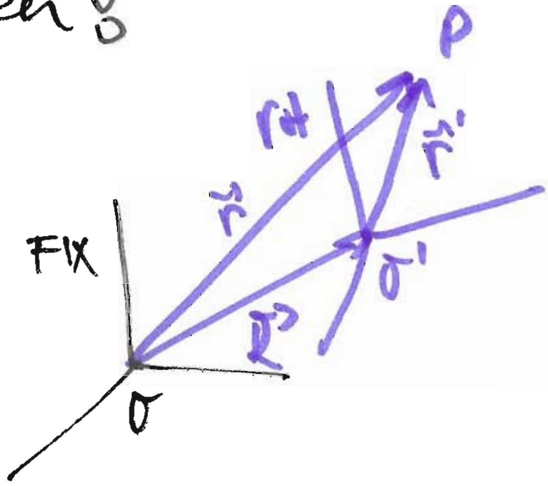
13.10.2008

Dobrý den!

1



$$\vec{r} = \vec{R} + \vec{r}'$$

$$(d\vec{r}')_{fix} = (d\vec{r}')_{rot} + d\vec{\Theta} \times \vec{r}'$$

$$(d\vec{r})_{fix} = (d\vec{R})_{fix} + (d\vec{r}')_{Fix}$$

$$\left(\frac{d\vec{r}'}{dt}\right)_{fix} = \left(\frac{d\vec{r}'}{dt}\right)_{rot} + \vec{\omega} \times \vec{r}'$$

$$\left(\frac{d\vec{Q}}{dt}\right)_{fix} = \left(\frac{d\vec{Q}}{dt}\right)_{rot} + \vec{\omega} \times \vec{Q}$$

$$\vec{Q} = \vec{\omega} \qquad \dot{\vec{\omega}}_{fix} = \dot{\vec{\omega}}_{rot}$$

$$\underbrace{\left(\frac{d\vec{r}}{dt}\right)_{fix}}_{\vec{v}_{fix}} = \left(\frac{d\vec{R}}{dt}\right)_{fix} + \left(\frac{d\vec{r}'}{dt}\right)_{fix} = \underbrace{\left(\frac{d\vec{R}}{dt}\right)_{fix}}_{\vec{V}_{fix}} + \underbrace{\left(\frac{d\vec{r}'}{dt}\right)_{rot}}_{\vec{v}_{rot}} + \vec{\omega} \times \vec{r}'$$

$$\left(\frac{d\vec{v}_{fix}}{dt}\right)_{fix} = \left(\frac{d\vec{V}_{fix}}{dt}\right)_{fix} + \left(\frac{d\vec{v}_{rot}}{dt}\right)_{fix} + \frac{d}{dt}(\vec{\omega} \times \vec{r}')_{fix}$$

$$\left(\frac{d\vec{v}_{rot}}{dt}\right)_{fix} = \left(\frac{d\vec{v}_{rot}}{dt}\right)_{rot} + \vec{\omega} \times \vec{V}_{rot}$$

$$\frac{d}{dt}(\vec{\omega} \times \vec{r}')_{fix} = \dot{\vec{\omega}} \times \vec{r}' + \vec{\omega} \times \left(\frac{d\vec{r}'}{dt}\right)_{fix} = \dot{\vec{\omega}} \times \vec{r}' + \vec{\omega} \times \left[\left(\frac{d\vec{r}'}{dt}\right)_{rot} + \vec{\omega} \times \vec{r}'\right]$$

$$\left(\frac{d\vec v_{fix}}{dt}\right)_{fix} = \left(\frac{d\vec V_{fix}}{dt}\right)_{fix} + \left(\frac{d\vec v_{rot}}{dt}\right)_{rot} + \vec\omega\times\vec v_{rot} + \dots$$

$$+ \dot{\vec\omega}\times\vec r' + \vec\omega\times\left(\frac{d\vec r'}{dt}\right)_{rot} + \vec\omega\times(\vec\omega\times\vec r')$$

$$\vec a_{fix} = +\vec A_{fix} + \dot{\vec\omega}\times\vec r' + \vec a_{rot} + 2\vec\omega\times\vec v_{rot} + \vec\omega\times\vec\omega\times\vec r'$$

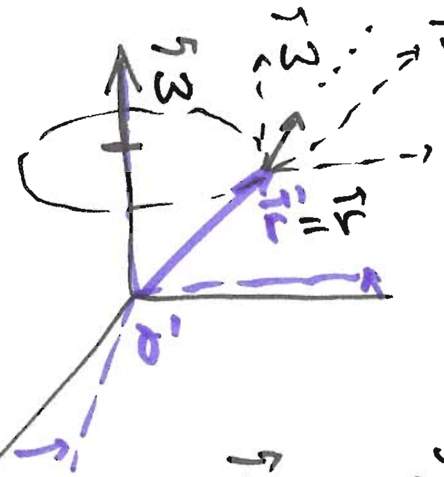
zrychlení P vůči fix

(O' vůči O ve fixní)

Eulerovo zrychlení

zrychlení P vůči rot

Coriolis



$$m\vec a_{rot} = m\vec a_{fix} - m\vec A_{fix} \underbrace{- m\dot{\vec\omega}\times\vec r'}_{+\text{ Euler}} \underbrace{- 2m\vec\omega\times\vec v_{rot}}_{+\text{ Coriolis}} - m\vec\omega\times(\vec\omega\times\vec r')$$

$+ \vec a^*_{od}$

DYNAMIKA

1 NPZ

Existence inerciální soustavy

2 NPZ zákon síly

nemožit používat

$$\vec F_V = m\vec a = m\ddot{\vec r}$$

$$\sum_{i=1}^{N} \vec F_i = m\vec a$$

Stav: $\vec r(t), \vec v(t)$

$\vec r_0, \vec v_0 \to \vec r(t_1), \vec v(t_1)$

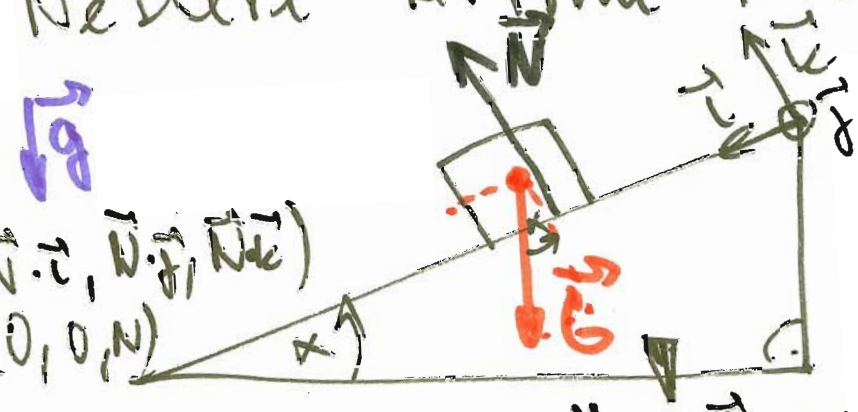
Počáteční podmínky

+

$\boxed{\vec F_V = m\vec a}$

3 zákon akce & reakce

Některé řešené úlohy:



$\vec N = (\vec N\cdot\vec i, \vec N\cdot\vec j, \vec N\cdot\vec k)$
$(0, 0, N)$

1) Vhodná volba vztažné soust.

II. NPZ $\vec F_V = m\vec a$

$\vec N + \vec G = m\vec a$

$\vec i$: $mg\sin\alpha = m\ddot x$ $\vec k$: $N - mg\cos\alpha = m\ddot z$

$\boxed{mg\sin\alpha = m\ddot{x}}$

$mg\cos\alpha - N = m\ddot{z}$ ← $z = \text{const.} \Rightarrow \dot{z} = 0 \Rightarrow \ddot{z} = 0$

$\boxed{mg\cos\alpha = N}$

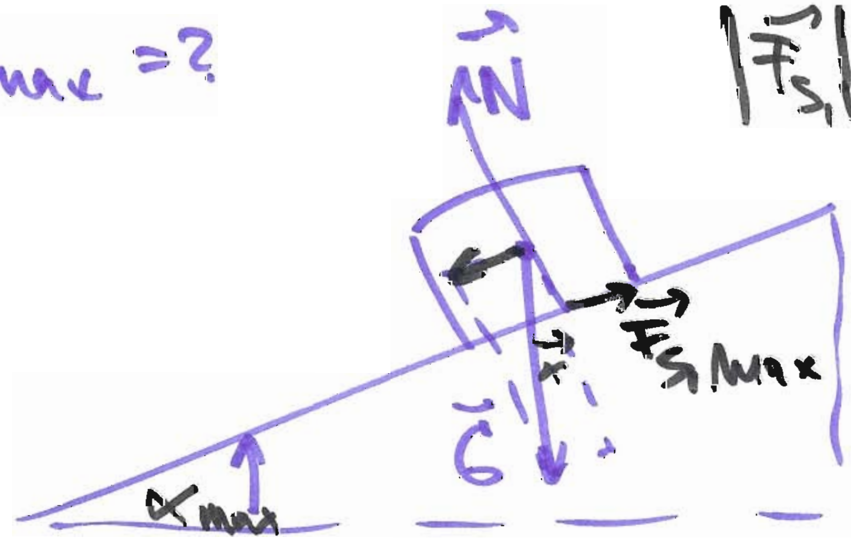
$x(t) = ?$

$\ddot{x} = g\sin\alpha \Rightarrow \dot{x}(t) = gt\sin\alpha$

$x(t) = \frac{1}{2} g t^2 \sin\alpha$

$f_{s,max} = ?$

$|\vec{F}_{s,max}| = f_s|\vec{N}|$



$mg\cos\alpha_m$

$mg\sin\alpha_m = F_{s,max} = N f_s$

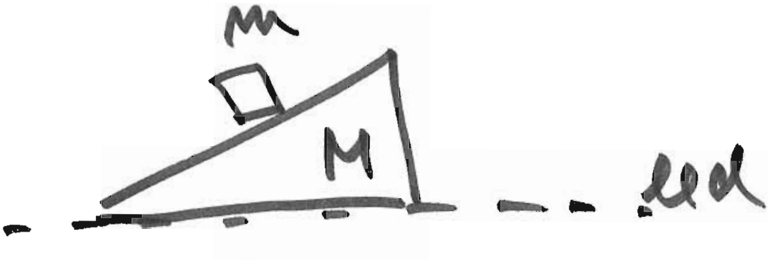
$mg\sin\alpha_m = mg\cos\alpha_m \, f_s \Rightarrow$

$\boxed{f_{s,max} = \mathrm{tg}\,\alpha_m}$

$f_d = \mathrm{tg}\,\alpha_d$

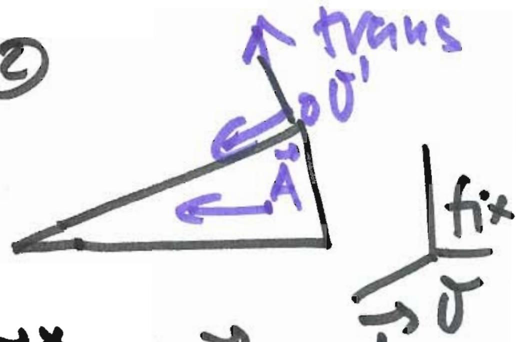
Nakloněná rovina 1000x jinak!

(1)



$\vec{F}_v = m\vec{a}$

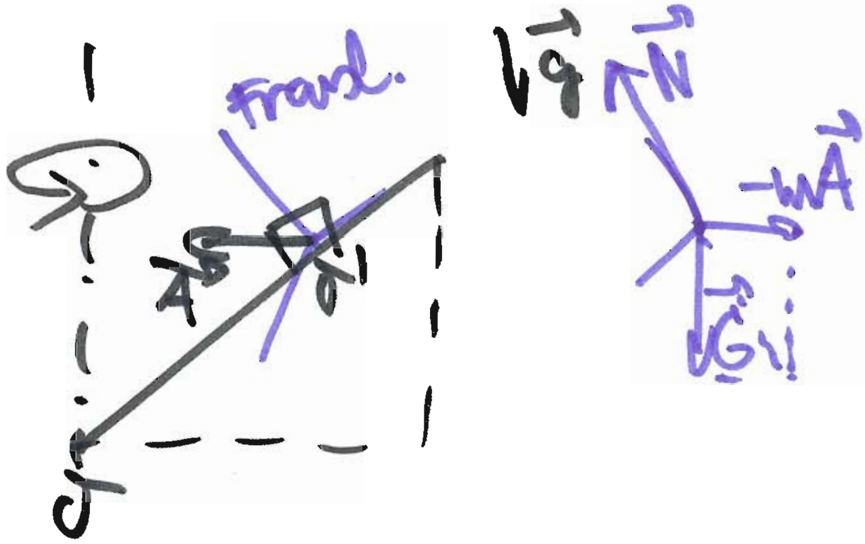
(2)



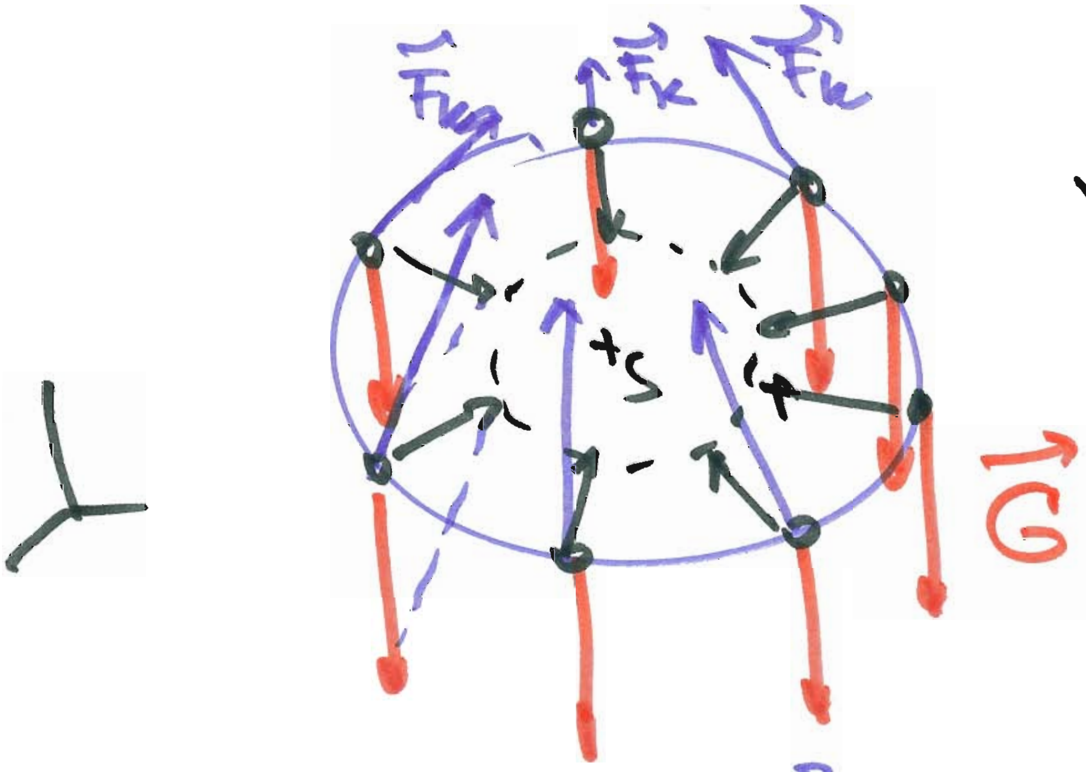
$m\vec{a}^* = m\vec{a} - m\vec{A}$

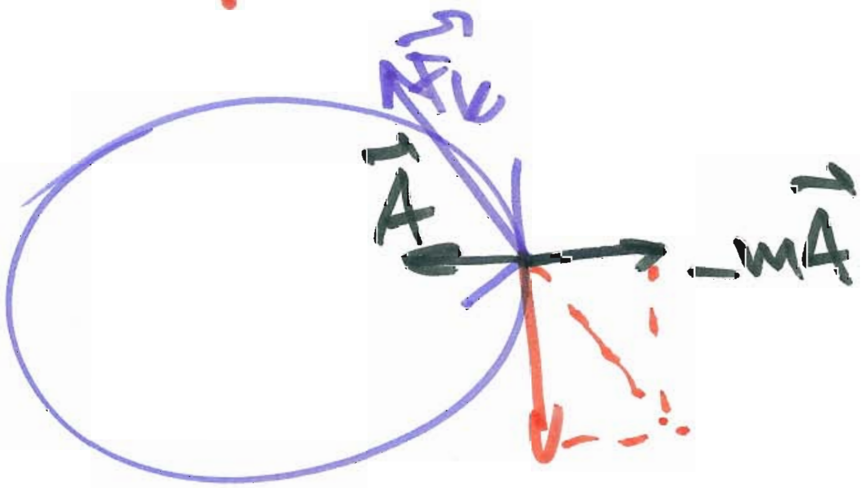
$m\vec{a}^* = m\vec{g} + \vec{N} - m\vec{A}$

(3)

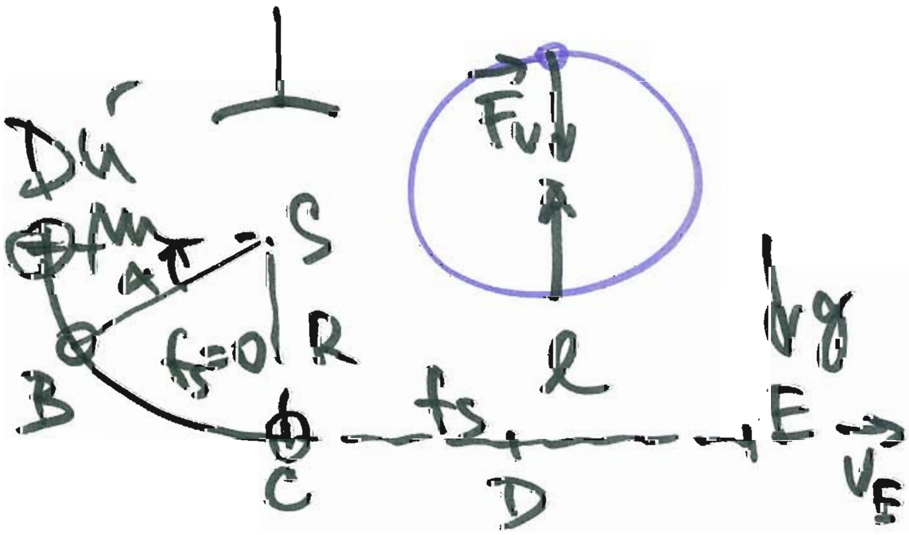


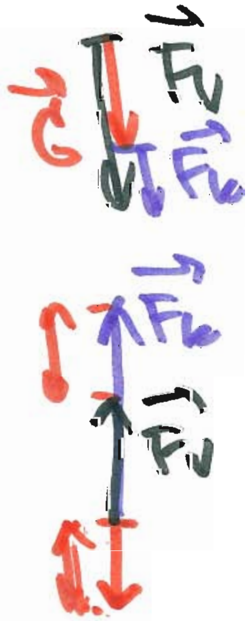
Ruské kolo





Rychlý kolotoč:





m = 85kg

ω =
ad
libitum