



SIGNÁLY A SOUSTAVY V MATEMATICKÉ BIOLOGII



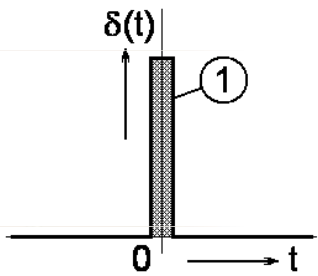
prof. Ing. Jiří Holčík, CSc.

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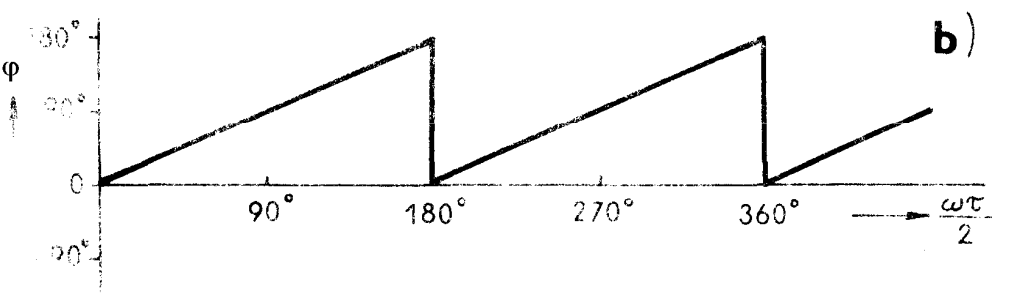
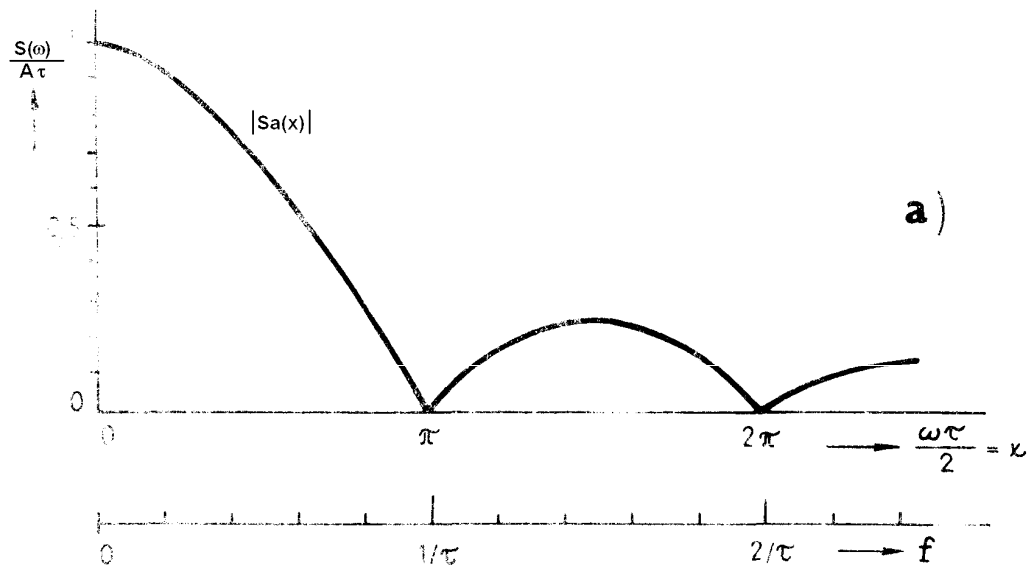
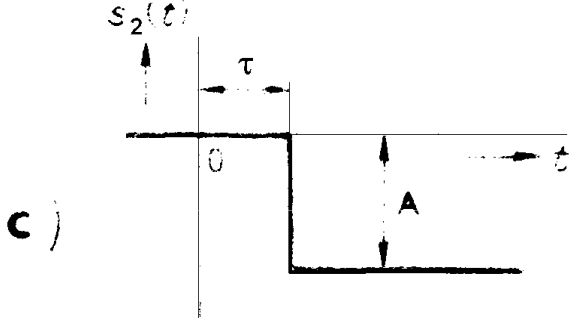
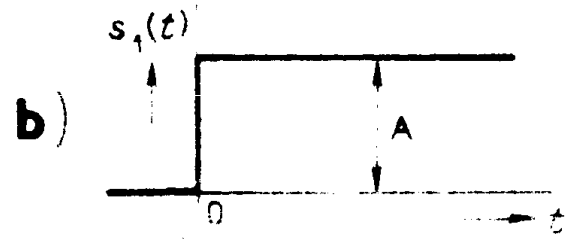
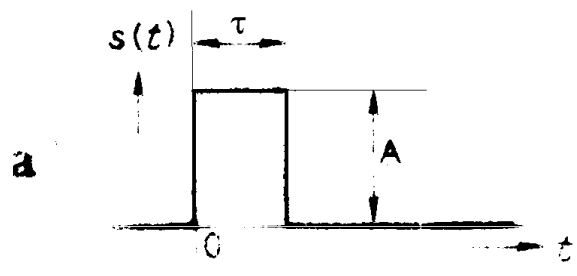
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IV. KONVOLUCE & VZORKOVACÍ TEORÉM

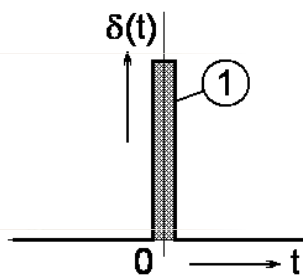
SPEKTRUM DIRACOVA IMPULZU



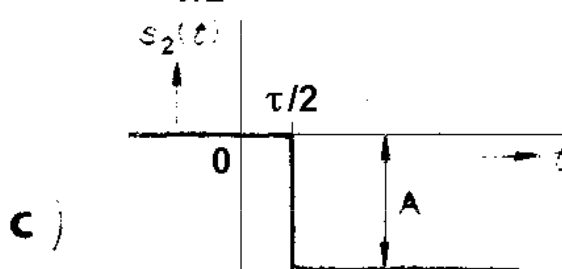
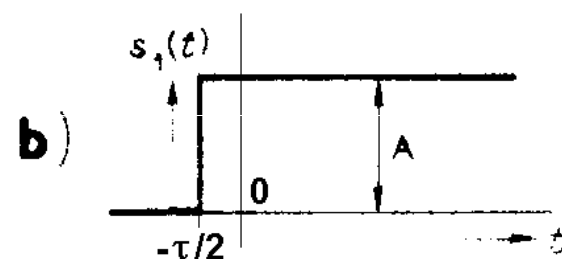
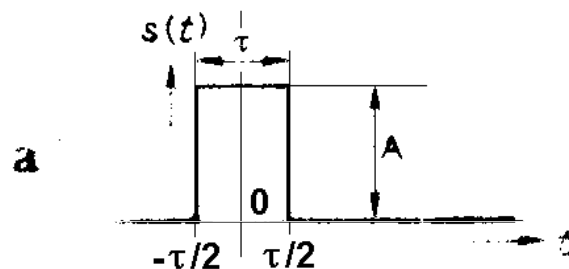
$\tau \rightarrow 0$
 $A \rightarrow \infty$
 $A \cdot \tau = 1$



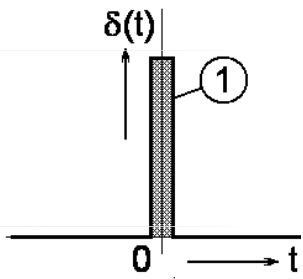
SPEKTRUM DIRACOVA IMPULZU



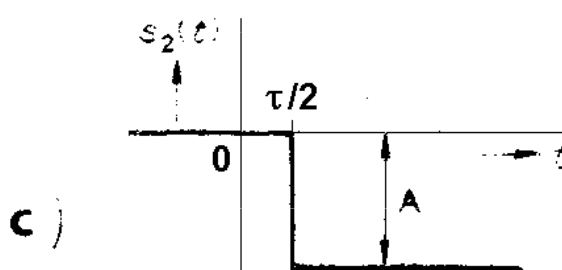
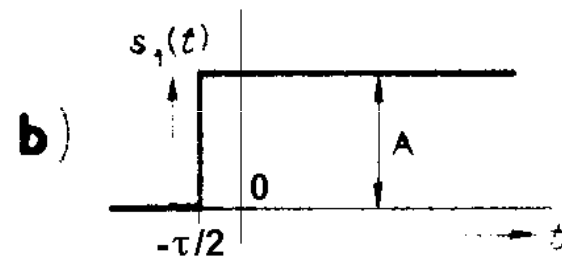
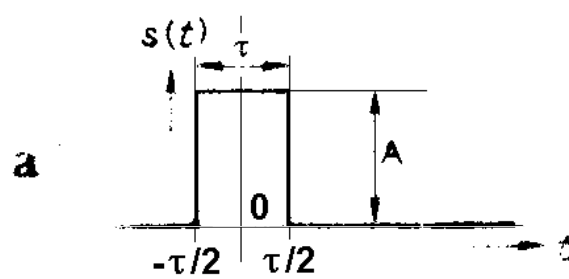
$$\begin{aligned}\tau &\rightarrow 0 \\ A &\rightarrow \infty \\ A \cdot \tau &= 1\end{aligned}$$



SPEKTRUM DIRACOVA IMPULZU

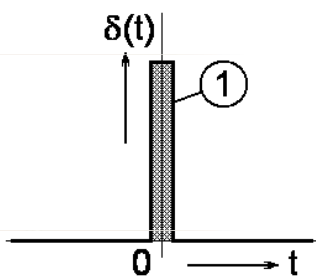


$$\begin{aligned} \tau &\rightarrow 0 \\ A &\rightarrow \infty \\ A \cdot \tau &= 1 \end{aligned}$$

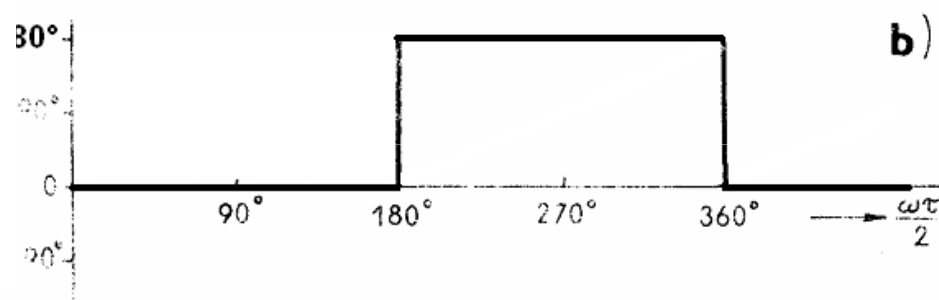
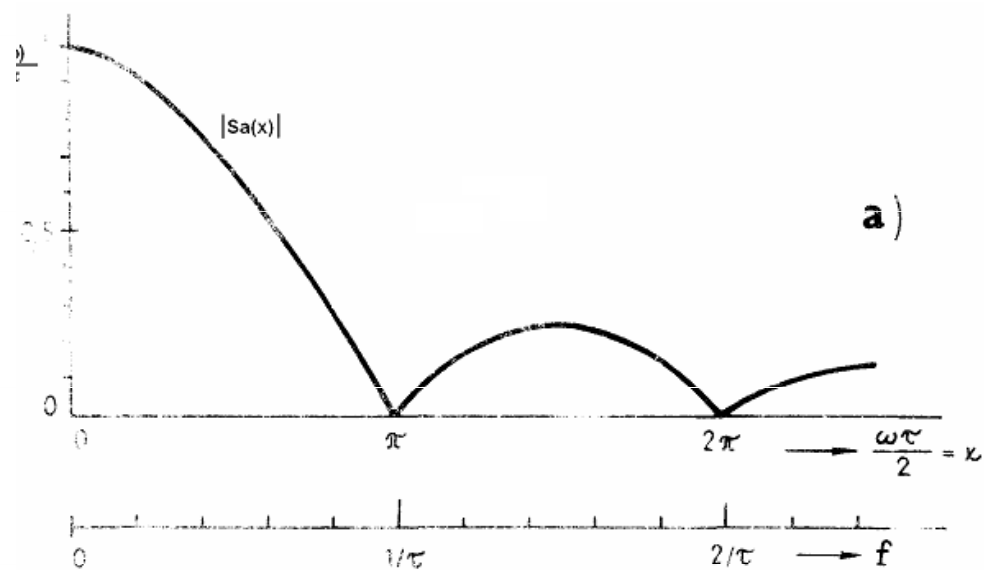
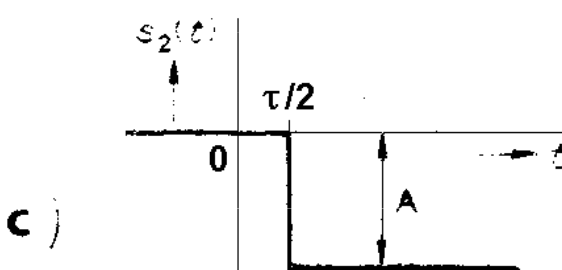
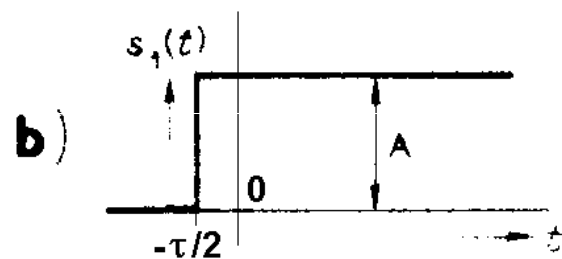
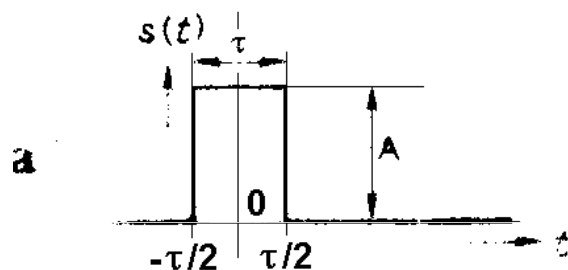


$$\begin{aligned} S(\omega) &= A \cdot \left(\frac{1}{j\omega} \cdot e^{j\omega\tau/2} - \frac{1}{j\omega} \cdot e^{-j\omega\tau/2} \right) = \\ &= A \cdot \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega} = \dots = \\ &= \frac{2A}{\omega} \cdot \sin \frac{\omega\tau}{2} = \dots = A \cdot \tau \cdot \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}. \end{aligned}$$

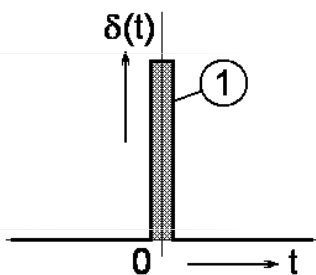
SPEKTRUM DIRACOVA IMPULZU



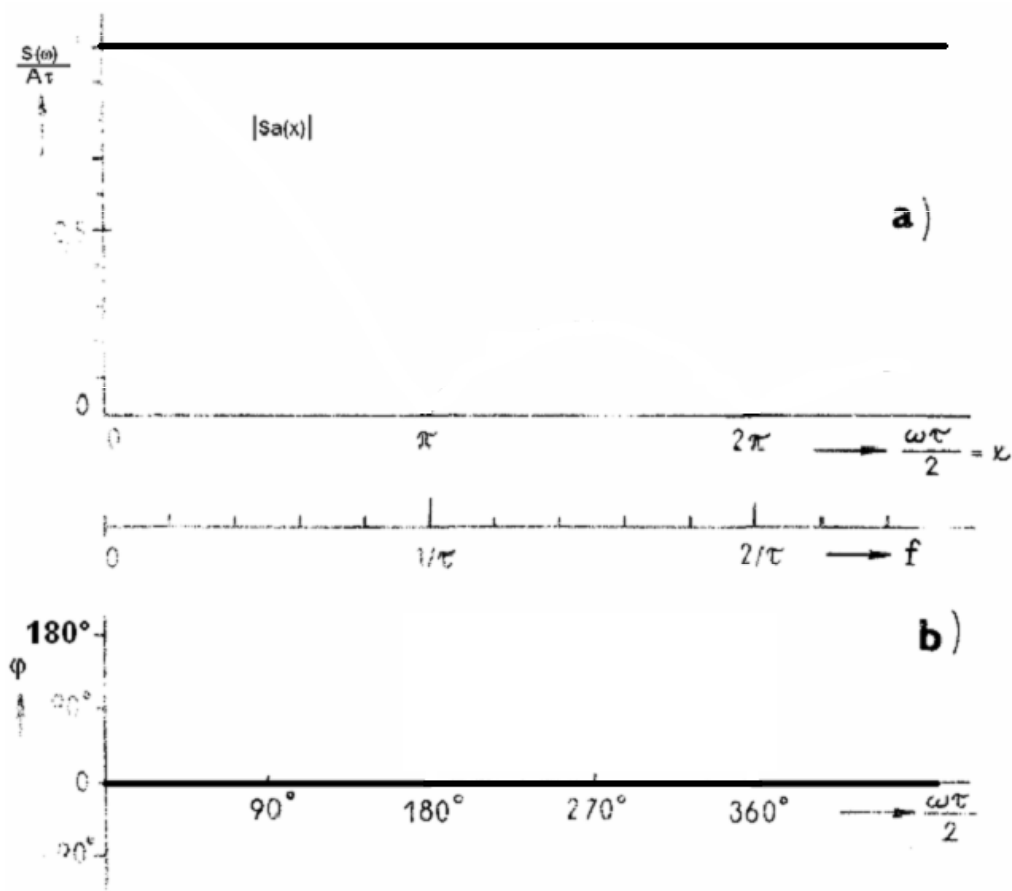
$$\begin{aligned} \tau &\rightarrow 0 \\ A &\rightarrow \infty \\ A \cdot \tau &= 1 \end{aligned}$$



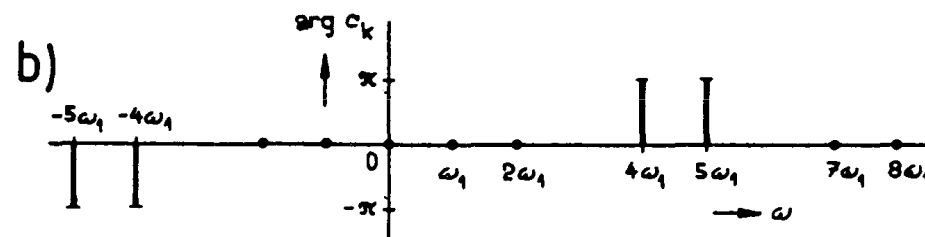
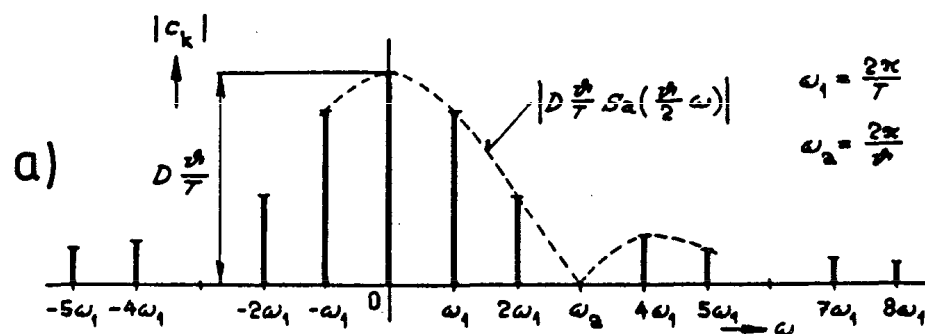
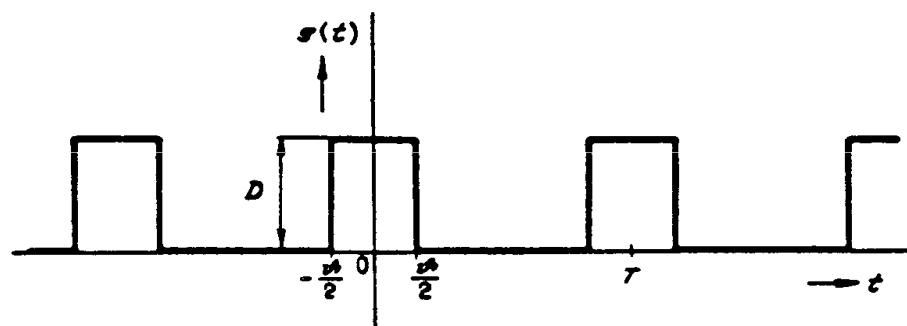
SPEKTRUM DIRACOVA IMPULZU



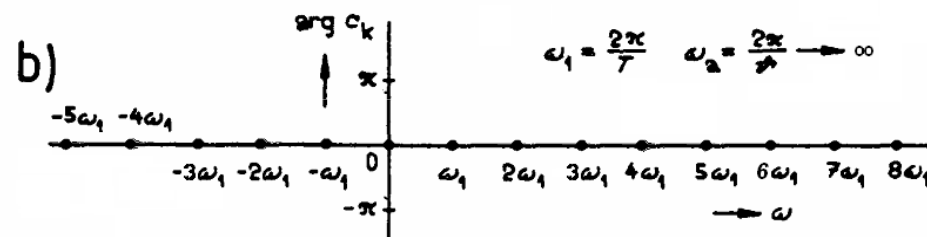
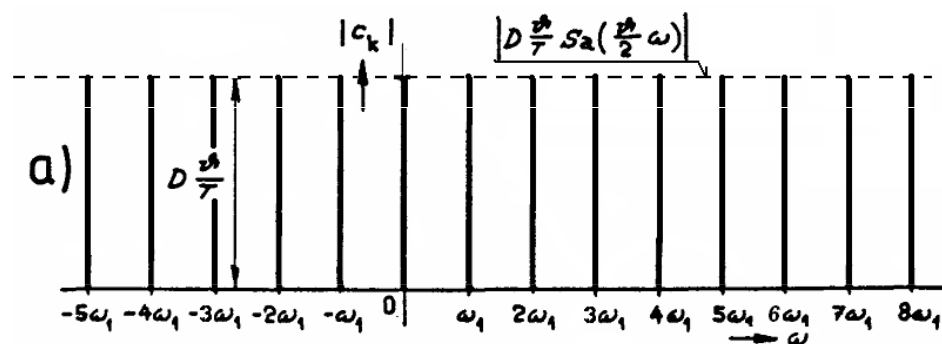
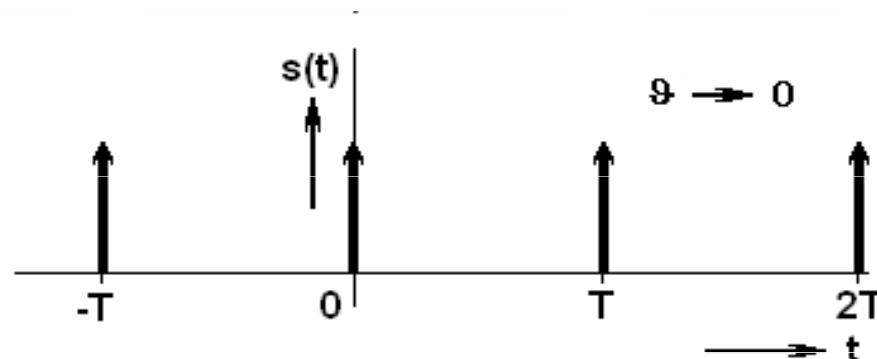
$$\begin{aligned}\tau &\rightarrow 0 \\ A &\rightarrow \infty \\ A \cdot \tau &= 1\end{aligned}$$



SPEKTRUM PULZU DIRACOVÝCH IMPULZŮ



SPEKTRUM PULZU DIRACOVÝCH IMPULZŮ



KONVOLUCE

$$\begin{aligned} s_1(t) * s_2(t) &= \int_{-\infty}^{\infty} s_1(\tau) \cdot s_2(t - \tau) \cdot d\tau = \\ &= \int_{-\infty}^{\infty} s_1(t - \tau) \cdot s_2(\tau) \cdot d\tau \quad \approx S_1(\omega) \cdot S_2(\omega) \end{aligned}$$

Důkaz:

$$\begin{aligned} s_1(t) * s_2(t) &= \int_{-\infty}^{\infty} s_1(\tau) \cdot s_2(t - \tau) \cdot d\tau = \left. \begin{array}{l} x = t - \tau \\ \tau = t - x \\ d\tau = -dx \end{array} \right| = \\ &= - \int_{\infty}^{-\infty} s_2(x) \cdot s_1(t - x) \cdot dx = s_2(t) * s_1(t) \end{aligned}$$

KONVOLUCE

Distributivní zákon:

$$f_1(t) * [f_2(t) * f_3(t)] = [f_1(t) * f_2(t)] * f_3(t)$$

Asociativní zákon:

$$f_1(t) * [f_2(t) + f_3(t)] = f_1(t) * f_2(t) + f_1(t) * f_3(t)$$

KONVOLUCE

Zákon o posunu v čase

Je – li

$$f_1(t) * f_2(t) = c(t),$$

pak

$$f_1(t) * f_2(t - T) = c(t - T),$$

$$f_1(t - T) * f_2(t) = c(t - T)$$

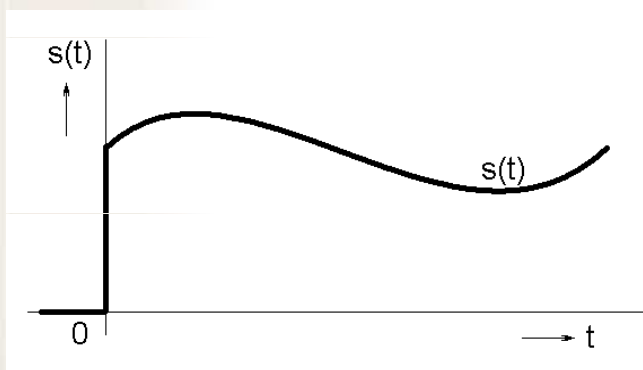
a

$$f_1(t - T_1) * f_2(t - T_2) = c(t - T_1 - T_2)$$

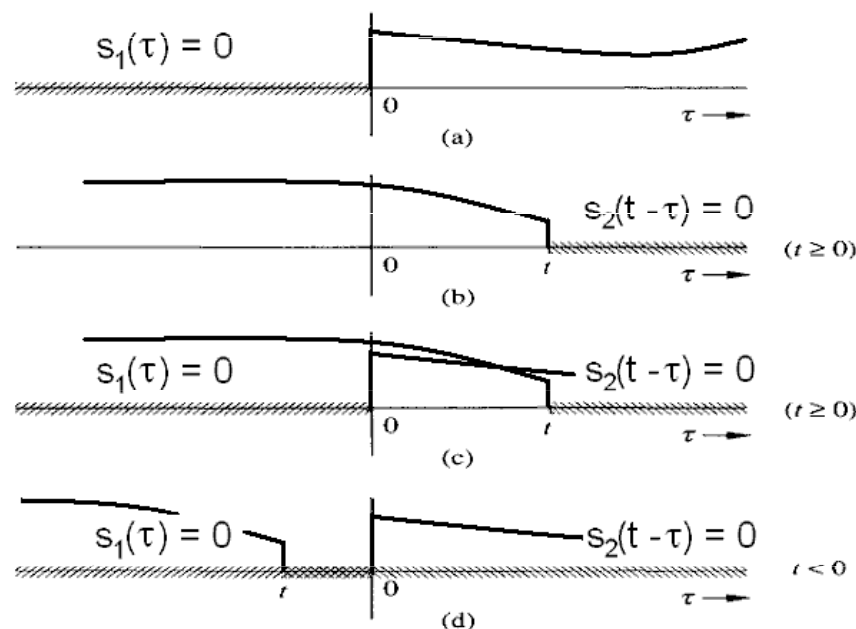
KONVOLUCE

Konvoluce kauzálních signálů:

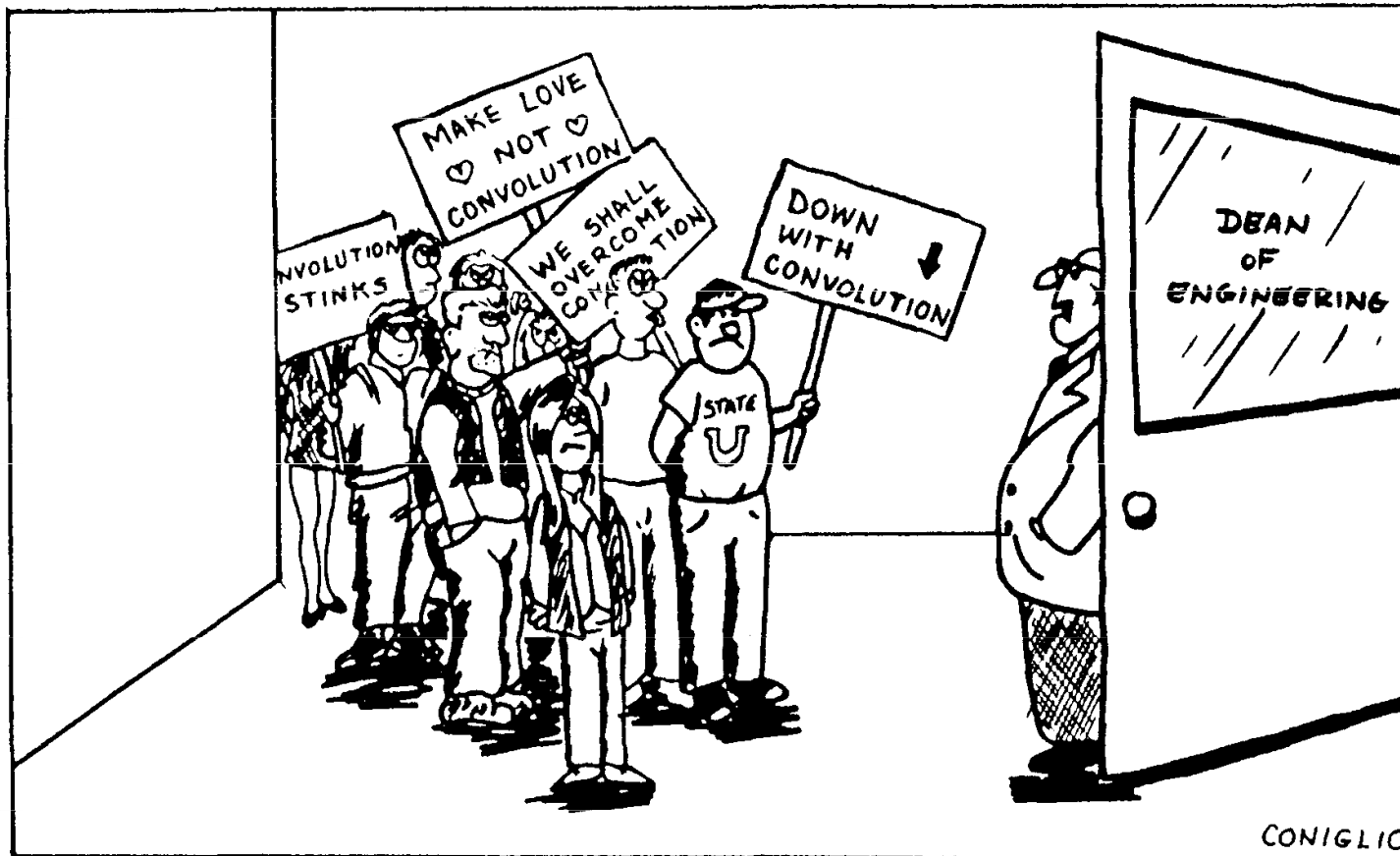
Pro kauzální signály platí $s(t) = 0$ pro $t < 0$



$$s_1(t) * s_2(t) = \int_0^t s_1(\tau) \cdot s_2(t - \tau) \cdot d\tau$$

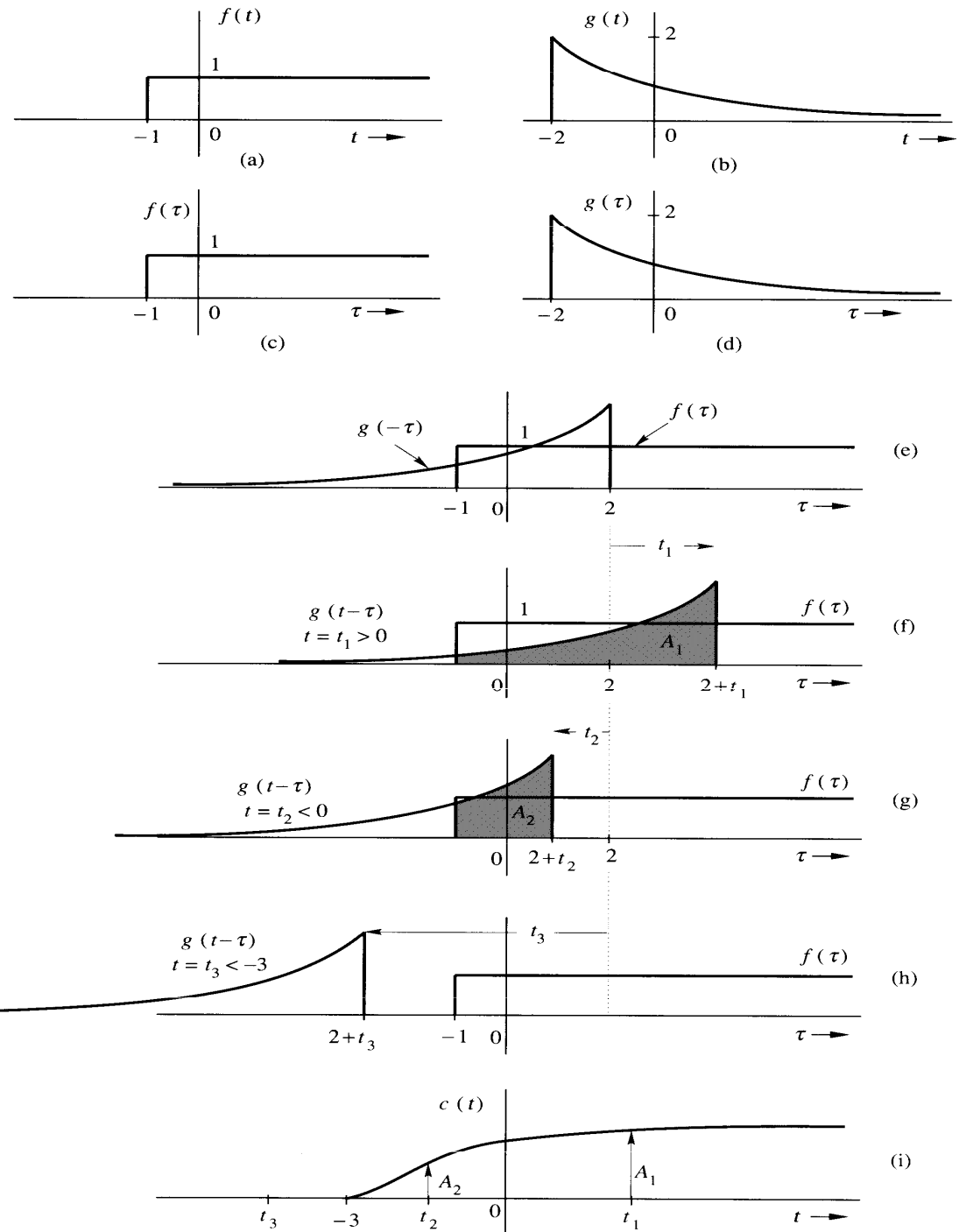


KONVOLUCE

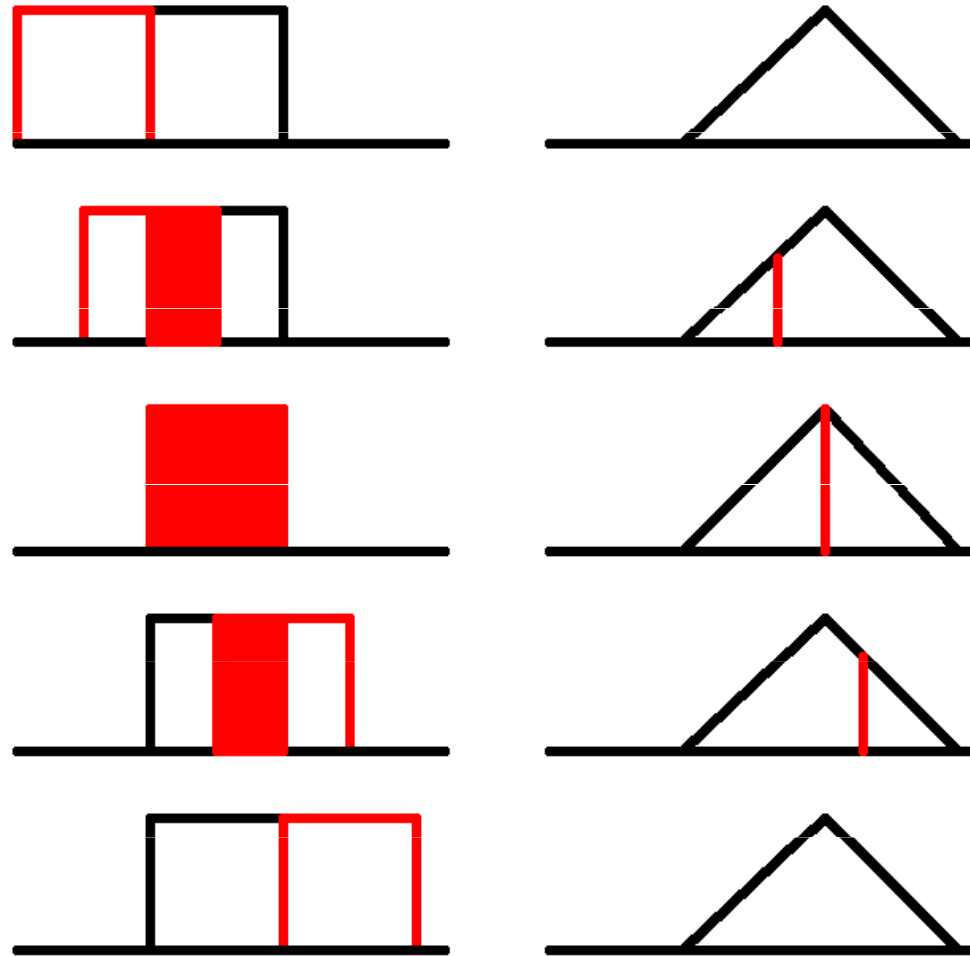


Convolution: its bark is worse than its bite!

KONVOLUCE



KONVOLUCE



KONVOLUCE

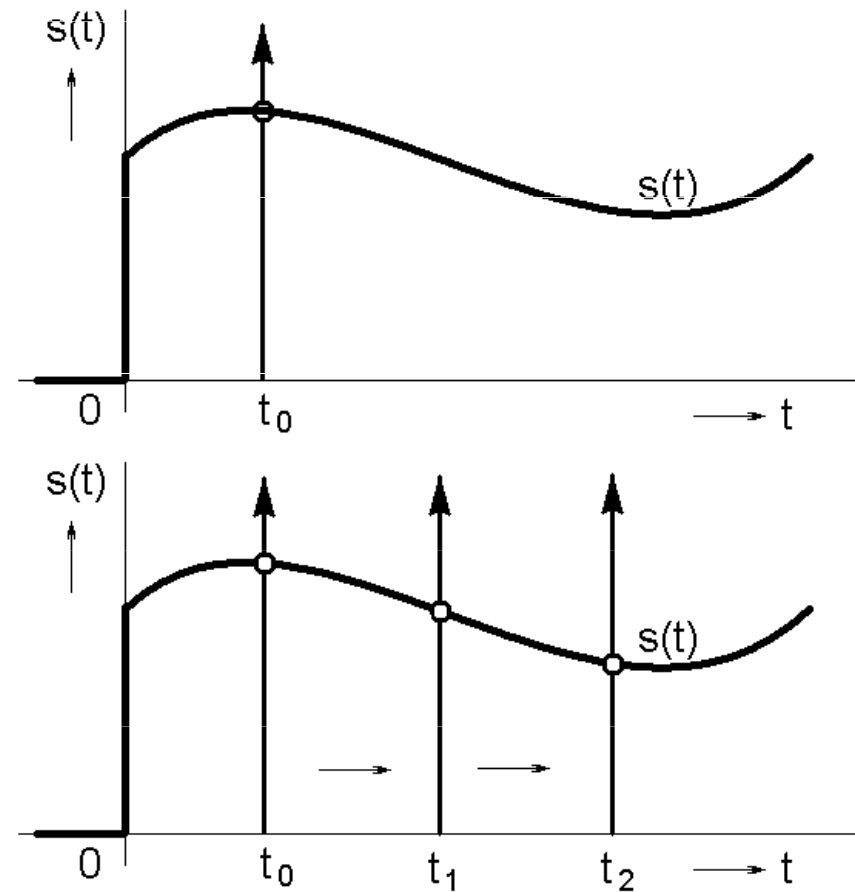
signálu s jednotkovým impulsem

definice:

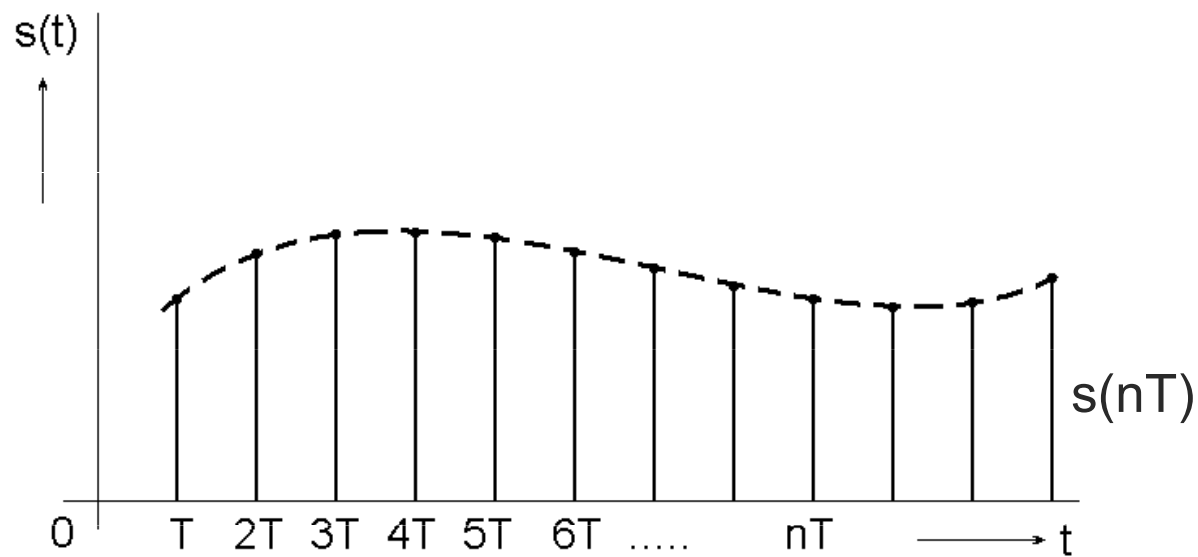
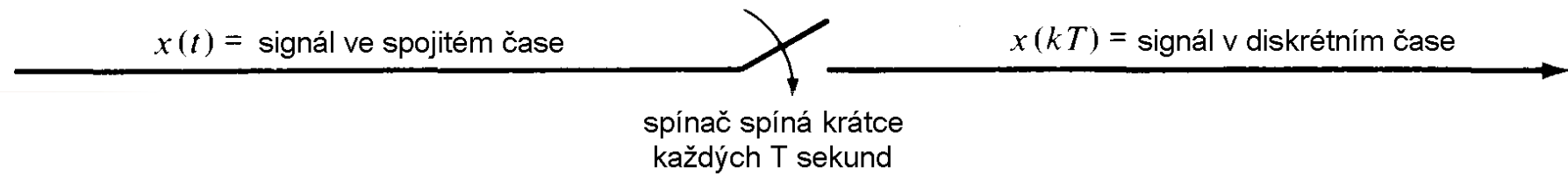
$$\int_{-\infty}^{\infty} s(t) \cdot \delta(t - t_0) dt = s(t_0)$$

konvoluce:

$$s(t) * \delta(t) = \int_{-\infty}^{\infty} s(t) \cdot \delta(t - \tau) d\tau = s(t)$$



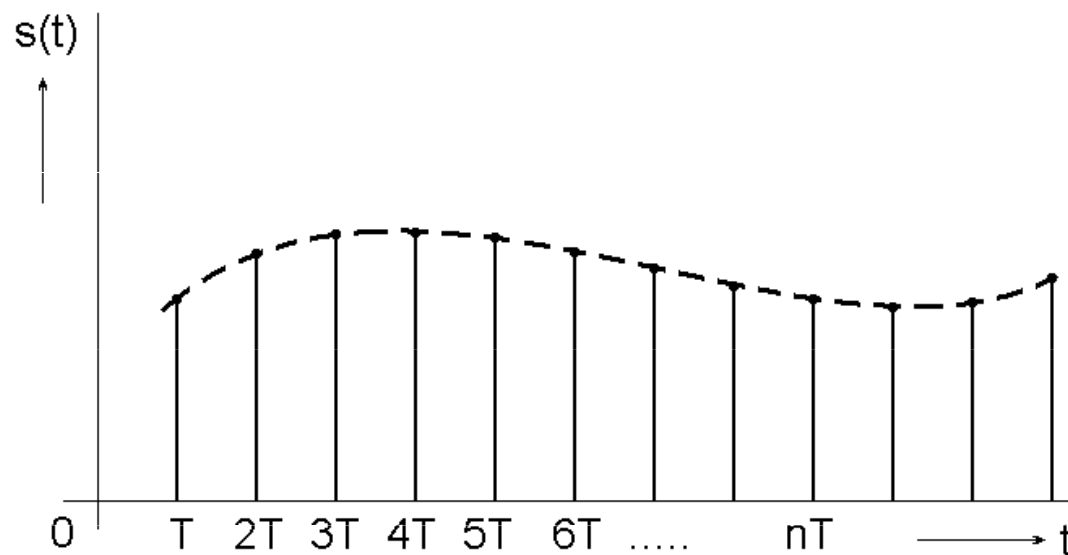
DISKRÉTNÍ SIGNÁL - VZORKOVÁNÍ



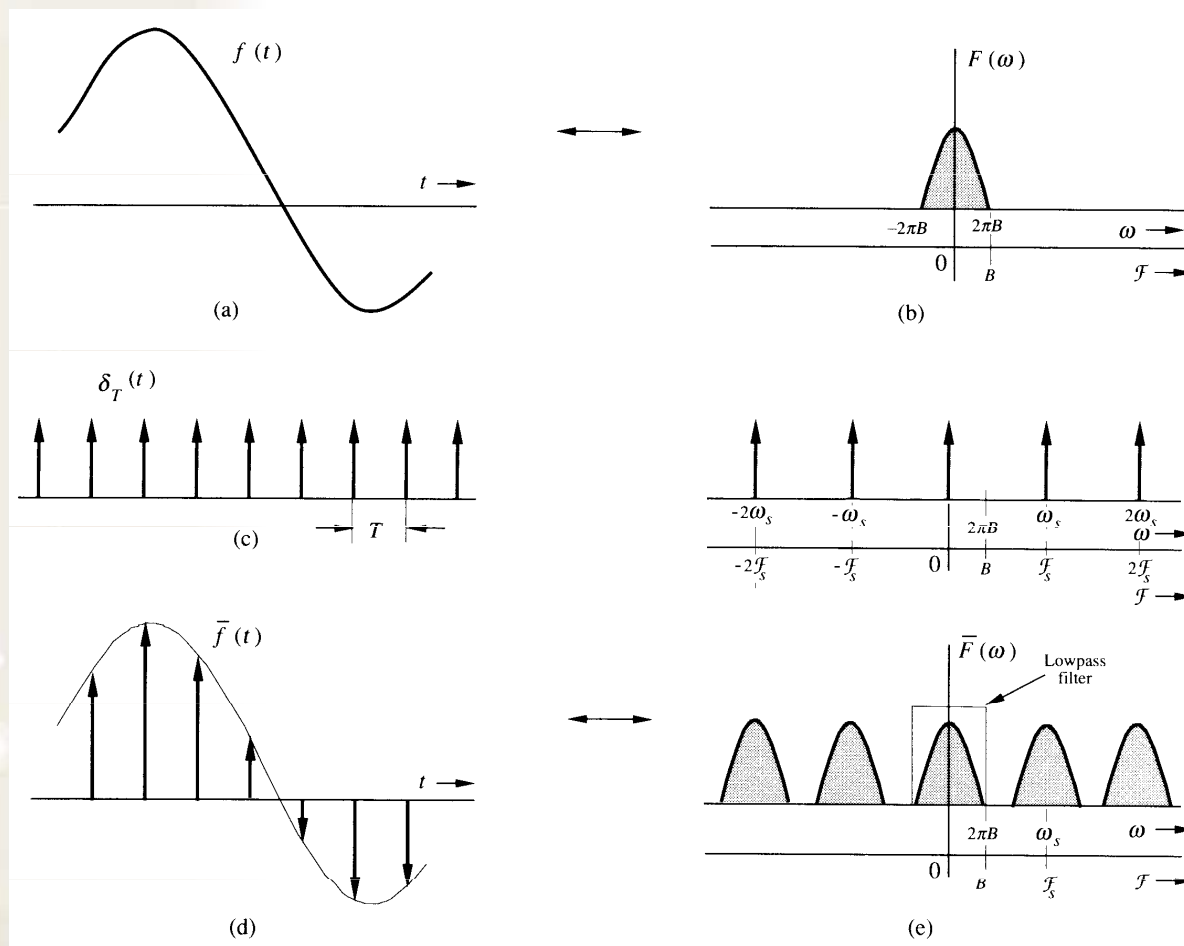
VZORKOVACÍ TEORÉM

$$s(t) \rightarrow s(T_1), s(T_2), s(T_3), \dots, s(T_n), \dots$$

$$s(t) \rightarrow s(T), s(2T), s(3T), \dots, s(nT), \dots$$



VZORKOVACÍ TEORÉM



Vzorkovací frekvence:

$$f_s \geq 2B = f_N,$$

kde B je maximální kmitočet ve vzorkovaném signálu

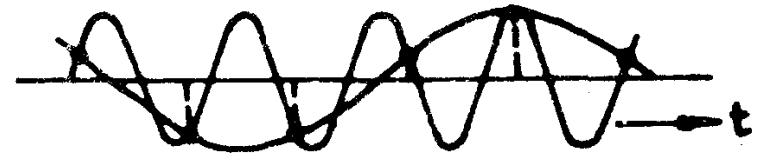
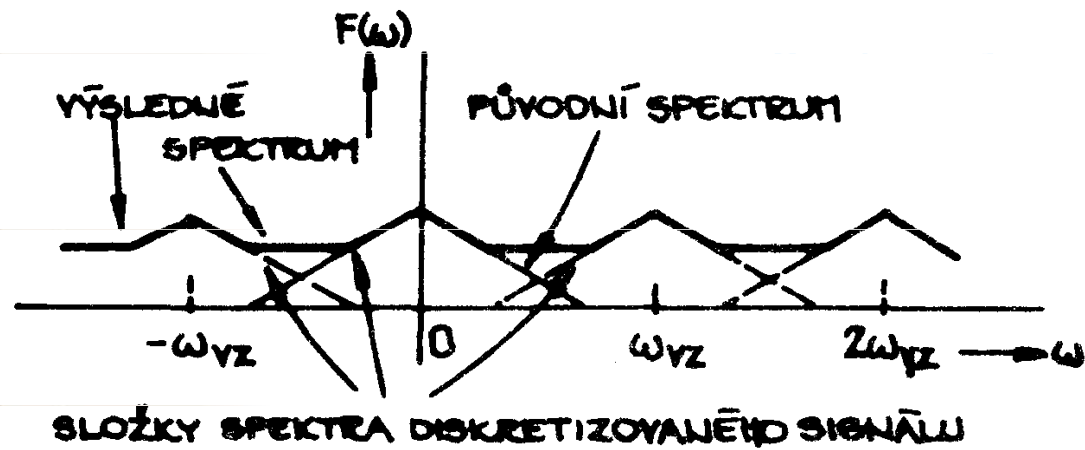
f_N –

Nyquistův, (Shannonův, Kotelnikovův) kmitočet

$$T_N = 1/f_N = 1/2B$$

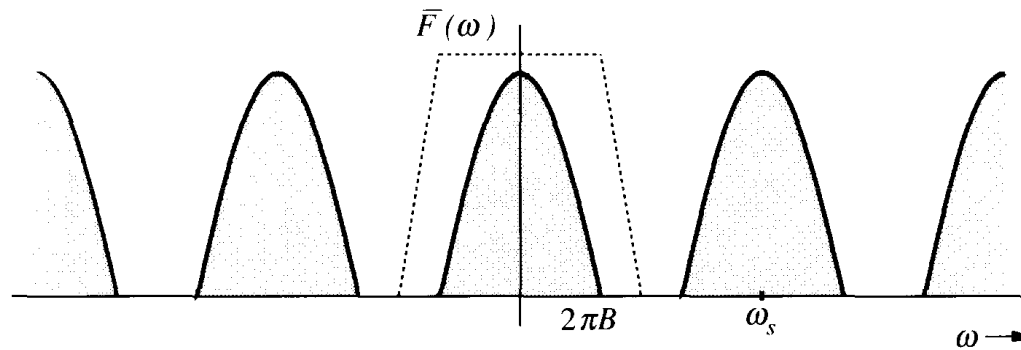
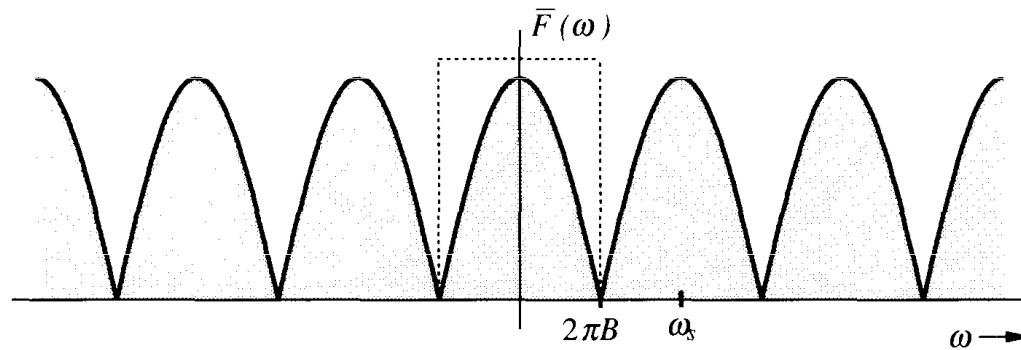
Nyquistův interval (perioda),
vzorkovací interval (perioda)

VZORKOVACÍ TEORÉM



VZORKOVACÍ TEORÉM

Reálné vzo

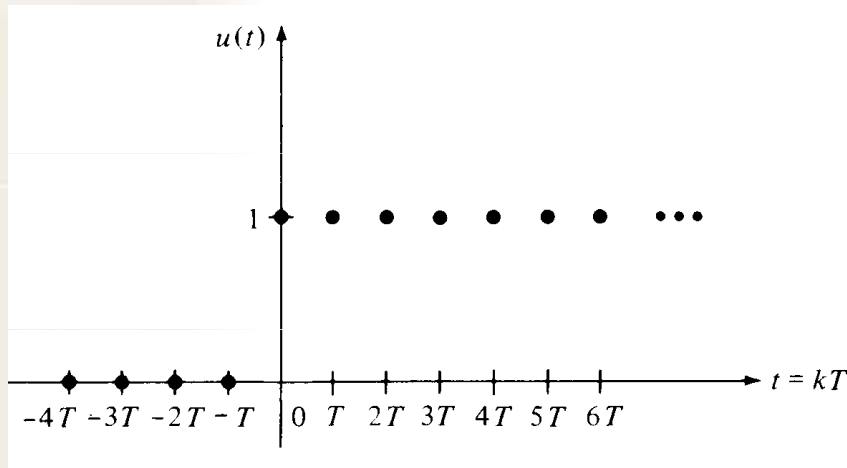


$$f_{sr} = (4 \div 5) \cdot f_N$$

V. DISKRÉTNÍ SIGNÁL

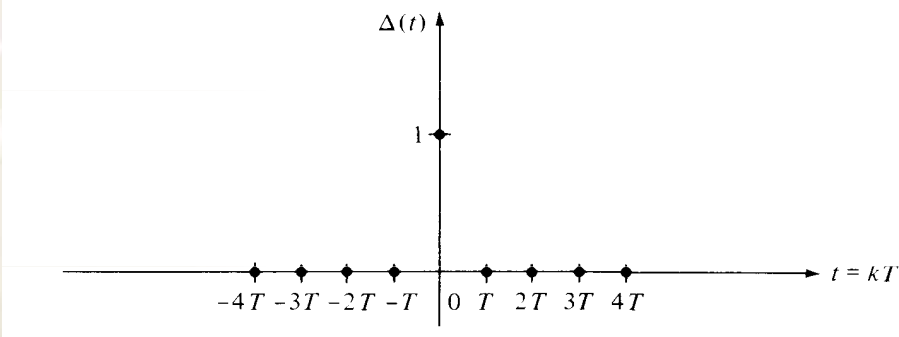
POPIS V ČASOVÉ OBLASTI

JEDNORÁZOVÉ DISKRÉTNÍ SIGNÁLY



☑ jednotkový skok

$$\Sigma(t) = \begin{cases} 0, & t = kT, k = \dots, -2, -1, \\ 1, & t = kT, k = 0, 1, 2, \dots \end{cases}$$



☑ jednotkový impuls

$$\Delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t = kT, k \neq 0 \end{cases}$$

PERIODICKÉ DISKRÉTNÍ SIGNÁLY

- ✓ diskretní signál $x(kT)$ je periodický s periodou NT , když platí

$$x[(k+N)T] = x(kT), \text{ pro } k = 0, \pm 1, \pm 2, \dots$$

- ✓ příklady

- $x(kT) = A \cdot \cos(2\pi k/N)$

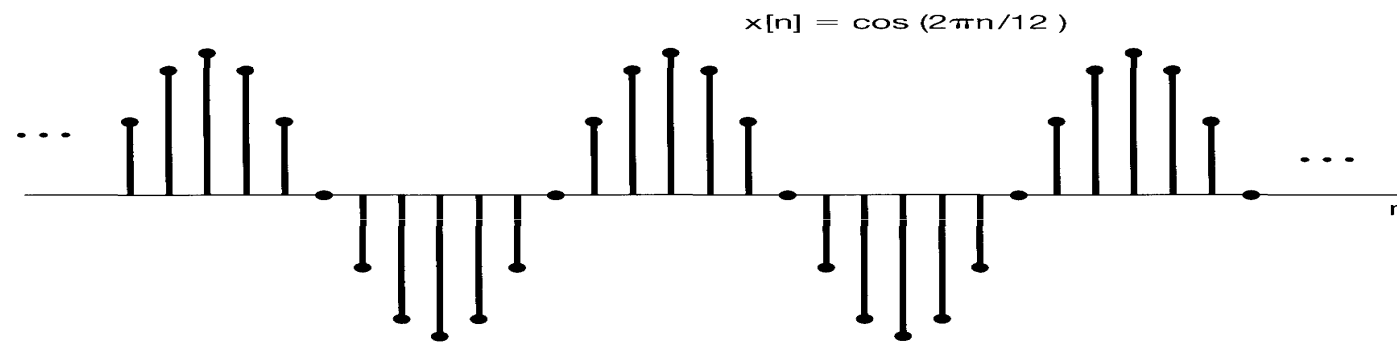
- $x(kT) = A \cdot \sin(2\pi k/N)$

- $x(kT) = A \cdot \exp(j2\pi k/N)$

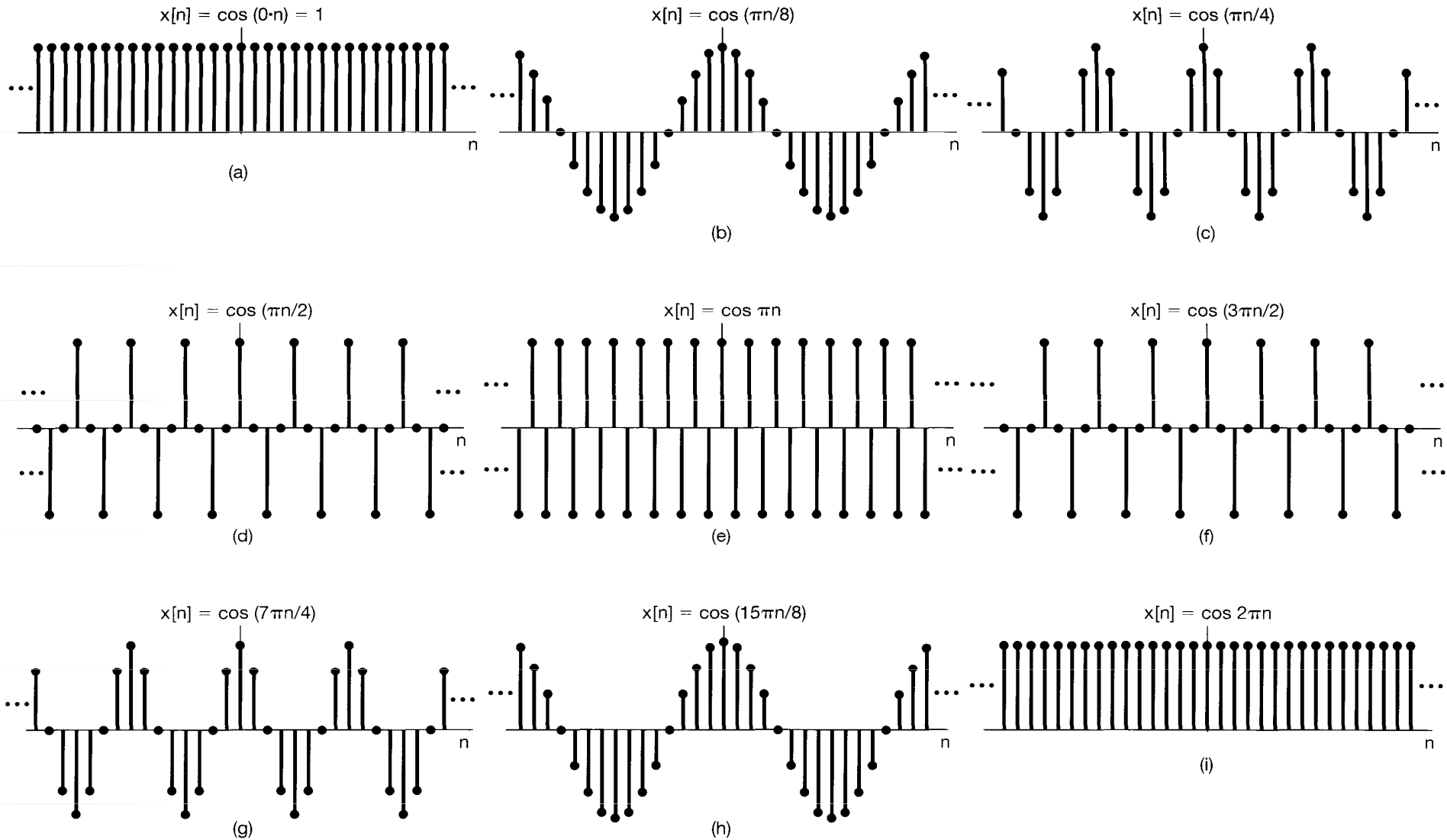
$$x[(k+N)T] = \exp\left(\frac{j2\pi(k+N)}{N}\right) = \exp\left(\frac{j2\pi k}{N}\right) \cdot \exp(j2\pi)$$

$$\exp(j2\pi) = \cos 2\pi + j \sin 2\pi$$

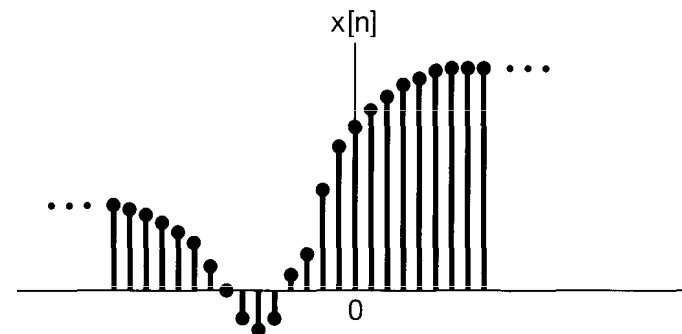
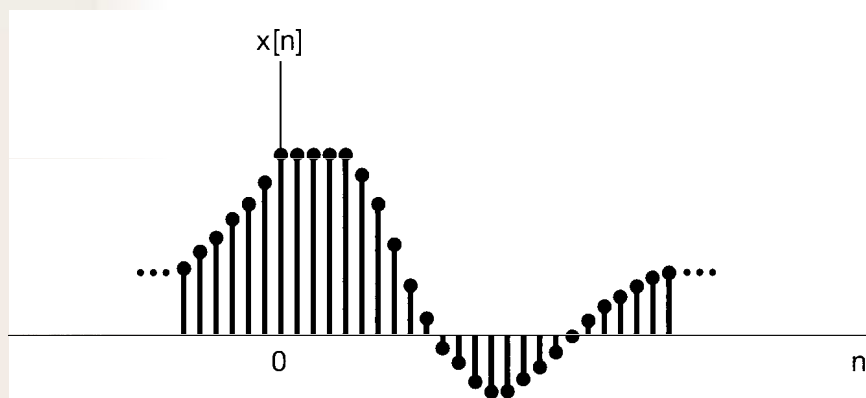
HARMONICKÝ DISKRÉTNÍ SIGNÁL



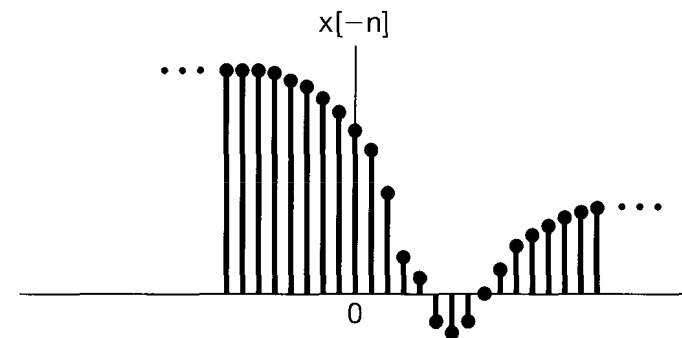
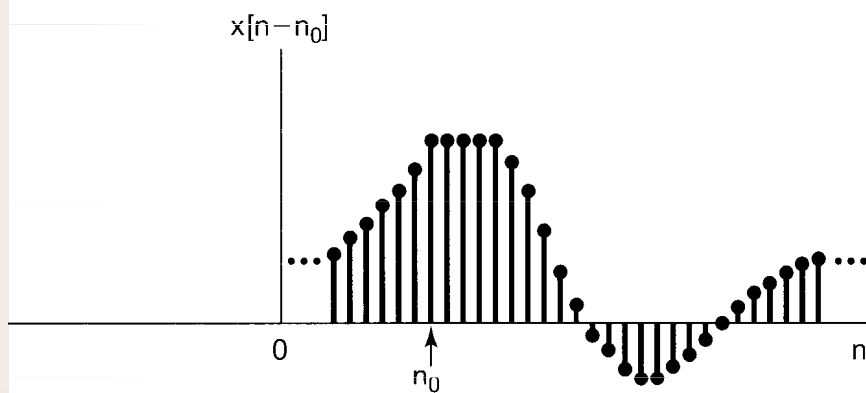
HARMONICKÝ DISKRÉTNÍ SIGNÁL



DISKRÉTNÍ SIGNÁLY



(a)



(b)

KONVOLUCE

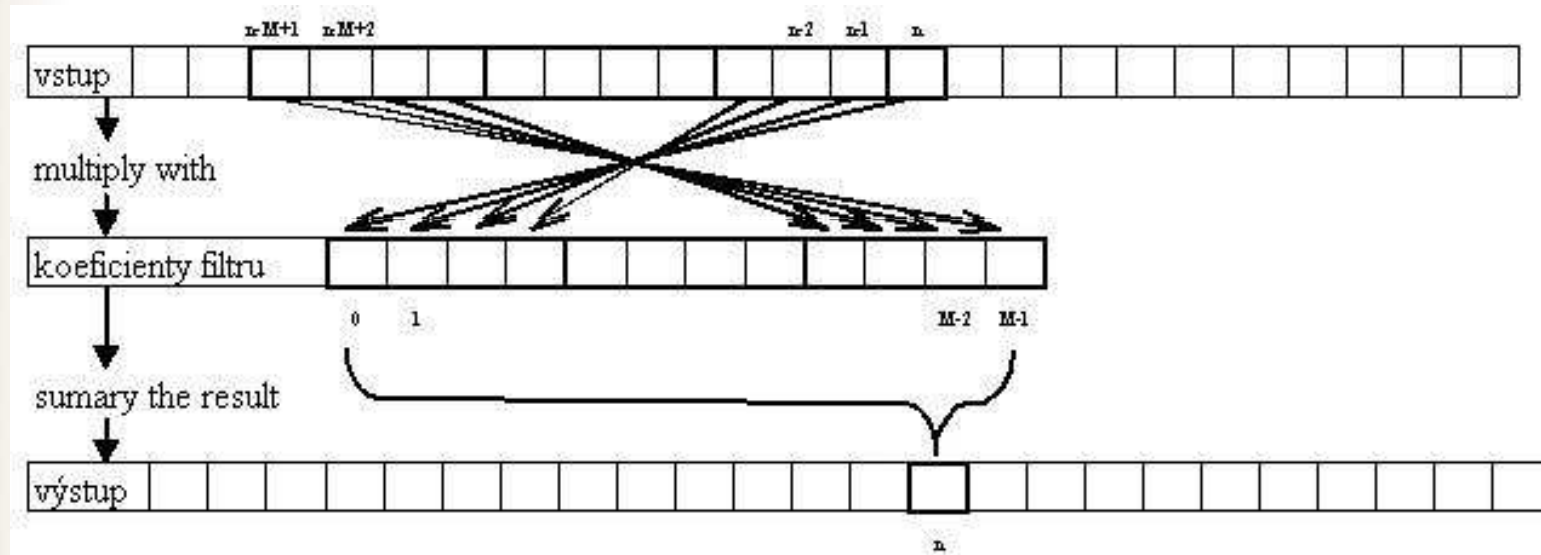
☑ spojité signály

$$s_1(t) * s_2(t) = \int_{-\infty}^t s_1(\tau) \cdot s_2(t - \tau) \cdot d\tau \approx S_1(\omega) \cdot S_2(\omega)$$

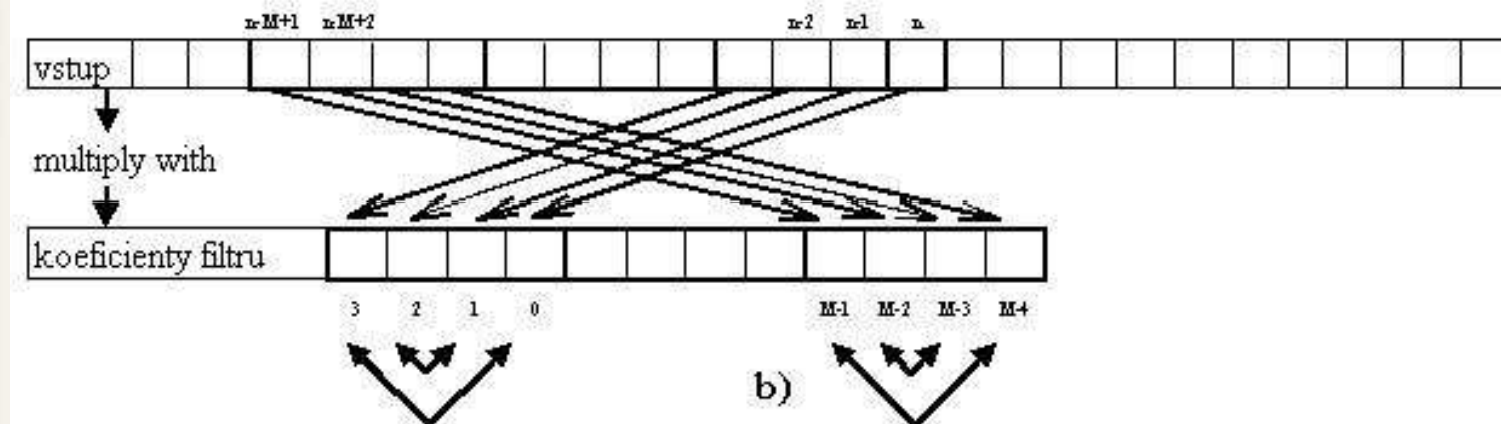
☑ diskrétní signály

$$s_1(nT) * s_2(nT) = \sum_{i=0}^n s_1(iT) \cdot s_2(nT - iT) \approx S_1(z) \cdot S_2(z)$$

DISKRÉTNÍ KONVOLUCE



a)



b)