DYNAMICAL SYSTEMS
SPECIAL NONLINEAR SYSTEMS AND NONEXISTENCE OF CYCLES
12.03.2025

PROBLEM 1: Determine the stationary points and classify their type.

$$x' = -y - x\sqrt{x^2 + y^2}$$
$$y' = x - y\sqrt{x^2 + y^2}.$$

PROBLEM 2: Analyze the simple predator-prey model:

$$K' = \alpha K - \beta K D$$
$$D' = -\gamma D + \delta K D,$$

where K represents the prey population size, D the predator population size, and α, β, γ, $\delta > 0$.

Also examine the modified system with harvesting:

$$K' = \alpha K - \beta K D - \varepsilon K$$
$$D' = -\gamma D + \delta K D - \varepsilon D,$$

where $\varepsilon > 0$.

PROBLEM 3: Show that the equation

$$x'' + \gamma x' + \omega^2 \sin x = 0, \qquad \gamma \geq 0,\ \omega \neq 0,$$

has no periodic solutions.

PROBLEM 4: Determine whether cycles exist for the system:

$$x' = x(a + bx + cy)$$
$$y' = y(\alpha + \beta x + \gamma y),$$

where a, b, c, α, β, $\gamma \in \mathbb{R}$.