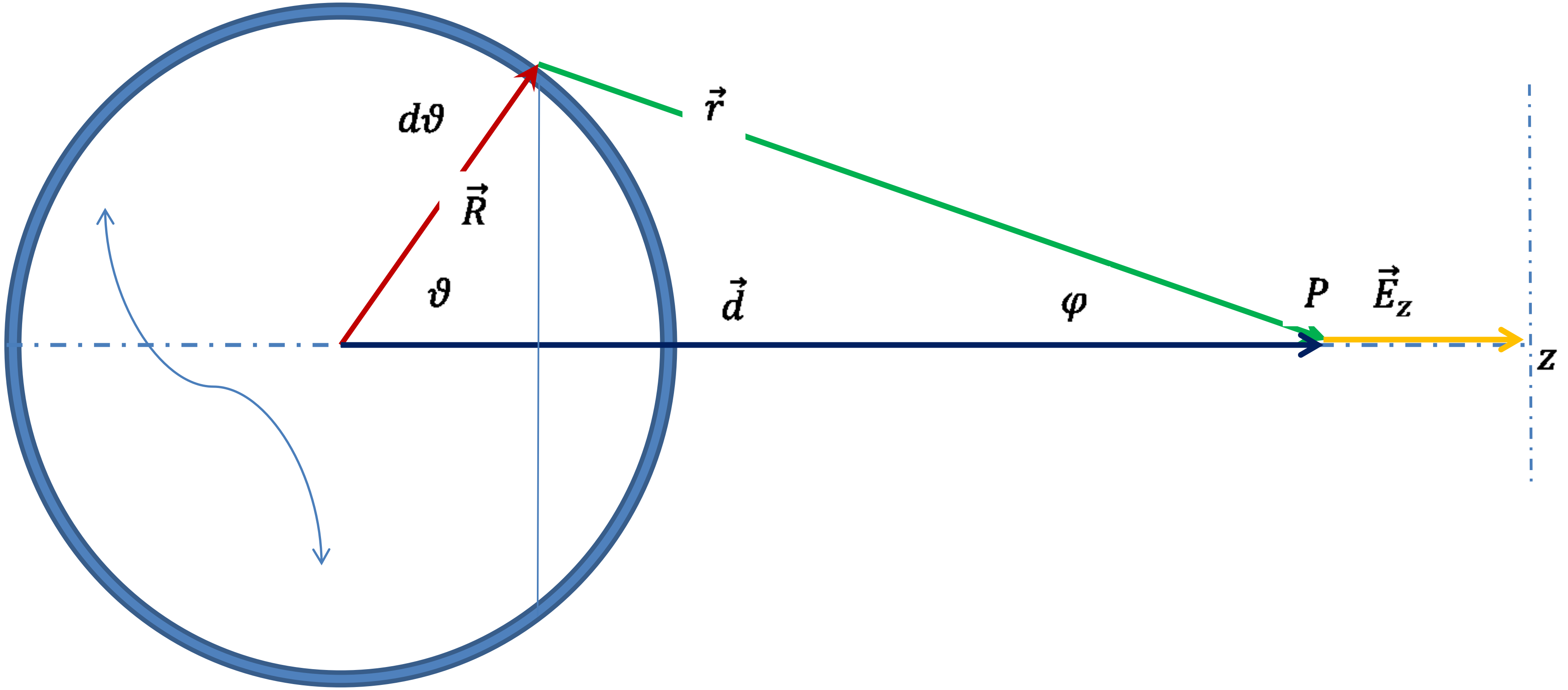


$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi}$$

$$\frac{E_z}{2} = \frac{\sigma}{4\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi} = \frac{\sigma}{2\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\vartheta_0}$$

$$\frac{1}{2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi} = \left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\vartheta_0}$$

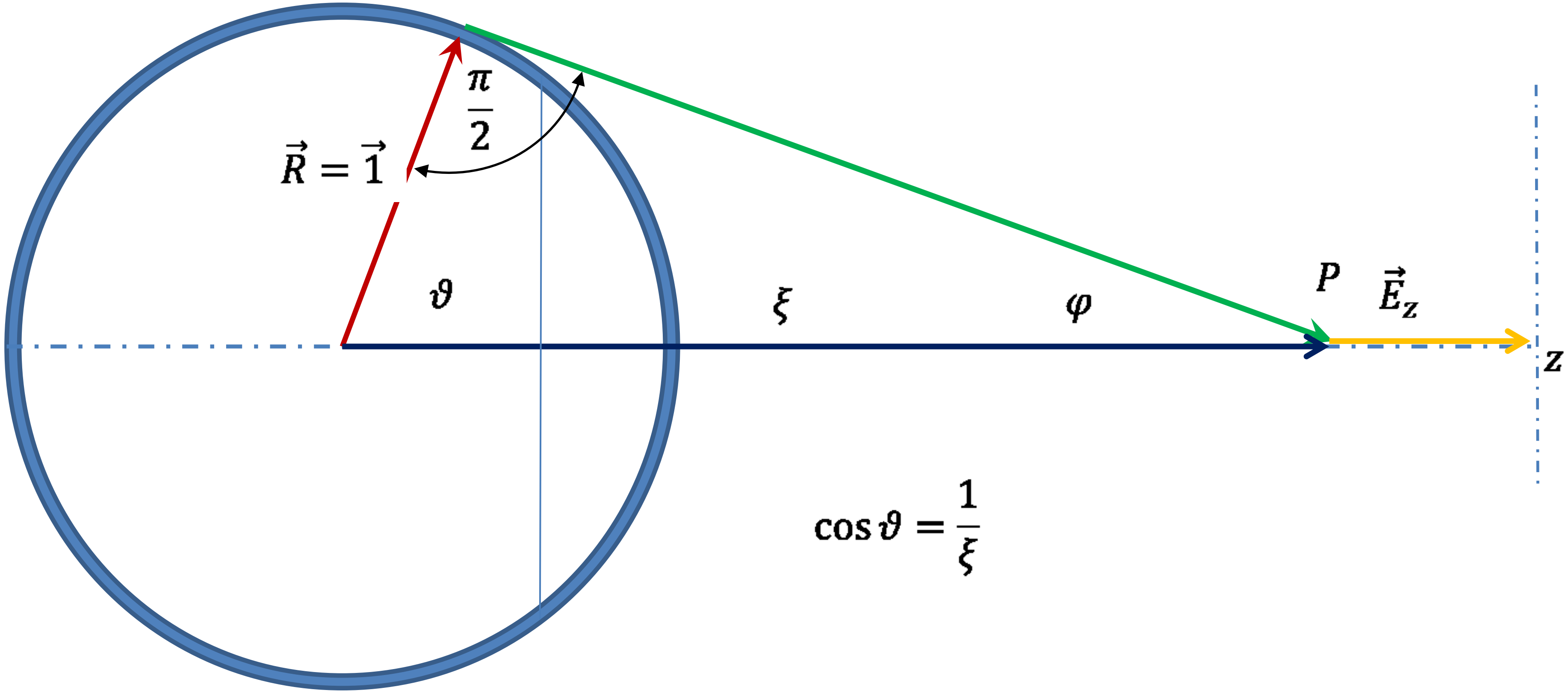
Příklad 1, řešení



$$\frac{1}{2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2}-\frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi}=\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2}-\frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\vartheta_0}$$

$$1+1=(\xi^2+1-2\xi\cos\vartheta)^{1/2}-\frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{\frac{1}{2}}}+2 \qquad \cos\vartheta=\frac{1}{\xi}$$

Příklad 1, řešení



$$\cos\vartheta = \frac{1}{\xi}$$