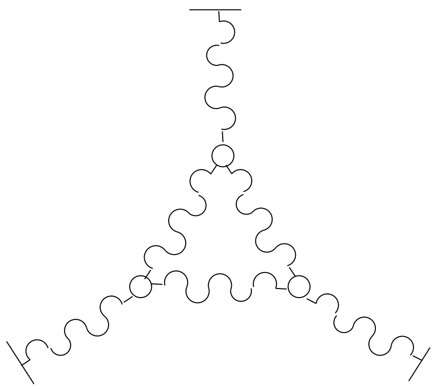
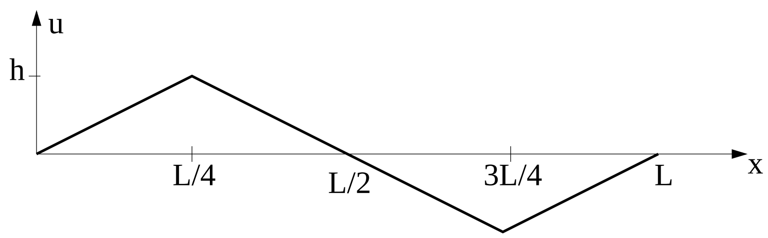
Oscillations, waves, optics 2007/2008 – problems, part 1.

1. Compose computer program, which finds and writes into the file displacement time dependence of anharmonic damped oscillations. Reverse force dependence on the displacement is $F_r(x) = -kx(1 + \alpha x^2)$, damping force depends on the velocity as $F_o(x) = -\beta \dot{x}$. Spring constants k, α, β, system weight m and initial conditions will be parameters of the program. Find dependence of system on the parameters, especially for harmonic undamped oscillations, harmonic damped oscillations, positive and negative values of anharmonic force parameter α. Graphs of time dependence of the mentioned states will be part of solution..

2. Find eigen-frequencies and eigen-modes of shown system of transverse oscillations (perpendicular to the picture plane). All balls have weight m and all spring transverse spring constant k.



3. A string of length L is tensile between solid points and displaced from its equilibrium position as shown in the figure. The string is released at time $t = 0$. Find function $u(x,t)$ describing time evolution of string shape. Wave phase velocity in the string is c.



4. A simple seismometer consists from weight hanging on a spring fastened to a solid construction fastened to ground. Weight movement is critically damped and vertical movement of the weight is registered. Show that measured amplitude of steady oscillations, which are caused by the ground vertical oscillations $H \cos \omega t$, is given by the relation $A/H = (\omega/2\omega_0)[R(\omega)]^{1/2}$, where ω_0 is angular frequency of weight eigen-oscillations, $R(\omega) = \gamma^2\omega^2/[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]$ is response function and γ is damping constant. Draw graph of A/H as a function of oscillation frequency ω.

5. A thin non-absorbing layer of thickness d and refractive index n_1 is deposited on a non-absorbing substrate of refractive index n. Calculate reflectivity at normal incidence of a system as a function of wavelength. Calculate multiple reflections and suppose that incoming light is coherent and refractive indeces are independent on the wavelength. Propose a way to obtain values of n, n_1, d from measured spectral dependence. Plot graph of reflectivity spectral dependence for parameters $n = 1.5$, $n_1 = 1.3$ a $d = 150\,\text{nm}$ in the visible light spectrum.

6. A diffraction screen is put between point monochromatic light source and an observing screen. The diffraction and observing screens are parallel. The diffraction screen is perforated with rectangle holes of sides a_1, a_2. Holes are aligned in a rectangle lattice, their centers have positions $\vec{R} = n_1\vec{d_1} + n_2\vec{d_2}$, where $0 \leq n_1 < N_1$, $0 \leq n_2 < N_2$ a n_1, n_2 are integers. Numbers N_1, N_2 describe macroscopic system size. The vectors $\vec{d_1}$, $\vec{d_2}$ are perpendicular and are parallel to rectangle sides. Calculate final amplitude and intensity on the observing screen. Show, that the result is a product of two terms, one of them depending only on the holes alignment (geometric factor) and the other only on the hole shape (structure factor). Plot graph of intensity distribution on the observing screen for suitably chosen parameters.

7. Plot optical schemes of three basic telescopes – Galilei, Kepler and Newton. For Galilei and Kepler telescope solve following problem. The telescope is focused in such a way, that we can see sharp Moon image by an eye accommodated to infinity. The screen is put in the distance d from the ocular. How much the ocular with focal distance f_{oc} has to be displaced from the initial position to observe sharp image of Moon on the screen? How big will be the Moon image, if the objective focal distance is f_{ob}? Solve general at first and then with the values $f_{ok} = 2$ cm, $f_{ob} = 30$ cm, $d = 16$ cm.

8. Small fish is in the cylindrical fish-tank, with curvature radius $R = 40\,\text{cm}$ oriented vertically, at a line connecting center of a fish-tank and observer at distance $x = 7{,}5\,\text{cm}$ from center of a fish-tank and is flowing with velocity $v = 3{,}0\,\text{cm.s}^{-1}$ perpendicular to this connecting line horizontally. Find image position of the fish, its magnification and the image velocity. The refractive index of a water is $n = 1{,}33$, glass fish-tank can be neglected in the first order approximation.

Figure 1: Two sharks in a cylindrical fish-tank. Illustrative photo.