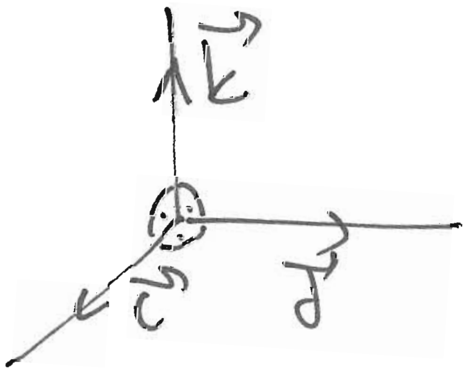
22.9.08

KINEMATIKA

Vztažná soustava



$\vec{i}\cdot\vec{j} = 0$ a cykl.

$\vec{i}\cdot\vec{i} = 1$

$\vec{a} = (a_x, a_y, a_z)$

$\vec{r}(t) = (x(t), y(t), z(t))$

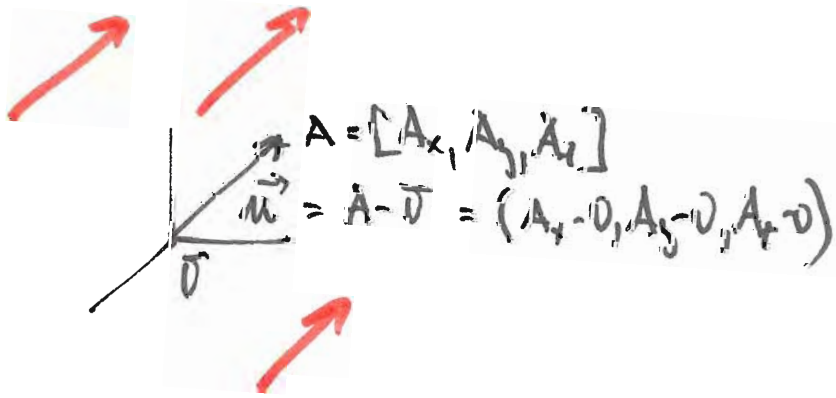
Γ

$$X = A + t\vec{u}$$

$x = A_x + t u_x$

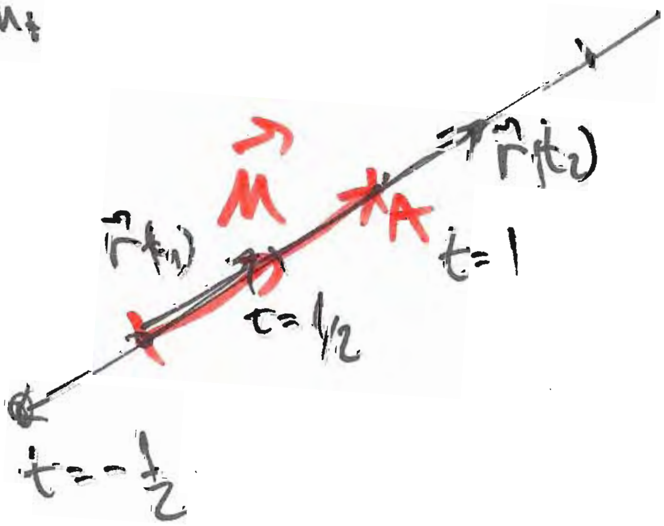
$y = A_y + t u_y$

$z = A_z + t u_z$

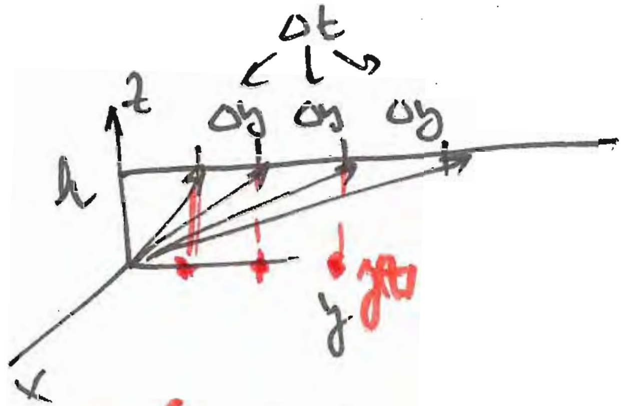


$t \in (0, \infty)$

$\vec{OA}$



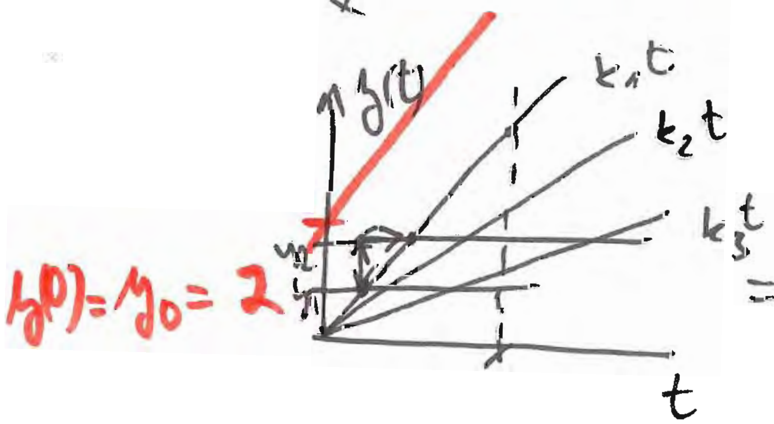
Pr 1



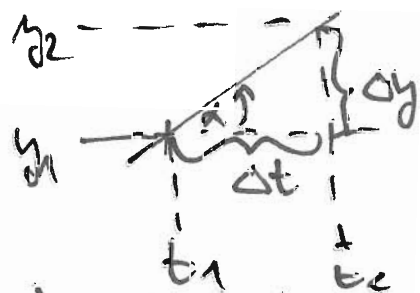
$x(t) = 0$ $\quad$ $\frac{m}{s}$ $\quad$ $k_1 > k_2$

$y(t) = 0 + 1 \cdot t = 0 + k_2 t$ $\quad$ s $\quad$ $k_3 < k_2$

$z(t) = h$



$\Longrightarrow$



$\underline{V_y = k = \text{konst}}$

Pr $\quad y = kx + q \quad y(x)$

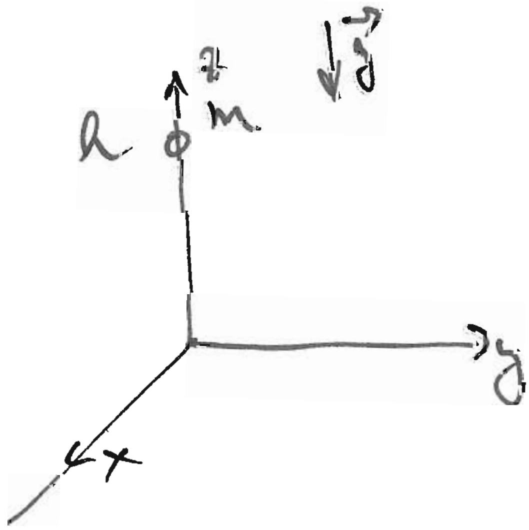
$y(t) = kt + q$

směrnice
k. čáry

$y_2 - y_1 = y(t_2) - y(t_1) = \Delta y$

$\frac{\Delta y}{\Delta t} = \text{tg}\,\alpha = k$

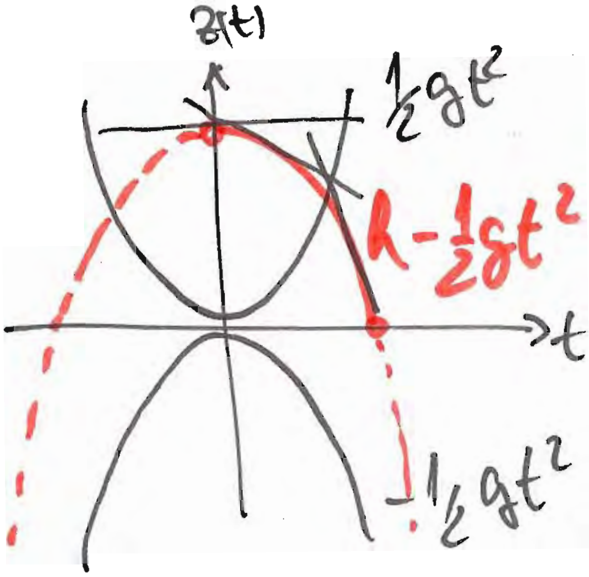
Pr 2

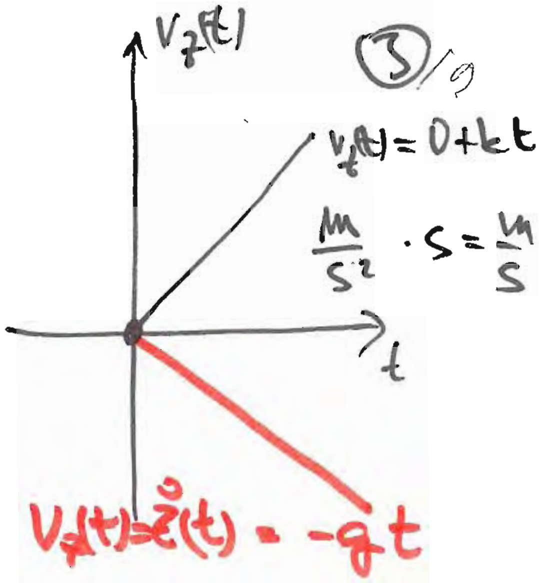


$x(t) = 0$

$y(t) = 0$

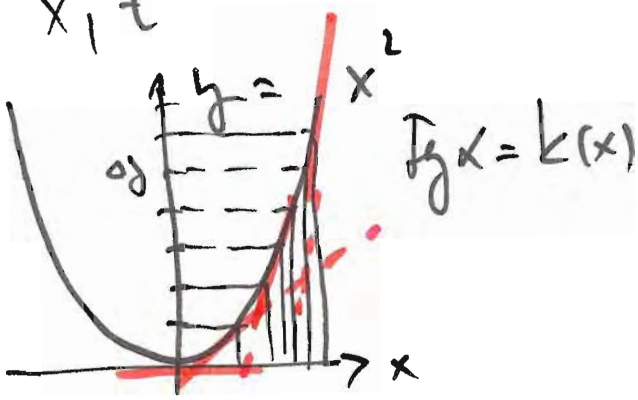
$z(t) = h - \frac{1}{2} g t^2$





$y = x^2$

! Grafy funkcí !



! derivace :

$$f(x) = x^2$$

$$\frac{df(x)}{dx} = y'(x) = \lim_{\varepsilon \to 0} \frac{y(x+\varepsilon) - y(x)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{x^2 + 2\varepsilon x + \varepsilon^2 - x^2}{\varepsilon} =$$

$\varepsilon \Rightarrow \Delta x$

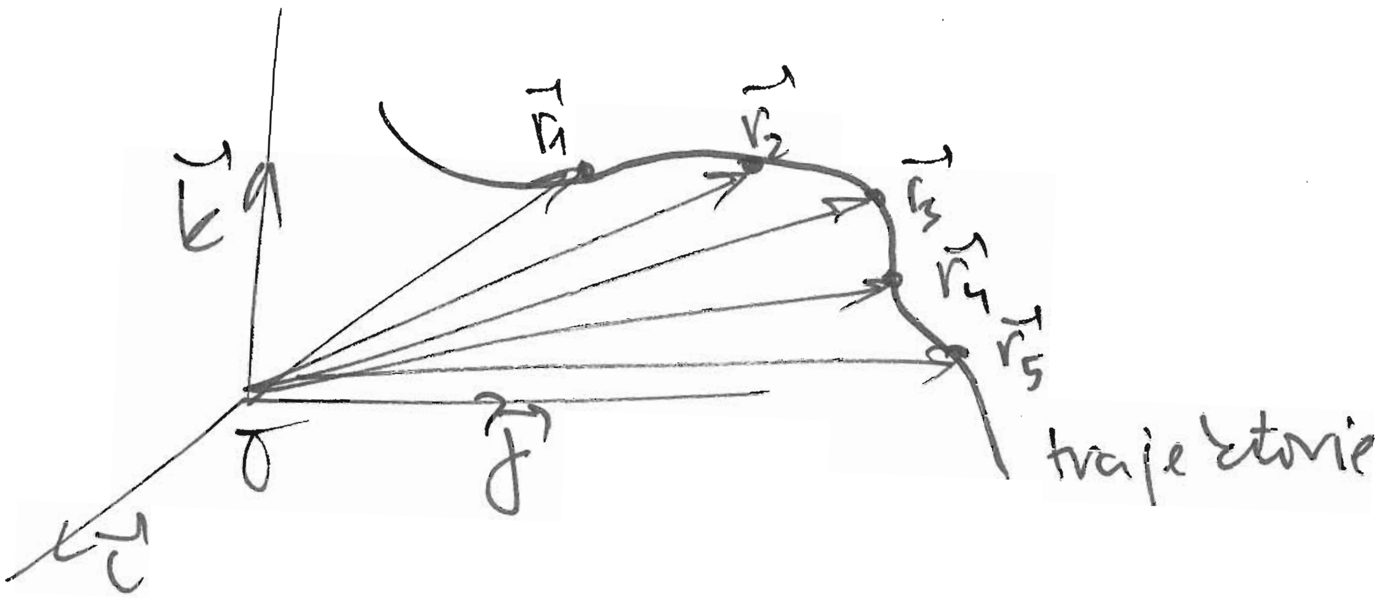
$$f(x+\varepsilon) = (x+\varepsilon)^2 = x^2 + 2\varepsilon x + \varepsilon^2$$

$$= \lim_{\varepsilon \to 0} \frac{2\varepsilon x + \varepsilon^2}{\varepsilon} = \lim_{\varepsilon \to 0} (2x + \varepsilon) = \lim_{\varepsilon \to 0} 2x + \lim_{\varepsilon \to 0} \varepsilon$$

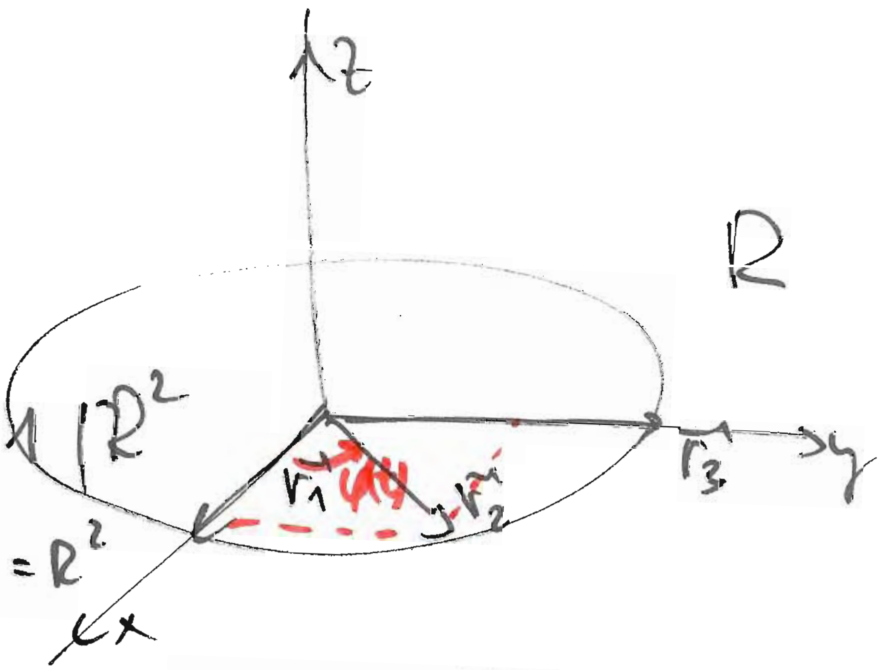
$$= 2x \qquad \frac{df}{dx} = y' = 2x$$

$$\boxed{dy = 2x\,dx} \qquad \boxed{\frac{dy}{dx} = 2x}$$

polohový vektor $\vec{r}(t) = (x(t), y(t), z(t))$



Př.:



$\varphi(t)$

$\sin^2\varphi(t) + \cos^2\varphi(t) = 1 \quad | R^2$

$R^2\sin^2\varphi(t) + R^2\cos^2\varphi(t) = R^2$

$\boxed{\varphi(t) = \varphi_0 + \omega t}$

$x(t) = R\cos\varphi(t)$

$y(t) = R\sin\varphi(t)$

$z(t) = 0$

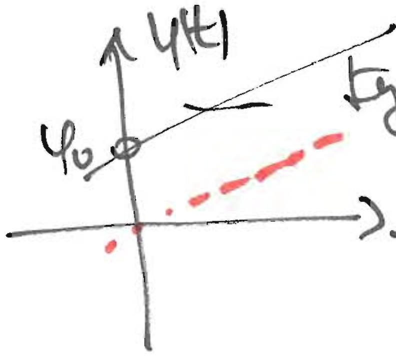
$\varphi(t) = ?$

rovnoměrné: $\frac{\Delta\varphi}{\Delta t} = k = \omega$

Γr $\varphi(t) = \varphi_0 + \omega t$

$$\Delta\varphi = \varphi_2 - \varphi_1 = \varphi(t+\Delta t) - \varphi(t) =$$
$$= \varphi_0 + \omega(t+\Delta t) - (\varphi_0 + \omega t) =$$
$$= \cancel{\varphi_0} + \cancel{\omega t} + \omega\Delta t - \cancel{\varphi_0} - \cancel{\omega t} = \omega\Delta t$$

$$\boxed{\frac{\Delta\varphi}{\Delta t} = \omega}$$



$\operatorname{tg}\alpha = \omega$

$\frac{d\varphi}{dt} = \dot{\varphi} = \omega$

$$\varphi = \varphi_0 + \omega t$$

° (rad) ° (rad) °/s $(\text{rad}\, s^{-1})$ s

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k} = (x(t), y(t), z(t))$$

$$\vec{r}(t) = R(\cos\omega t, \sin\omega t, 0)$$

Spousta
UČO 70369

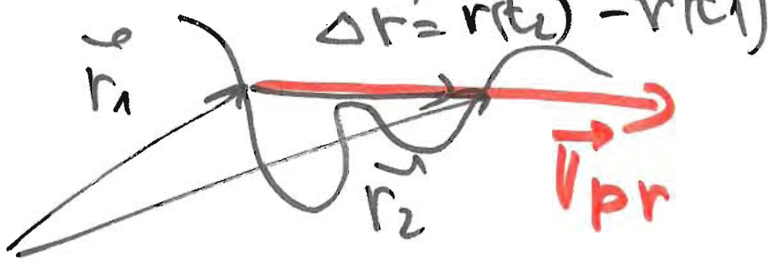
Polohový vektor $\vec{r}(t)$ m

rychlost HmB $\vec{v}(t)$ m/s

průměrné rychlost

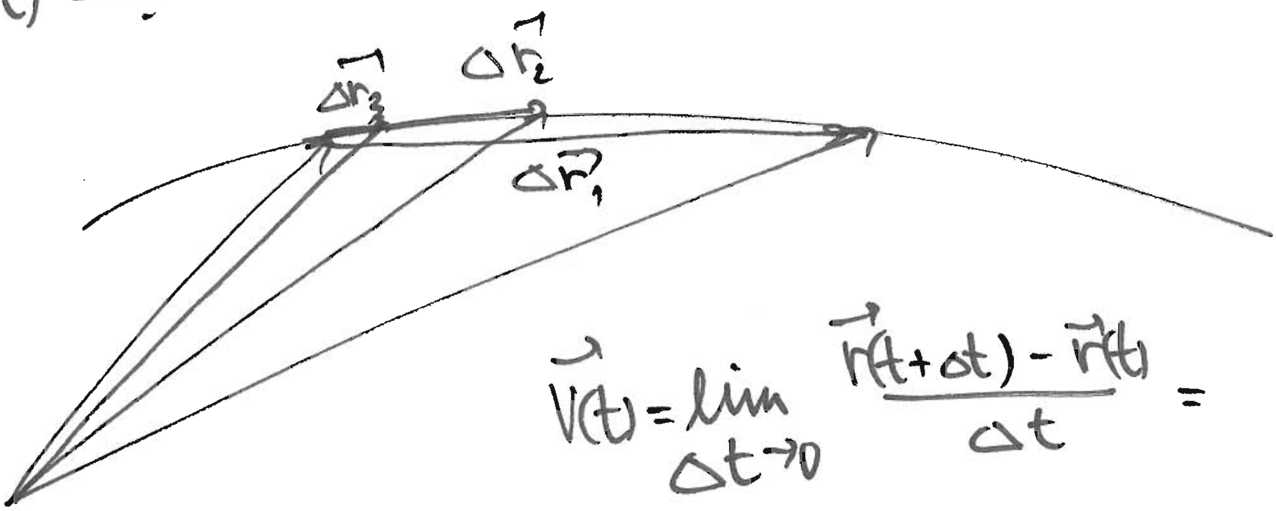
okamžitá

$$\vec{v}_{pr}(\Delta t) = \frac{\vec{r}(t_1+\Delta t)-\vec{r}(t_1)}{\Delta t} = \frac{\Delta\vec{r}}{\Delta t} \qquad t_2 = t_1+\Delta t$$



$\Delta t = \frac{1}{2}$ s

$\vec{v}(t) = \vec{?}$



$$\vec{v}(t) = \lim_{\Delta t\to 0} \frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t} =$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{d}{dt}\big(\underbrace{x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}}_{\vec{r}(t)}\big) =$$

$$= \frac{d}{dt}(x(t)\vec{i}) + \frac{d}{dt}(y(t)\vec{j}) + \frac{d}{dt}(z(t)\vec{k}) =$$

$$= \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} = \dot{x}(t)\vec{i} + \dot{y}(t)\vec{j} + \dot{z}(t)\vec{k} =$$

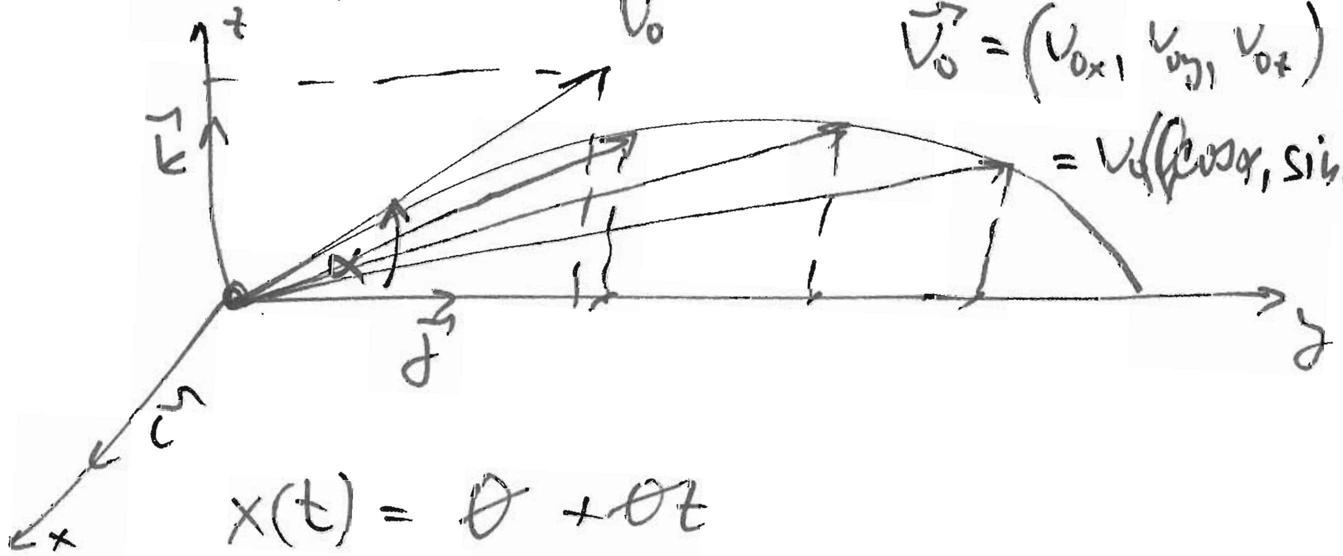
$$\vec{v}(t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t)) = (v_x(t), v_y(t), v_z(t))$$

Př.:

šikmý vrh

$$\vec{r}_0 = (\overset{x_0}{0}, \overset{y_0}{0}, \overset{z_0}{0})$$

$$\vec{v}_0 = (v_{0x}, v_{0y}, v_{0z}) = v_0(0, \cos\alpha, \sin\alpha)$$



$$x(t) = 0 + 0t$$

$$y(t) = 0 + v_{0y}\, t = (v_0 \cos\alpha)\, t$$

$$z(t) = 0 + v_{0z}\, t - \frac{1}{2} g t^2$$

$$\dot{x}(t) = v_x(t) = 0$$

$$\dot{y}(t) = v_{0y} = v_0 \cos\alpha$$

$$\dot{z}(t) = v_{0z} - gt$$

Velikost $|\vec{v}(t)| = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)} = v(t)$

Zrychlení

$\vec{a}(t)$ ..... $\vec{a}_{pr}(\Delta t)$ průměrné

okamžité

m/s^2

$$\vec{a}_{pr}(\Delta t) = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$$

$$\vec{a}(t) = \lim_{\Delta t \to 0} \vec{v}_{pr}(\Delta t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$$

$$= \frac{d\vec{v}(t)}{dt} = \dot{\vec{v}}(t) = \frac{d^2\vec{r}(t)}{dt^2} = \ddot{\vec{r}}(t)$$

$$\vec{a}(t) = (a_x(t), a_y(t), a_z(t)) = (\dot{v}_x(t), \dot{v}_y(t), \dot{v}_z(t)) =$$

$$= (\ddot{x}(t), \ddot{y}(t), \ddot{z}(t))$$

☺ znám $\vec{r}(t) \xrightarrow{\frac{d\vec{r}}{dt}}$ chci $\vec{v}(t)$, znám $\vec{v}(t) \xrightarrow{\frac{d\vec{v}}{dt}}$ chci $\vec{a}(t)$

☹ znám $\vec{a}(t) \xrightarrow{+\vec{v}_0} \vec{v}(t) \xrightarrow{+\vec{r}_0} \vec{r}(t)$

Př.:

$$\boxed{\vec{a} = (0, 0, -g)}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} \Rightarrow \vec{a}(t)\,dt = d\vec{v}(t)$$

$$\int \vec{a}(t)\,dt = \int d\vec{v}$$

$$\int a_x(t)\,dt = \int dv_x + C_x$$

$$\int a_y(t)\,dt = \int dv_y + C_y$$

$$\int a_z(t)\,dt = \int dv_z + C_z$$

$\vdots$

$\vec{a} = \overrightarrow{konst}$ $(\neq \vec{f}(t))$

$$a_x t = v_x + v_{0x}$$

$$a_y t = v_y + v_{0y}$$

$$a_z t + v_{0z} = v_z$$

$$a_x = 0 = \frac{dv_x}{dt} \Rightarrow v_x(t) = C_1 \Rightarrow x(t) = C_1 t + B_1$$

$$a_y = 0 = \frac{dv_y}{dt} \Rightarrow v_y(t) = C_2 \Rightarrow y(t) = C_2 t + B_2$$

$$a_z = -g = \frac{dv_z}{dt} \Rightarrow v_z(t) = C_3 - gt$$

$$\Rightarrow z(t) = C_3 t - \frac{1}{2} g t^2 + B_3$$

$\vec{v}_0 = (C_1, C_2, C_3)$

$\vec{r}_0 = (B_1, B_2, B_3)$

DÚ $\vec{r}_0, \vec{v}_0$ / $\vec{r}(t), \vec{v}(t)$ VRHY