Cold atoms

Lecture 3.

18^{th} October, 2006

Non-interacting bosons in a trap

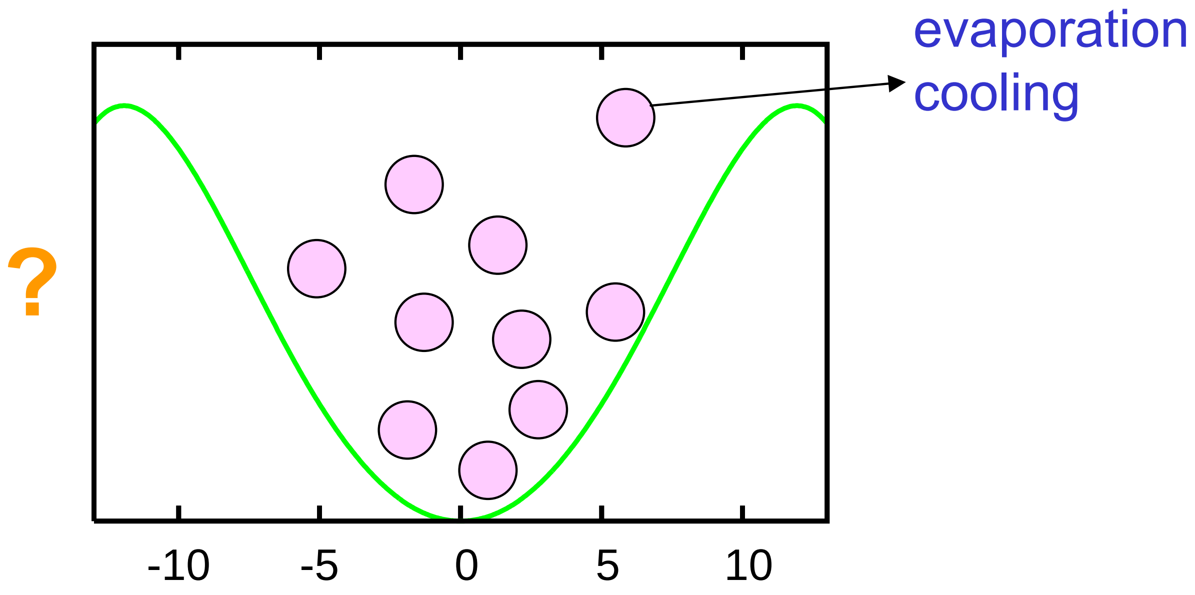
Useful digression: energy units

energy	1K	1eV	s^{-1}
1K	k_B/J	k_B/e	k_B/h
1eV	e/k_B	e/J	e/h
s^{-1}	h/k_B	h/e	h/J

energy	1K	1eV	s^{-1}
1K	1.38×10^{-23}	8.63×10^{-05}	$2.08\times10^{+10}$
1eV	$1.16\times10^{+04}$	1.60×10^{-19}	$2.41\times10^{+14}$
s^{-1}	4.80×10^{-11}	4.14×10^{-15}	6.63×10^{-34}

Trap potential

Typical profile



coordinate/ microns $\rightarrow$

This is just one direction

Presently, the traps are mostly 3D

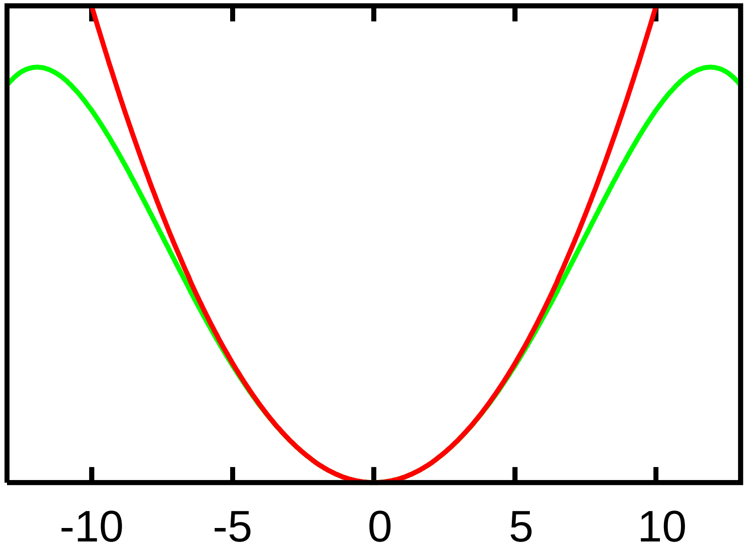
The trap is clearly from the real world, the atomic cloud is visible almost by a naked eye

Trap potential

Parabolic approximation

in general, an anisotropic harmonic oscillator

usually with axial symmetry



$$H = \frac{1}{2m}\boldsymbol{p}^2 + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2 + \frac{1}{2}m\omega_z^2 z^2$$

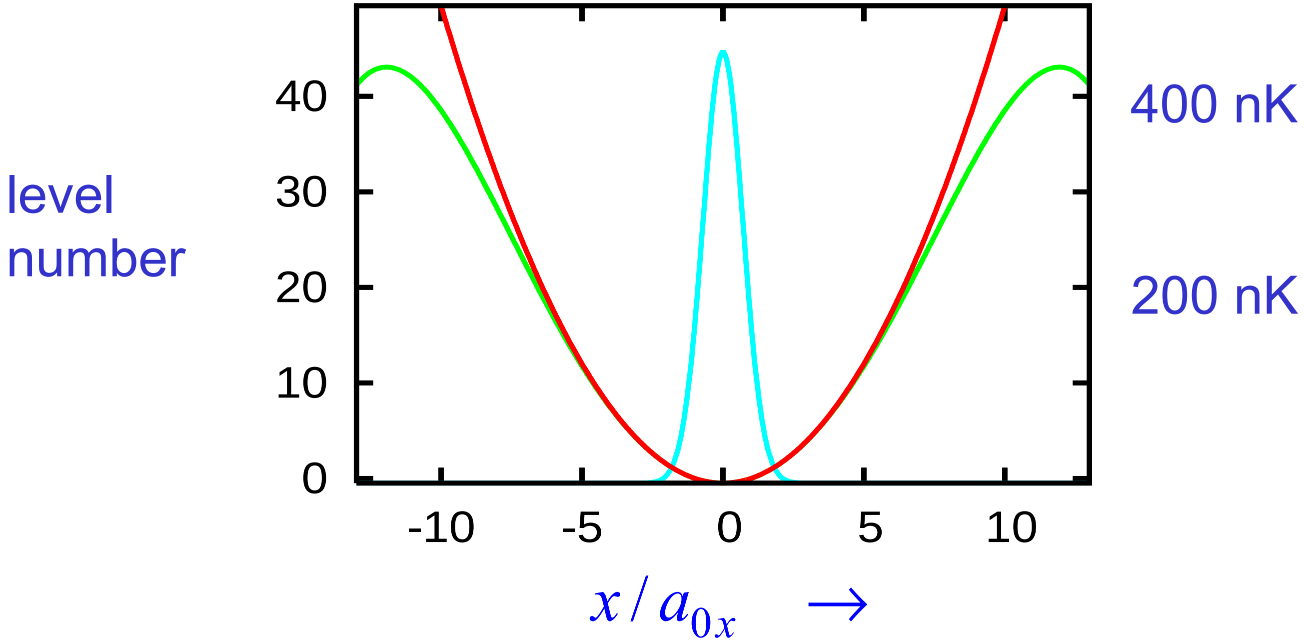
$$= H_x + H_y + H_z$$

1D

2D

3D

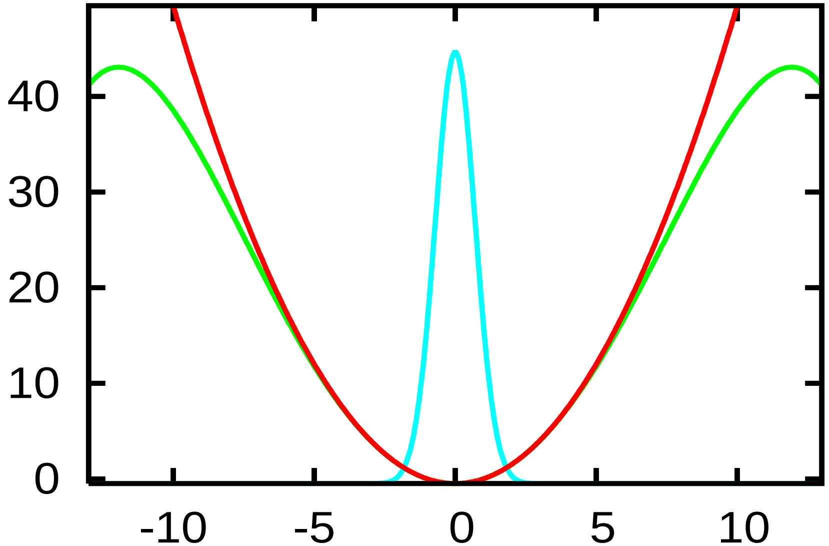
Ground state orbital and the trap potential



$$\psi_0(x,y,z) = \phi_{0x}(x)\,\phi_{0y}(y)\,\phi_{0z}(z)$$

$$\phi_0(u) = \frac{1}{\sqrt{a_0\pi}}\mathrm{e}^{-\frac{u^2}{2a_0^2}}, \quad \boxed{a_0 = \sqrt{\frac{\hbar}{m\omega}}}, \quad E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}\cdot\frac{\hbar^2}{ma_0^2} = \frac{1}{2}\cdot\frac{\hbar^2}{Mu_m a_0^2}$$

Ground state orbital and the trap potential



^{87}Rb
$a_0 = 1\,\mu\mathrm{m}$
$\hbar\omega = 10$ nK
$N \sim 10^6$ at.

$$\psi_0(x,y,z) = \phi_{0x}(x)\,\phi_{0y}(y)\,\phi_{0z}(z)$$

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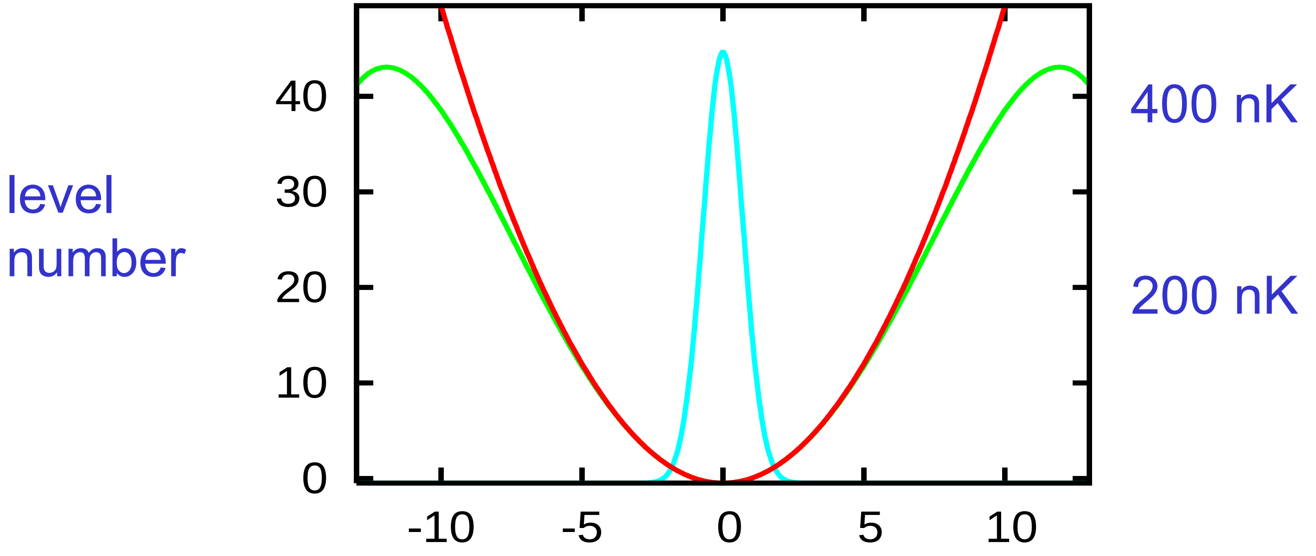
Ground state orbital and the trap potential



$x / a_{0x} \quad \rightarrow$

$^{87}\mathrm{Rb}$

$a_0 = 1\,\mu\mathrm{m}$

$\hbar\omega = 10\ \mathrm{nK}$

$N \sim 10^6\ \mathrm{at.}$

$$\psi_0(x,y,z) = \phi_{0x}(x)\,\phi_{0y}(y)\,\phi_{0z}(z)$$

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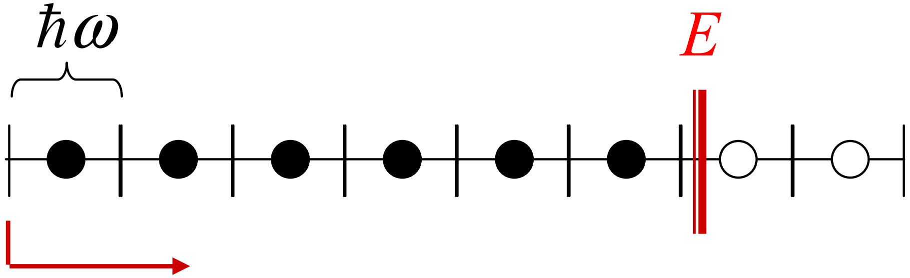
$$\boxed{V(u) = \frac{1}{2}m\omega^2 u^2 = \frac{1}{2}\hbar\omega\left(\frac{u}{a_0}\right)^2}$$

- characteristic energy ($\hbar\omega$)
- characteristic length (a_0)

Filling the trap with particles: IDOS, DOS

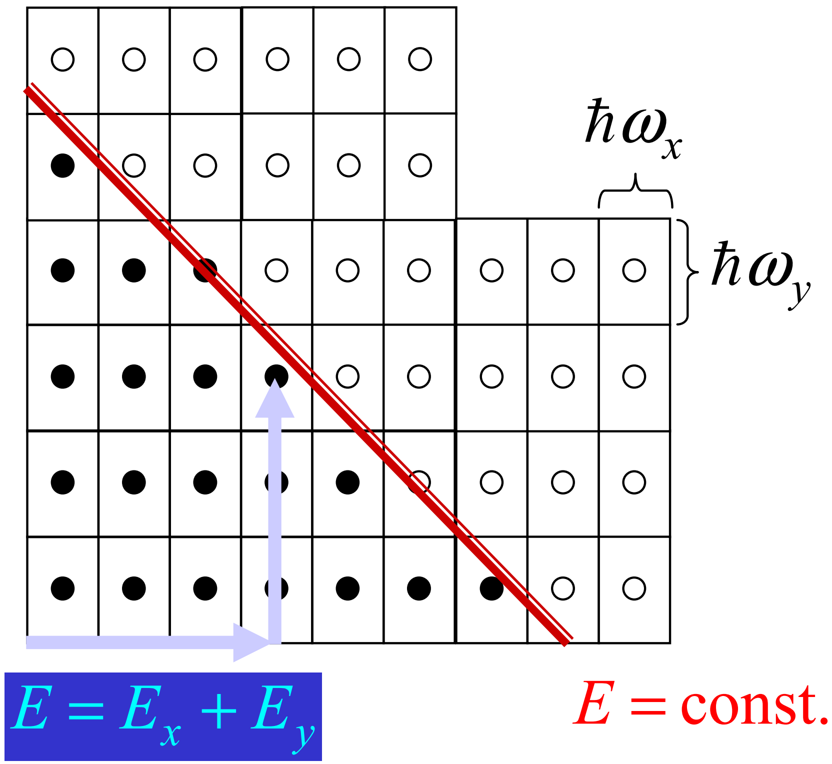


1D

$$\Gamma(E) = \mathrm{int}(E/\hbar\omega) \approx E/\hbar\omega$$

$$\mathcal{D}(E) = \Gamma'(E) = \left(\hbar\omega\right)^{-1}$$

For the finite trap, unlike in the extended gas, $\mathcal{D}(E)$ is **not** divided by volume !!



2D

$$\Gamma(E) \approx \tfrac{1}{2} E^2 / (\hbar\omega_x \cdot \hbar\omega_y)$$

$$\mathcal{D}(E) = \Gamma'(E) = E / (\hbar\omega_x \cdot \hbar\omega_y)$$

"thermodynamic limit"

only approximate … finite systems

better for small $\hbar\omega$

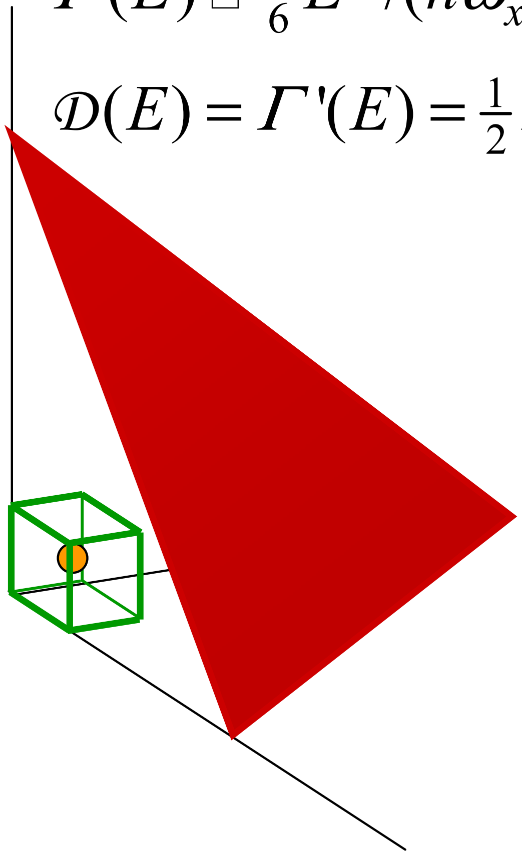
meaning wide trap potentials

Filling the trap with particles

3D

$$\Gamma(E) \approx \tfrac{1}{6} E^3 / (\hbar\omega_x \cdot \hbar\omega_y \cdot \hbar\omega_z)$$

$$\mathcal{D}(E) = \Gamma'(E) = \tfrac{1}{2} E^2 / (\hbar\omega_x \cdot \hbar\omega_y \cdot \hbar\omega_z)$$



<u>Estimate for the transition temperature</u>

particle number comparable with
the number of states in the thermal shell

$$N \approx \Gamma\left(k_B T\right)$$

2D $\quad T_c \approx \hbar\tilde{\omega}/k_B \cdot N^{\frac{1}{2}} \qquad \tilde{\omega} = (\omega_x \cdot \omega_y)^{\frac{1}{2}}$

3D $\quad T_c \approx \hbar\tilde{\omega}/k_B \cdot N^{\frac{1}{3}} \qquad \tilde{\omega} = (\omega_x \cdot \omega_y \cdot \omega_z)^{\frac{1}{3}}$

For 10^6 particles,

characteristic energy

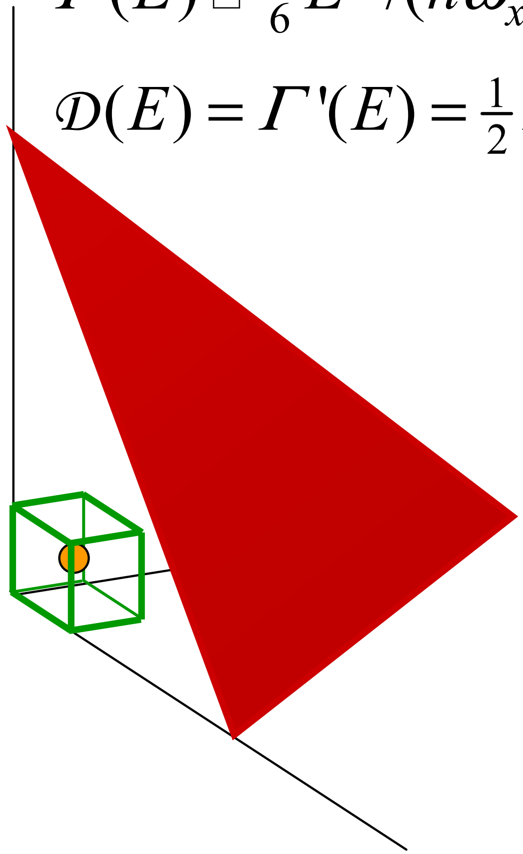
$$k_B T_c \approx 10^2 \hbar\tilde{\omega}$$

Filling the trap with particles

3D

$$\Gamma(E) \simeq \tfrac{1}{6} E^3 / (\hbar\omega_x \cdot \hbar\omega_y \cdot \hbar\omega_z)$$

$$\mathcal{D}(E) = \Gamma'(E) = \tfrac{1}{2} E^2 / (\hbar\omega_x \cdot \hbar\omega_y \cdot \hbar\omega_z)$$



Estimate for the transition temperature

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the number of states in the thermal shell

$$N \approx \Gamma\left(k_{\mathrm{B}} T\right)$$

$$\boxed{2\mathrm{D}} \quad T_c \approx \hbar\tilde{\omega}/k_{\mathrm{B}} \cdot N^{\frac{1}{2}} \qquad \tilde{\omega} = (\omega_x \cdot \omega_y)^{\frac{1}{2}}$$

$$\boxed{3\mathrm{D}} \quad T_c \approx \hbar\tilde{\omega}/k_{\mathrm{B}} \cdot N^{\frac{1}{3}} \qquad \tilde{\omega} = (\omega_x \cdot \omega_y \cdot \omega_z)^{\frac{1}{3}}$$

For 10^6 particles,

characteristic energy

$$k_{\mathrm{B}} T_c \approx 10^2 \hbar\tilde{\omega} \gg \hbar\tilde{\omega}$$ important for therm. limit

Exact expressions for critical temperature etc.

The general expressions are the same like for the homogeneous gas.

Working with discrete levels, we have

$$N = \mathcal{N}(T, \mu) = \sum_j \left\langle n(\varepsilon_j) \right\rangle = \sum_j \frac{1}{\mathrm{e}^{\beta(\varepsilon_j - \mu)} - 1}$$

and this can be used for numerics without exceptions.

In the approximate thermodynamic limit, the old equation holds, only the volume V does not enter as a factor:

$$N = \mathcal{N}(T, \mu) = \frac{1}{\mathrm{e}^{\beta(\varepsilon_0 - \mu)} - 1} + \cancel{V} \int_0^\infty \mathrm{d}\varepsilon \frac{1}{\mathrm{e}^{\beta(\varepsilon - \mu)} - 1} \mathcal{D}(\varepsilon)$$

$\mu \to 0 \quad \text{for} \quad T \leq T_C$

In 3D,

$$T_c = (\zeta(3))^{-\frac{1}{3}} \hbar\tilde{\omega}/k_\mathrm{B} \cdot N^{\frac{1}{3}} = 0.94 \hbar\tilde{\omega}/k_\mathrm{B} \cdot N^{\frac{1}{3}}$$

$$N_\mathrm{BE} = N \cdot \left(1 - (T/T_c)^3\right), \quad T < T_c$$

How good is the thermodynamic limit

1D illustration (almost doable)

$$N = \sum_j \frac{1}{e^{\beta(\hbar\omega \times j - \mu)} - 1} \overset{?}{=} \frac{1}{e^{-\beta\mu} - 1} + \int_0^\infty d\varepsilon \frac{1}{e^{\beta(\varepsilon - \mu)} - 1} \frac{1}{\hbar\omega}$$

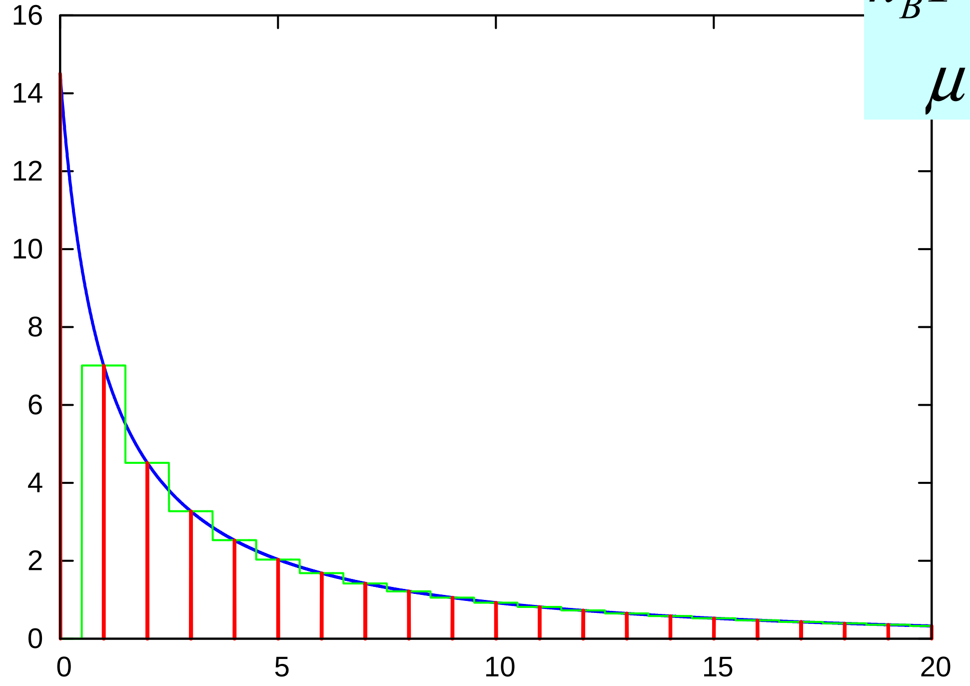
How good is the thermodynamic limit

1D illustration

$$N = \sum_j \frac{1}{e^{\beta(\hbar\omega \times j - \mu)} - 1} \overset{?}{=} \frac{1}{e^{-\beta\mu} - 1} + \int_0^\infty d\varepsilon \frac{1}{e^{\beta(\varepsilon - \mu)} - 1} \frac{1}{\hbar\omega}$$

$$k_B T = 15\hbar\omega$$

$$\mu = -\hbar\omega$$



How good is the thermodynamic limit

1D illustration

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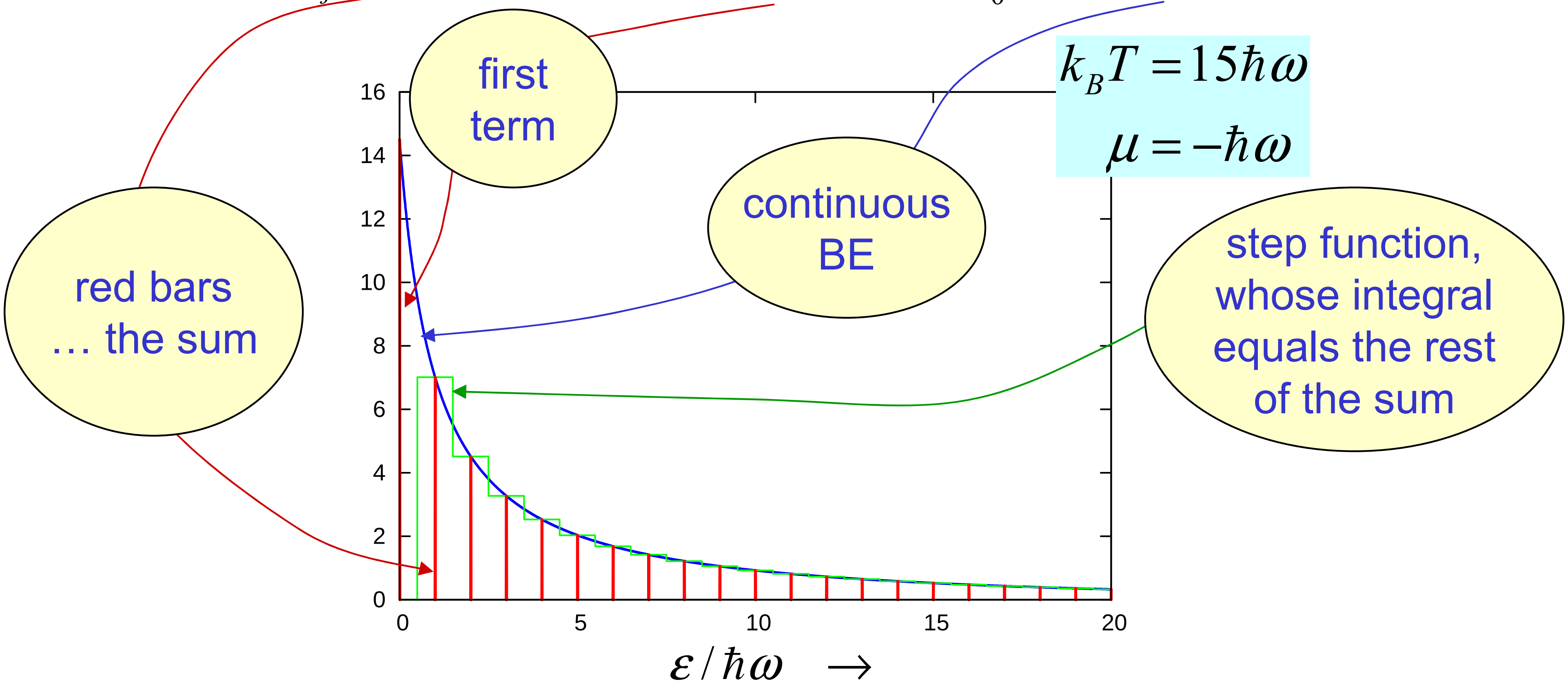

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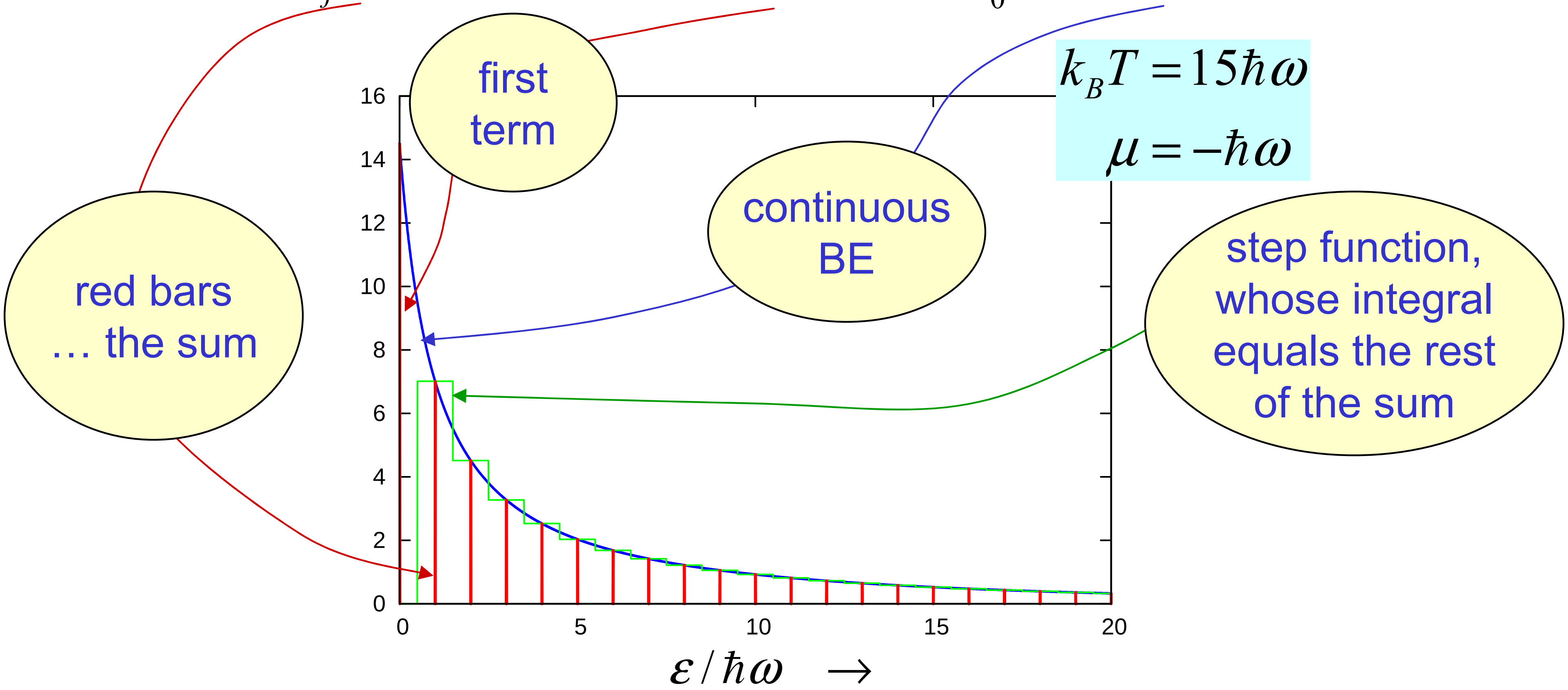
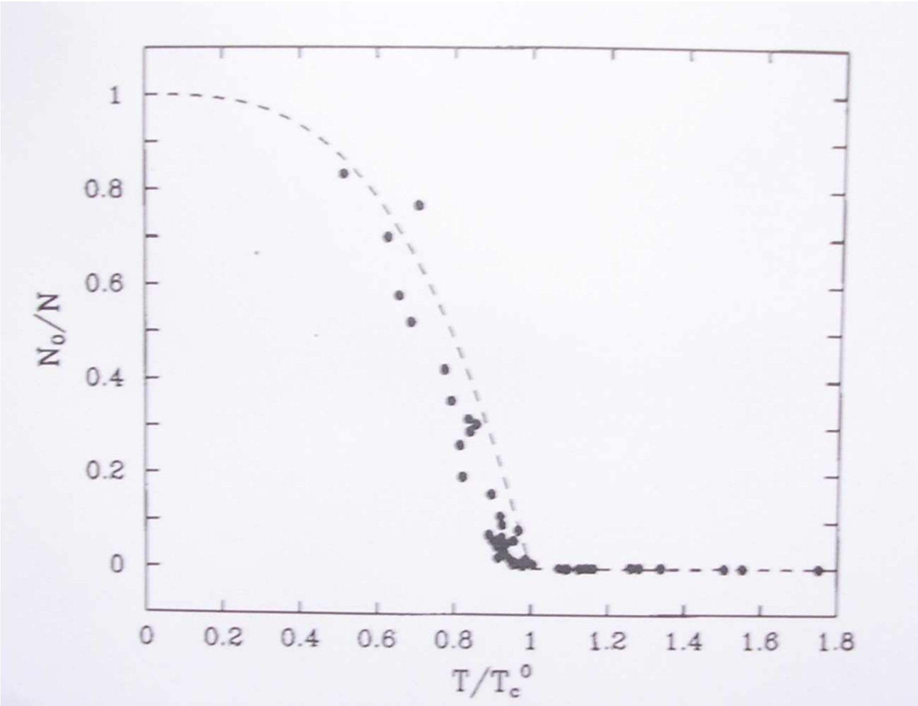


The quantitative criterion for the thermodynamic limit

$$\frac{k_B T_C}{\hbar\omega} \gg 1$$

How sharp is the transition



These are experimental data

fitted by the formula

$$N_{\rm BE} = N \cdot \left(1 - (T/T_c)^3\right), \quad T < T_c$$

The rounding is apparent,

but not really an essential feature

Seeing the condensate – reminescence of **L2**

Without field-theoretical means, the coherence of the condensate may be studied using the **one-particle density matrix.**

Definition of OPDM for non-interacting particles: Take an additive observable, like local density, or current density. Its average value for the whole assembly of atoms in a given equilibrium state:

$$\langle X\rangle = \sum_\alpha \langle\alpha|X|\alpha\rangle\langle n_\alpha\rangle \quad \text{double average, quantum and thermal}$$

$$= \sum_\alpha \langle\alpha|X\sum_\beta|\beta\rangle\langle\beta|\alpha\rangle\langle n_\alpha\rangle \quad \text{insert unit operator}$$

$$= \sum_\beta \langle\beta|\boxed{\sum_\alpha|\alpha\rangle\langle n_\alpha\rangle\langle\alpha|}X|\beta\rangle \quad \text{change the summation order}$$

$$= \sum_\beta \langle\beta|\rho X|\beta\rangle \quad \text{define the one-particle density matrix}$$

$$= \mathrm{Tr}\,\rho X \qquad \rho = \sum_\alpha|\alpha\rangle\langle n_\alpha\rangle\langle\alpha|$$

OPDM in the Trap

- Use the eigenstates of the 3D oscillator
- Use the BE occupation numbers

$$\rho = \sum_{\tilde{\nu}} |\tilde{\nu}\rangle \langle n_{\tilde{\nu}} \rangle \langle \tilde{\nu}| \qquad \tilde{\nu} = (\nu_x, \nu_y, \nu_z), \quad \nu_w = 0, 1, 2, 3, \cdots$$

$$= \sum_{\tilde{\nu}} |\tilde{\nu}\rangle \frac{1}{e^{\beta(E_{\tilde{\nu}}-\mu)} - 1} \langle \tilde{\nu}|, \qquad |\tilde{\nu}\rangle = |\nu_x\rangle |\nu_y\rangle |\nu_z\rangle$$

$$E_{\tilde{\nu}} = E_{\nu_x} + E_{\nu_y} + E_{\nu_z} = \hbar\omega_x \nu_x + \hbar\omega_x \nu_x + \hbar\omega_x \nu_x$$

zero point oscillations absorbed in the chemical potential

- Single out the ground state

$$\rho = |000\rangle \frac{1}{e^{-\beta\mu} - 1} \langle 000| + \sum_{\tilde{\nu} \neq (000)} |\tilde{\nu}\rangle \frac{1}{e^{\beta(E_{\tilde{\nu}}-\mu)} - 1} \langle \tilde{\nu}|$$

$$\equiv \qquad \rho_{\mathrm{BEC}} \qquad + \qquad \rho_{\mathrm{TERM}}$$

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$$E_{\tilde{\nu}} = E_x + E_y + E_z = \hbar\omega\nu_x + \hbar\omega\nu_y + \hbar\omega\nu_z$$

...int oscillations ... in the ... tial

Coherent component,
be it condensate or not.
At $T \ll \hbar\omega/k_B$, it contains
ALL atoms in the cloud

Incoherent thermal component,
coexisting with the condensate.
At $T \gg \hbar\omega/k_B$ it freezes out
and contains NO atoms

-

$$\rho = |000\rangle \frac{1}{\mathrm{e}^{-\beta\mu} - 1} \langle 000| + \sum_{\tilde{\nu} \neq (000)} |\tilde{\nu}\rangle \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}} - \mu)} - 1} \langle \tilde{\nu}|$$

$$\equiv \qquad \rho_{\mathrm{BEC}} \qquad + \qquad \rho_{\mathrm{TERM}}$$

OPDM in the Trap, Particle Density in Space

The spatial distribution of atoms in the trap is inhomogeneous.
Proceed by definition:

$$n(\boldsymbol{r}) = \mathrm{Tr}\,\rho\delta(\boldsymbol{r}_{\mathrm{op}} - \boldsymbol{r})$$

$$= \mathrm{Tr}\,\rho \int \mathrm{d}\overline{\boldsymbol{r}} \left|\overline{\boldsymbol{r}}\right\rangle \delta(\overline{\boldsymbol{r}} - \boldsymbol{r}) \left\langle \overline{\boldsymbol{r}}\right| = \mathrm{Tr}\,\rho \left|\boldsymbol{r}\right\rangle\left\langle \boldsymbol{r}\right|$$

$$= \left\langle \boldsymbol{r}\right|\rho\left|\boldsymbol{r}\right\rangle = \sum_{\tilde{\nu}} \left\langle \boldsymbol{r}\middle|\tilde{\nu}\right\rangle \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}}-\mu)}-1} \left\langle \tilde{\nu}\middle|\boldsymbol{r}\right\rangle$$

$$= \sum_{\tilde{\nu}} \left|\phi_{\tilde{\nu}}(\boldsymbol{r})\right|^2 \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}}-\mu)}-1}$$

as we would write down naively at once

Split into the two parts, the coherent and the incoherent phase

$$n(\boldsymbol{r}) = \left\langle \boldsymbol{r}\right|\rho\left|\boldsymbol{r}\right\rangle = \left\langle \boldsymbol{r}\right|\rho_{\mathrm{BEC}}\left|\boldsymbol{r}\right\rangle + \left\langle \boldsymbol{r}\right|\rho_{\mathrm{THERM}}\left|\boldsymbol{r}\right\rangle$$

OPDM in the Trap, Particle Density in Space

Split into the two parts, the coherent and the incoherent phase

$$n(\boldsymbol{r}) = \langle \boldsymbol{r} | \rho | \boldsymbol{r} \rangle = \langle \boldsymbol{r} | \rho_{\mathrm{BEC}} | \boldsymbol{r} \rangle + \langle \boldsymbol{r} | \rho_{\mathrm{THERM}} | \boldsymbol{r} \rangle$$

$$= \langle \boldsymbol{r} | 000 \rangle \frac{1}{\mathrm{e}^{-\beta\mu} - 1} \langle 000 | \boldsymbol{r} \rangle + \sum_{\tilde{\nu} \neq (000)} \langle \boldsymbol{r} | \tilde{\nu} \rangle \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}} - \mu)} - 1} \langle \tilde{\nu} | \boldsymbol{r} \rangle$$

$$= \underbrace{\left|\phi_{000}(\boldsymbol{r})\right|^2 \frac{1}{\mathrm{e}^{-\beta\mu} - 1}}_{\text{known} \quad n_{\mathrm{BEC}}(\boldsymbol{r})} + \underbrace{\sum_{\tilde{\nu} \neq (000)} \left|\phi_{\tilde{\nu}}(\boldsymbol{r})\right|^2 \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}} - \mu)} - 1}}_{\text{laborious} \quad n_{\mathrm{THERM}}(\boldsymbol{r})}$$

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$$n_{\mathrm{BEC}}(\boldsymbol{r}) = \left|\phi_{0x}(x)\right|^2 \left|\phi_{0y}(y)\right|^2 \left|\phi_{0z}(z)\right|^2 \frac{1}{\mathrm{e}^{-\beta\mu} - 1}$$

$$= \frac{1}{a_{0x} a_{0y} a_{0z} \pi^3} \mathrm{e}^{-\frac{x^2}{a_{0x}^2} - \frac{y^2}{a_{0y}^2} - \frac{z^2}{a_{0z}^2}} \frac{1}{\mathrm{e}^{-\beta\mu} - 1}$$

The characteristic lengths directly observable

Particle Density in Space: Boltzmann Limit

We approximate the thermal distribution by its classical limit.

Boltzmann distribution in an external field:

$$f_B(\boldsymbol{r},\boldsymbol{p}) = \mathrm{e}^{\beta(\mu - W - U(\boldsymbol{r}))}$$

$$n_{\mathrm{THERM}}(\boldsymbol{r}) = \int \mathrm{d}^3\boldsymbol{p} \cdot f_B(\boldsymbol{r},\boldsymbol{p})$$

$$\propto \mathrm{e}^{-\beta U(\boldsymbol{r})}$$

$$= \mathrm{e}^{-\frac{1}{2}\beta m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)}$$

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For comparison:

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$$= \frac{1}{a_{0x} a_{0y} a_{0z} \pi^3} \mathrm{e}^{-\frac{x^2}{a_{0x}^2} - \frac{y^2}{a_{0y}^2} - \frac{z^2}{a_{0z}^2}} \frac{1}{\mathrm{e}^{-\beta\mu} - 1}$$

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Two directly observable characteristic lengths

$$a_0 = (a_{0x}a_{0y}a_{0z})^{\frac{1}{3}} = \sqrt{\frac{\hbar}{m\tilde{\omega}}},$$

$$R_T = 1\Big/\sqrt{\beta m\tilde{\omega}^2}$$

$$= a_0\sqrt{k_B T/\hbar\tilde{\omega}} \gg a_0$$

$$\tilde{\omega} = (\omega_x\cdot\omega_y\cdot\omega_z)^{\frac{1}{3}}$$

For comparison:

$$n_{\mathrm{BEC}}(\boldsymbol{r}) = |\phi_{0x}(x)|^2\,|\phi_{0y}(y)|^2\,|\phi_{0z}(z)|^2\,\frac{1}{\mathrm{e}^{-\beta\mu}-1}$$

$$= \frac{1}{a_{0x}a_{0y}a_{0z}\pi^3}\,\mathrm{e}^{-\frac{x^2}{a_{0x}^2}-\frac{y^2}{a_{0y}^2}-\frac{z^2}{a_{0z}^2}}\,\frac{1}{\mathrm{e}^{-\beta\mu}-1}$$

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$$R_T = 1 \Big/ \sqrt{\beta m \tilde{\omega}^2}$$

$$= a_0 \sqrt{k_B T / \hbar \tilde{\omega}} \gg a_0$$

$$\tilde{\omega} = (\omega_x \cdot \omega_y \cdot \omega_z)^{\frac{1}{3}}$$

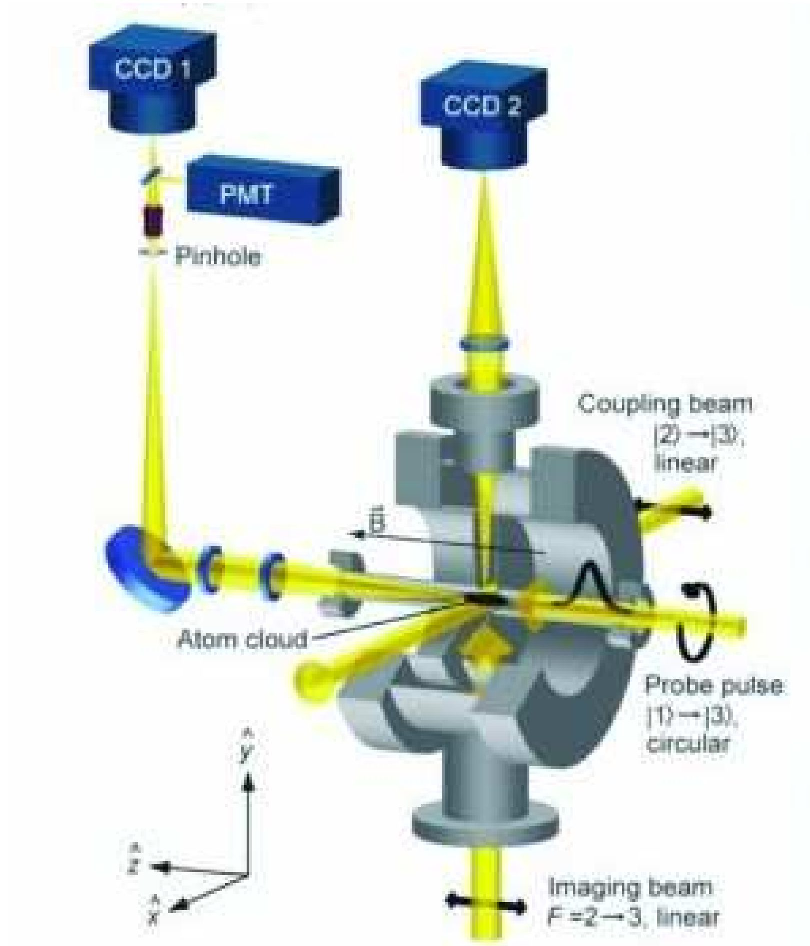
For comparison:

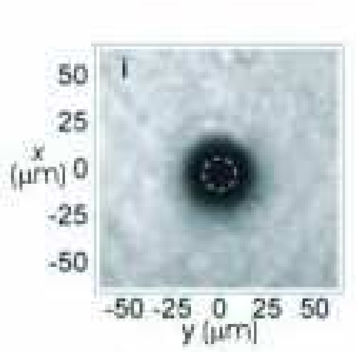
$$n_{\mathrm{BEC}}(\boldsymbol{r}) = \left|\phi_{0x}(x)\right|^2 \left|\phi_{0y}(y)\right|^2 \left|\phi_{0z}(z)\right|^2 \frac{1}{\mathrm{e}^{-\beta\mu} - 1}$$

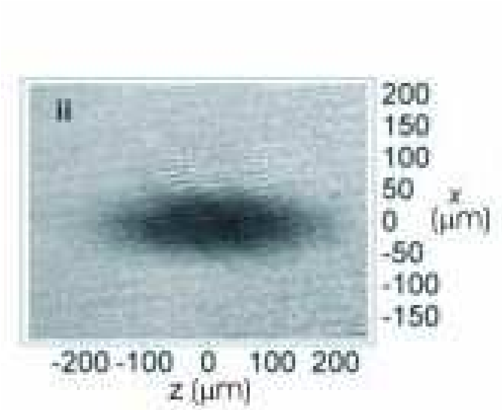
$$= \frac{1}{a_{0x} a_{0y} a_{0z} \pi^3} \mathrm{e}^{-\frac{x^2}{a_{0x}^2} - \frac{y^2}{a_{0y}^2} - \frac{z^2}{a_{0z}^2}} \frac{1}{\mathrm{e}^{-\beta\mu} - 1}$$

anisotropy
given by analogous
definitions of the
two lengths
for each direction

Real space Image of an Atomic Cloud







1999 - Nature 18 Feb.,(Vol.397, p.594)

L.V.Hau, S.E.Harris, Z.Dutton, C.H.Behrozi

Light speed reduction to 17 metres per second in an ultracold atomic gas

Na - atomy, $T = 450$ nK (15 nK nad T_c), 17 m/s (32 m/s)

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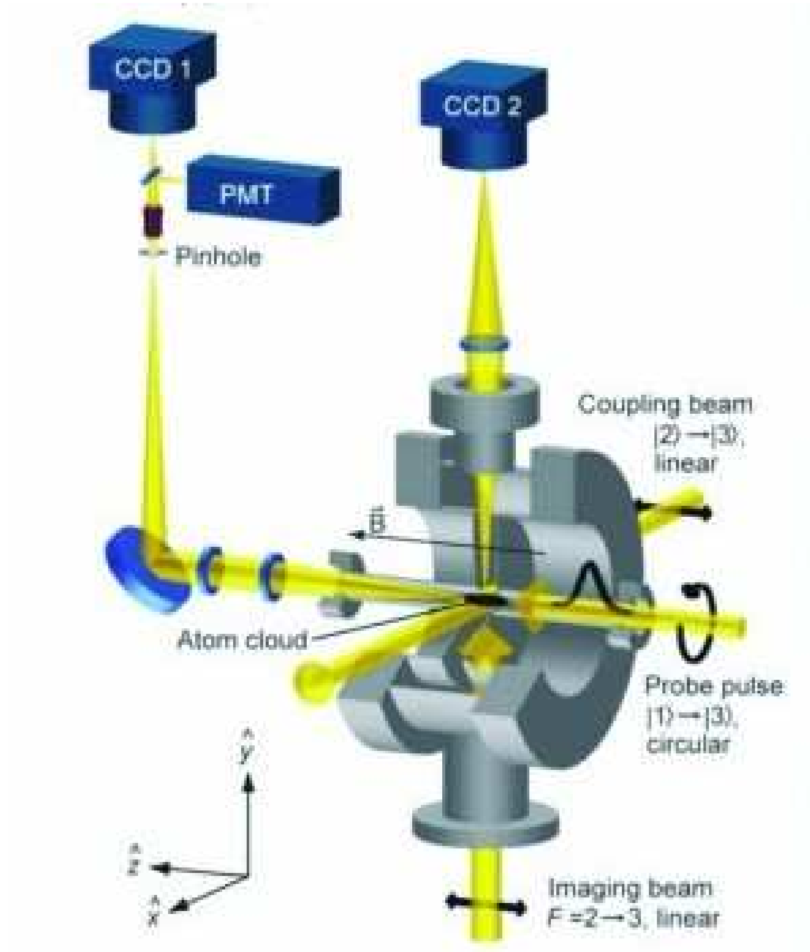

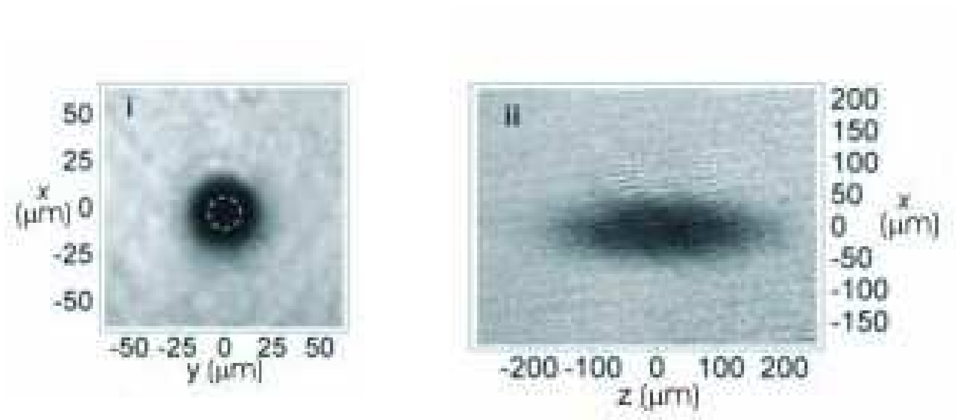

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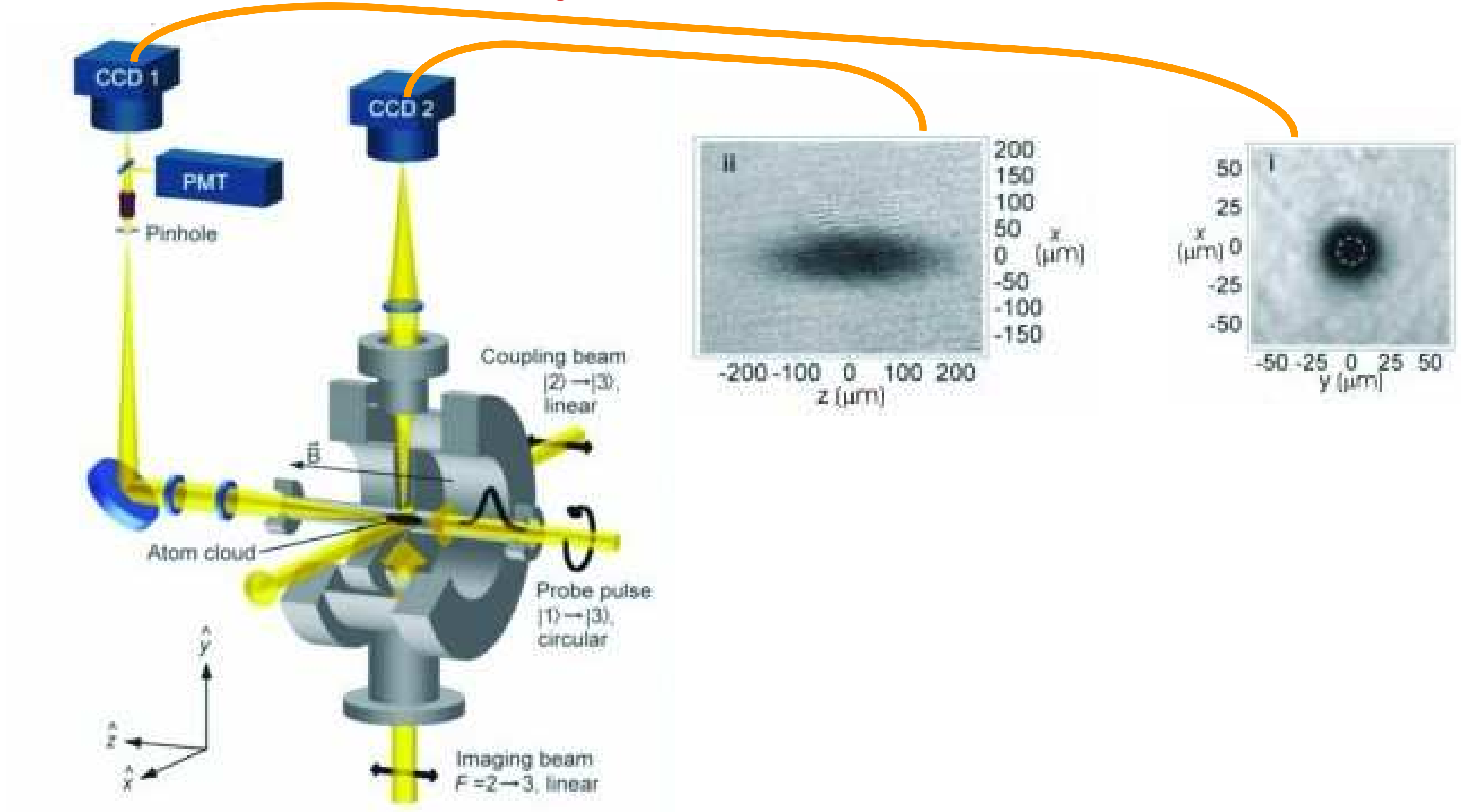

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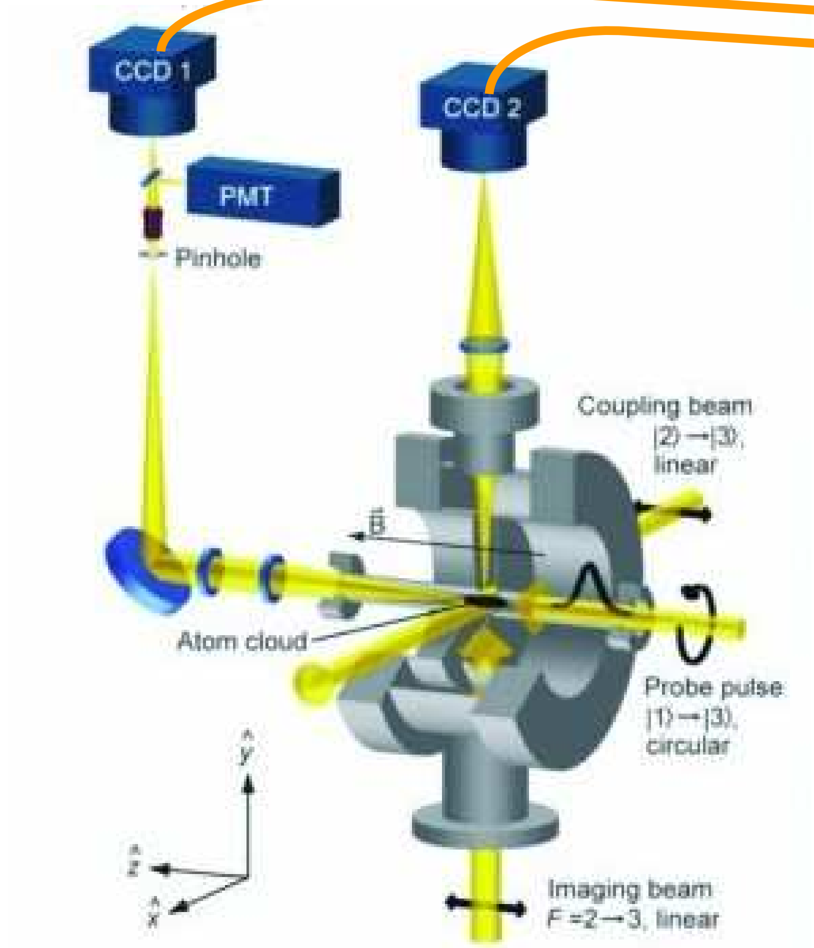

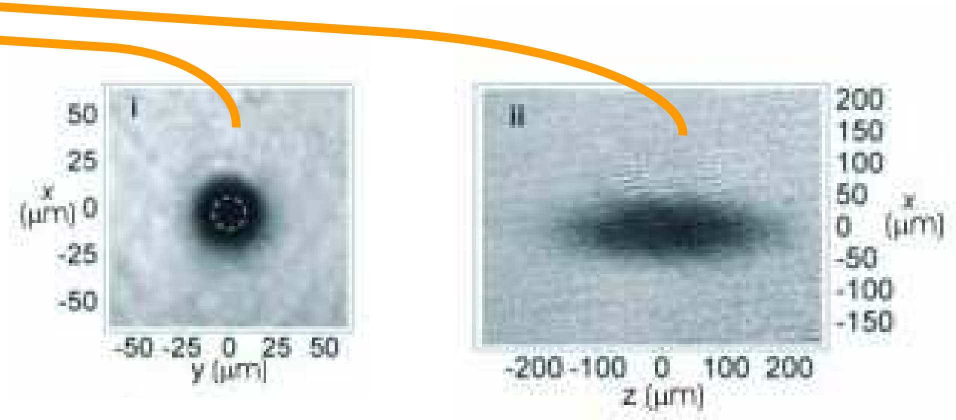

- the cloud is *macroscopic*
- basically, we see the *thermal distribution*
- a cigar shape: *prolate rotational ellipsoid*
- diffuse contours: *Maxwell – Boltzmann distribution in a parabolic potential*

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Particle Velocity (Momentum) Distribution

The procedure is similar, do it quickly:

$$f(\boldsymbol{p}) = \langle \boldsymbol{p} | \rho | \boldsymbol{p} \rangle = \langle \boldsymbol{p} | \rho_{\text{BEC}} | \boldsymbol{p} \rangle + \langle \boldsymbol{p} | \rho_{\text{THERM}} | \boldsymbol{p} \rangle$$

$$= \langle \boldsymbol{p} | 000 \rangle \frac{1}{\mathrm{e}^{-\beta\mu} - 1} \langle 000 | \boldsymbol{p} \rangle + \sum_{\tilde{\nu} \neq (000)} \langle \boldsymbol{p} | \tilde{\nu} \rangle \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}} - \mu)} - 1} \langle \tilde{\nu} | \boldsymbol{p} \rangle$$

$$= \underbrace{\left| \breve{\phi}_{000}(\boldsymbol{p}) \right|^2 \frac{1}{\mathrm{e}^{-\beta\mu} - 1}}_{\text{known} \quad f_{\text{BEC}}(\boldsymbol{r})} + \underbrace{\sum_{\tilde{\nu} \neq (000)} \left| \breve{\phi}_{\tilde{\nu}}(\boldsymbol{p}) \right|^2 \frac{1}{\mathrm{e}^{\beta(E_{\tilde{\nu}} - \mu)} - 1}}_{\text{laborious} \quad f_{\text{THERM}}(\boldsymbol{r})}$$

$$f_{\text{BEC}}(\boldsymbol{p}) = \left| \breve{\phi}_{0x}(p_x) \right|^2 \left| \breve{\phi}_{0y}(p_y) \right|^2 \left| \breve{\phi}_{0z}(p_z) \right|^2 \frac{1}{\mathrm{e}^{-\beta\mu} - 1}$$

$$\propto \mathrm{e}^{-\frac{p_x^2}{b_{0x}^2} - \frac{p_y^2}{b_{0y}^2} - \frac{p_z^2}{b_{0z}^2}} \frac{1}{\mathrm{e}^{-\beta\mu} - 1}, \qquad b_{0w} = \frac{\hbar}{a_{0w}}$$

Thermal Particle Velocity (Momentum) Distribution

Again, we approximate the thermal distribution by its classical limit.

Boltzmann distribution in an external field:

$$f_B(\boldsymbol{r},\boldsymbol{p}) = \mathrm{e}^{\beta(\mu - W - U(\boldsymbol{r}))}$$

$$f_{\mathrm{THERM}}(\boldsymbol{r}) = \int \mathrm{d}^3\boldsymbol{r} \cdot f_B(\boldsymbol{r},\boldsymbol{p})$$

$$\propto \mathrm{e}^{-\beta W}$$

$$= \mathrm{e}^{-\frac{1}{2}\beta m^{-1}(p_x^2 + p_y^2 + p_z^2)}$$

Two directly observable characteristic lengths

$$b_0 = (b_{0x} b_{0y} b_{0z})^{\frac{1}{3}} = \frac{\hbar}{a_0},$$

$$B_T = 1\Big/\sqrt{\beta m}$$

$$= b_0\sqrt{k_B T/\hbar\tilde{\omega}} \gg b_0$$

$$f_{\mathrm{BEC}}(\boldsymbol{p}) = \left|\breve{\phi}_{0x}(p_x)\right|^2 \left|\breve{\phi}_{0y}(p_y)\right|^2 \left|\breve{\phi}_{0z}(p_z)\right|^2 \frac{\quad}{\mathrm{e}}$$

$$\propto \mathrm{e}^{-\frac{p_x^2}{b_{0x}^2} - \frac{p_y^2}{b_{0y}^2} - \frac{p_z^2}{b_{0z}^2}} \frac{1}{\mathrm{e}^{-\beta\mu} - 1},$$

$b_{0w} =$

<u>Remarkable:</u>

$$B_T = b_0\sqrt{k_B T/\hbar\tilde{\omega}} \gg b_0$$

$$R_T = a_0\sqrt{k_B T/\hbar\tilde{\omega}} \gg a_0$$

Thermal and condensate lengths in the same ratio for positions and momenta

BEC observed by TOF in the velocity distribution

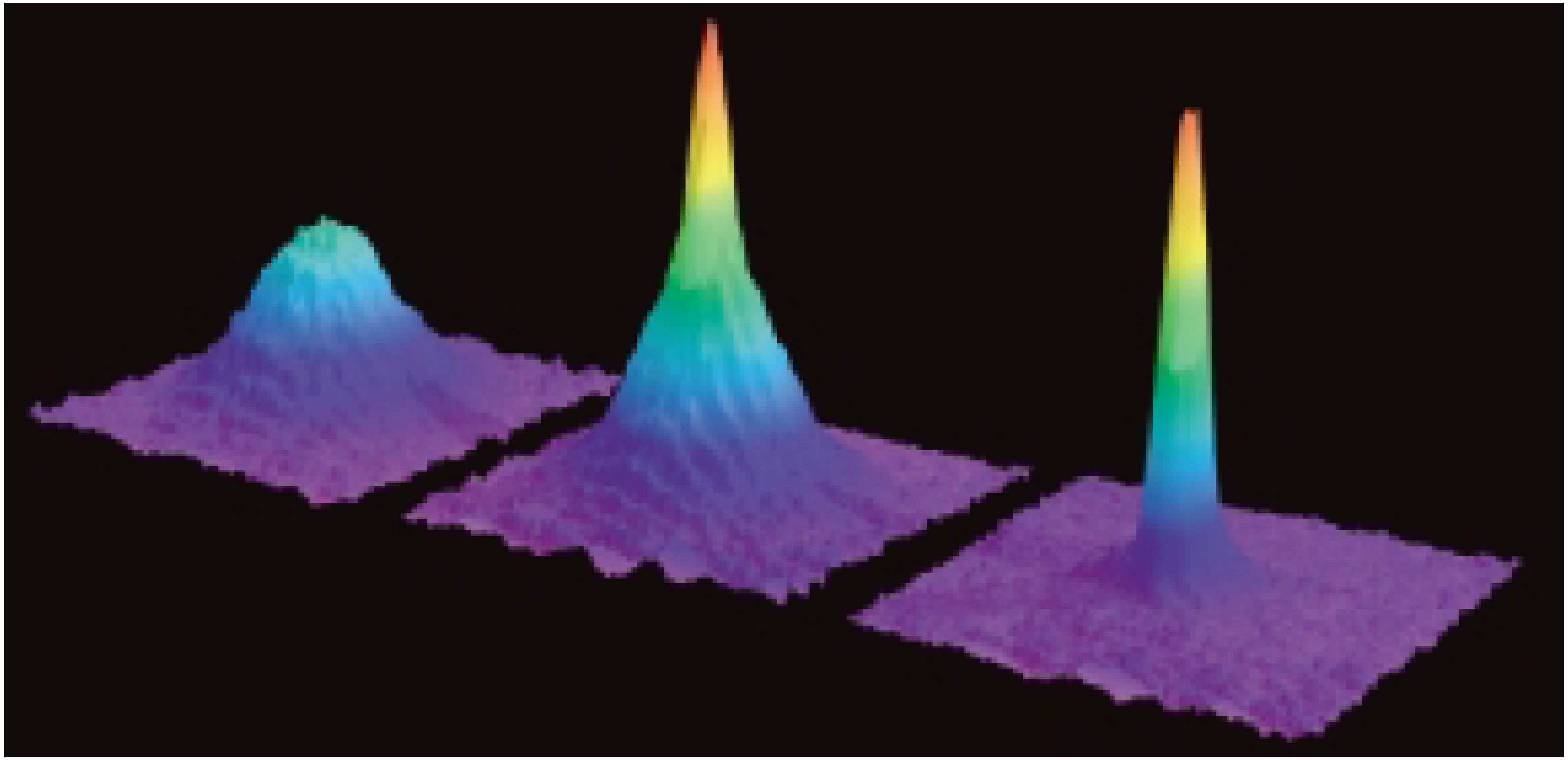


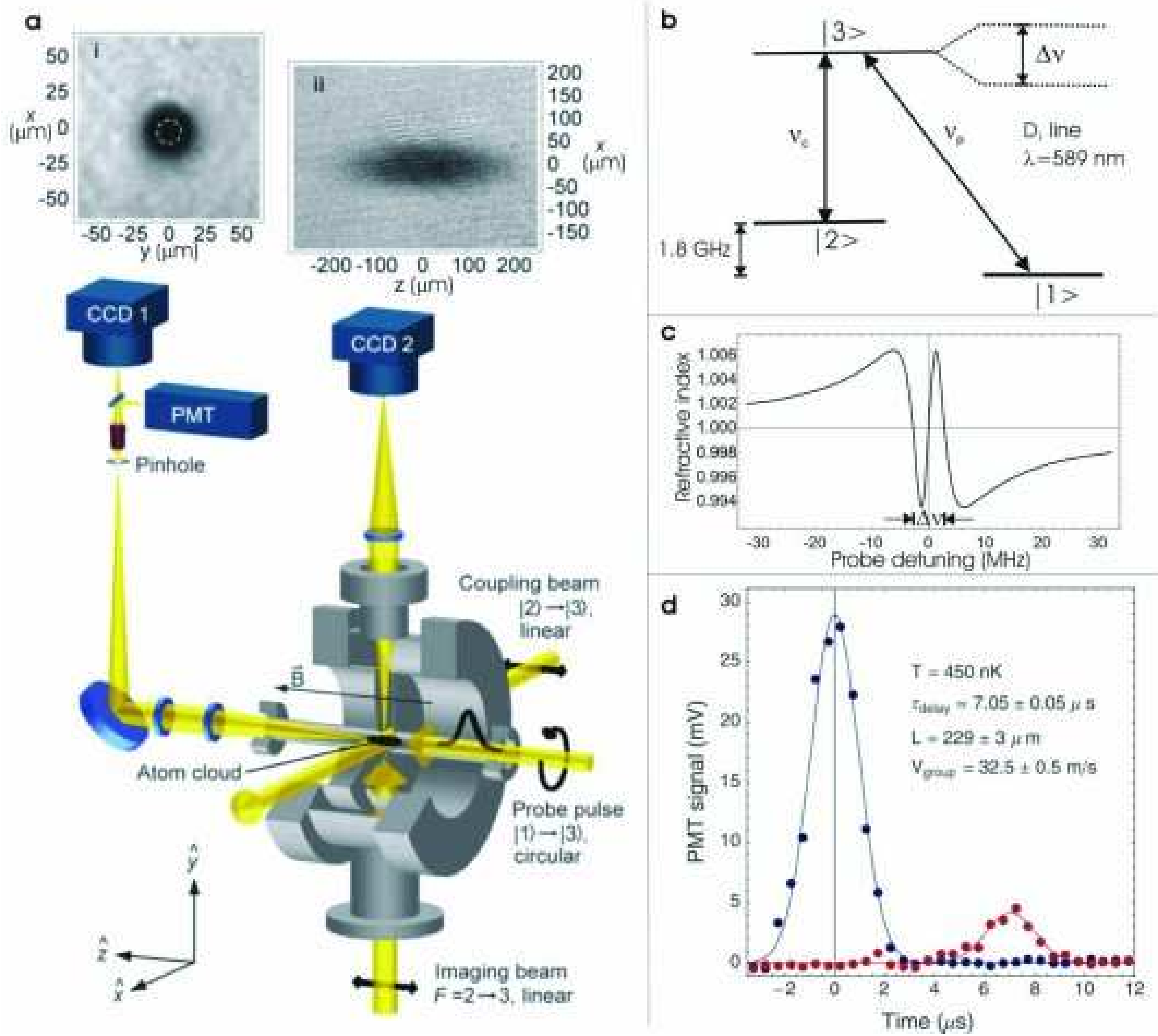
Figure 7. Observation of Bose-Einstein condensation by absorption imaging. Shown is absorption vs. two spatial dimensions. The Bose-Einstein condensate is characterized by its slow expansion observed after 6 ms time-of-flight. The left picture shows an expanding cloud cooled to just above the transition point; middle: just after the condensate appeared; right: after further evaporative cooling has left an almost pure condensate. The total number of atoms at the phase transition is about 7×10^5, the temperature at the transition point is 2 μK.

Qualitative features: ♠ all Gaussians

♠wide vz.narrow

♠ isotropic vs. anisotropic

The end


a
i
ii
x (μm)
y (μm)
z (μm)
CCD 1
CCD 2
PMT
Pinhole
Coupling beam
|2⟩→|3⟩,
linear
Atom cloud
Probe pulse
|1⟩→|3⟩,
circular
Imaging beam
F =2→3, linear
b
|3>
|2>
|1>
Δν
1.8 GHz
D1 line
λ=589 nm
c
Refractive index
Probe detuning (MHz)
Δν
d
PMT signal (mV)
Time (μs)
T = 450 nK
τdelay = 7.05 ± 0.05 μs
L = 229 ± 3 μm
Vgroup = 32.5 ± 0.5 m/s