20.10.2008

①

Dobrý den!

Užití Newtonových pohybových zákonů!

i) Volba vhodné vztažné soustavy.

ii) Inerciální or neinerciální*?

iii) Nakreslit pěkný obrázek.

iv) Zakreslit všechny* síly, působící na hm B

v) $\vec{F}_V = m\vec{a}$ rozepsat do složek (do os $\vec{i}, \vec{j}, \vec{k}$).

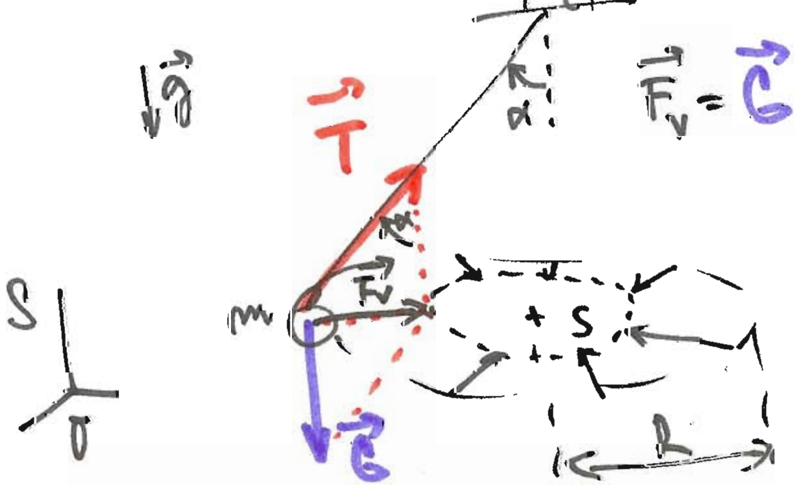
vi) najít $\vec{a}$ → poč. podmínky → $\vec{v}$ → p.p. → $\vec{r}$

vi) znám pohyb ($\vec{a}$) ... dopočtu jednu z působících sil.

vii) radostně podtrhnu (správný) výsledek.

Příklady:

① KÓNICKÉ KYVADLO I. $\left[T^2 = F_V^2 + G^2 \right]$



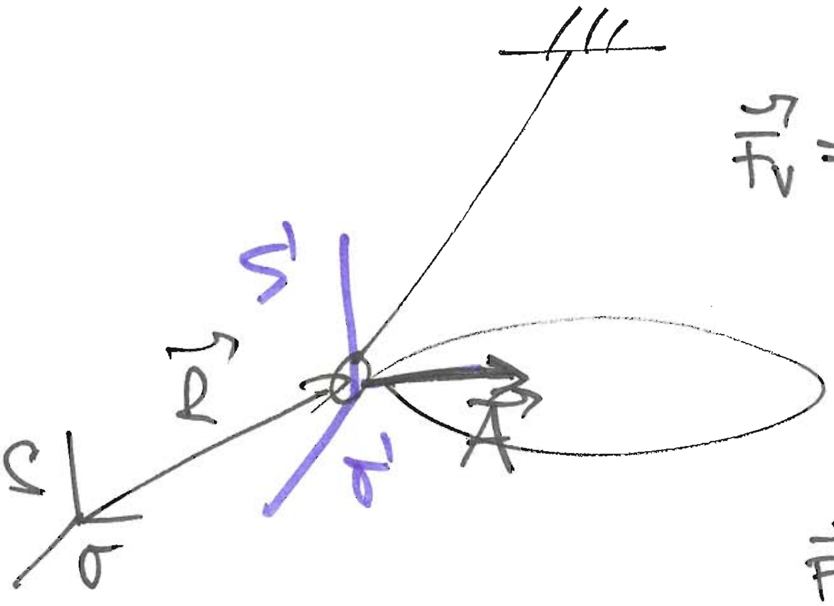
$$\vec{F}_V = \vec{G} + \vec{T}$$

$$F_V = ma = ma_n = m\frac{v^2}{R}$$

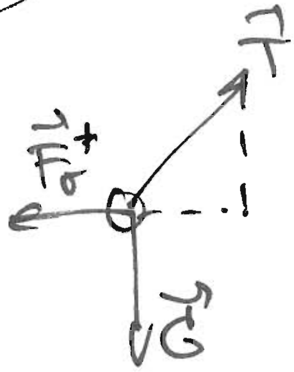
$$G = mg$$

Otázka $\vec{T} = ?$

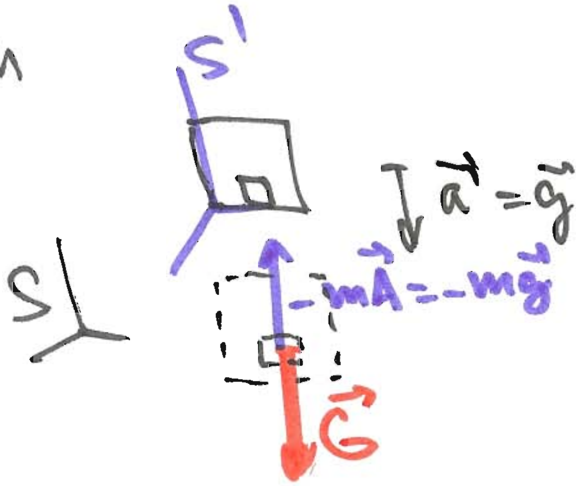
Kónické kyvadlo II :



$$\vec F_v = \cancel{\vec B \ldots} = \vec G + \vec T + \vec F_o^* = m\vec g + \vec T - m\vec A$$

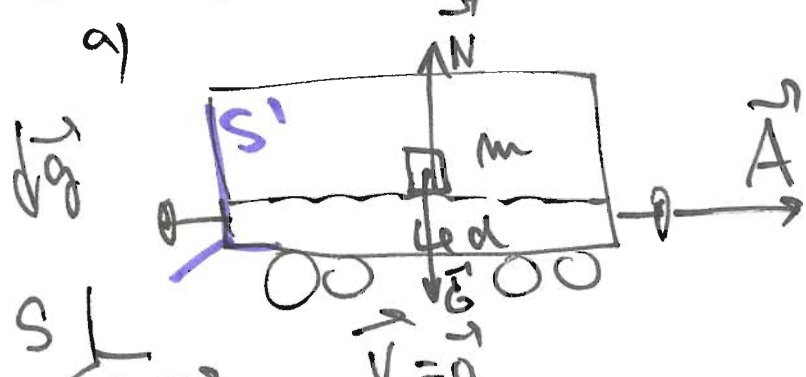


② Utržený výtah



③ Fyzikální vozík

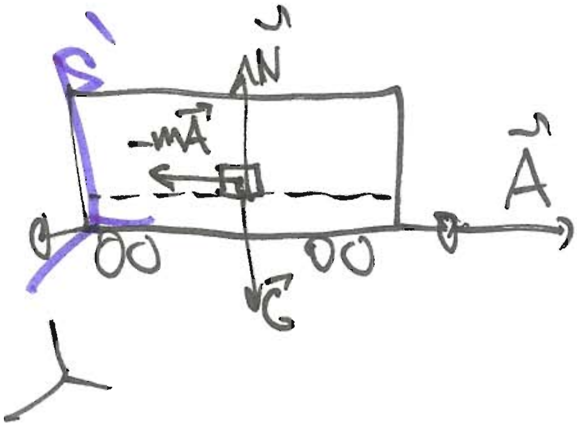
a)



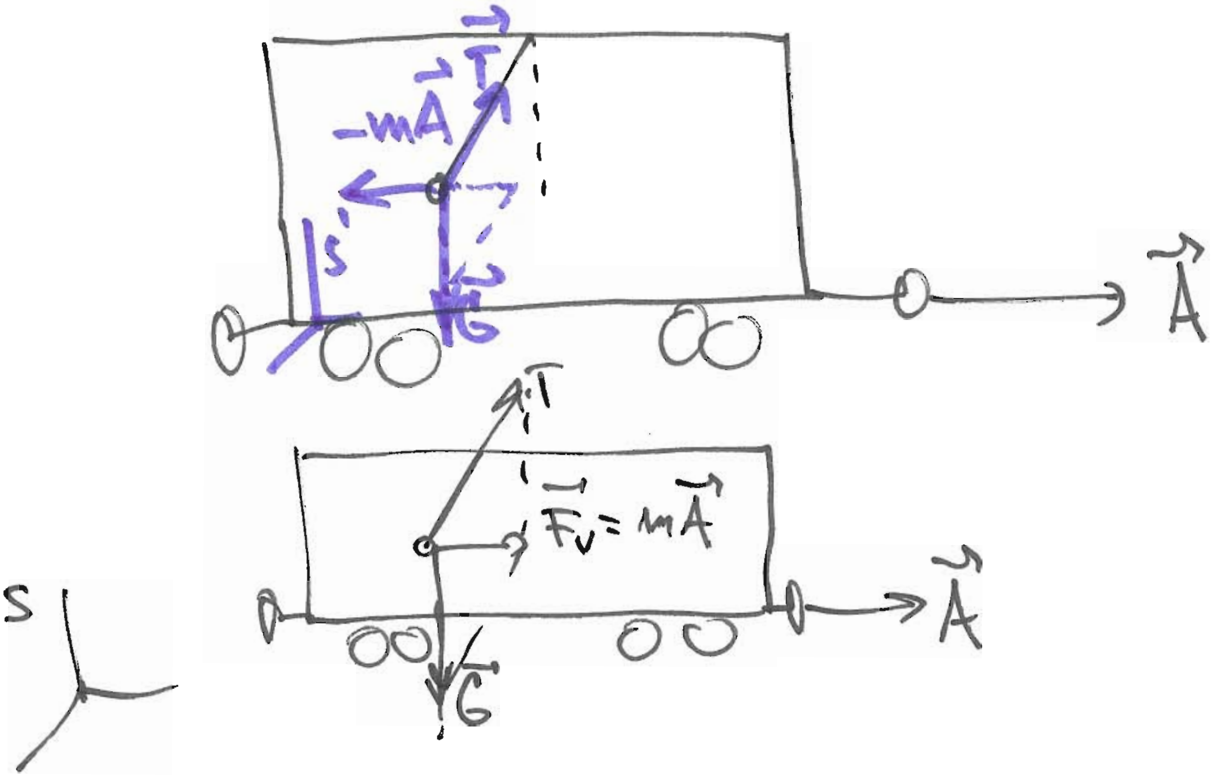
$\vec F_v = \vec 0$

$\vec v_0 = \vec 0$

$\vec r_0 = \vec k = \vec r(t)$

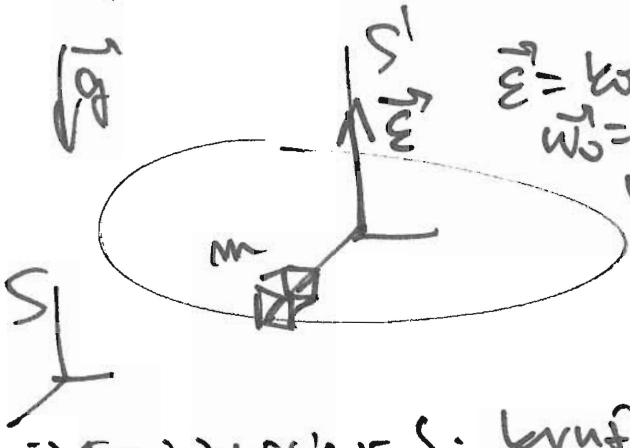


5)



④ Hrnčiarsky kruh

a)

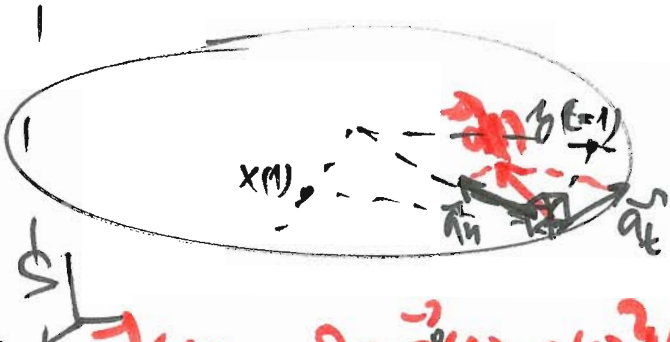


$\vec{\varepsilon}$ = konšt = 0,5 s^{-2}

$\vec{\omega}_0 = 0$

$\varphi_0 = 0$

b) $t = 1\,s$



(i) Trajektória v S: kružnica

(ii) $\vec{a} = \vec{a}_t + \vec{a}_n$; $a_t = R\varepsilon$

$a_n = \omega^2 R = \frac{v^2}{R}$

... $\omega(t) = \omega_0 + \varepsilon t$

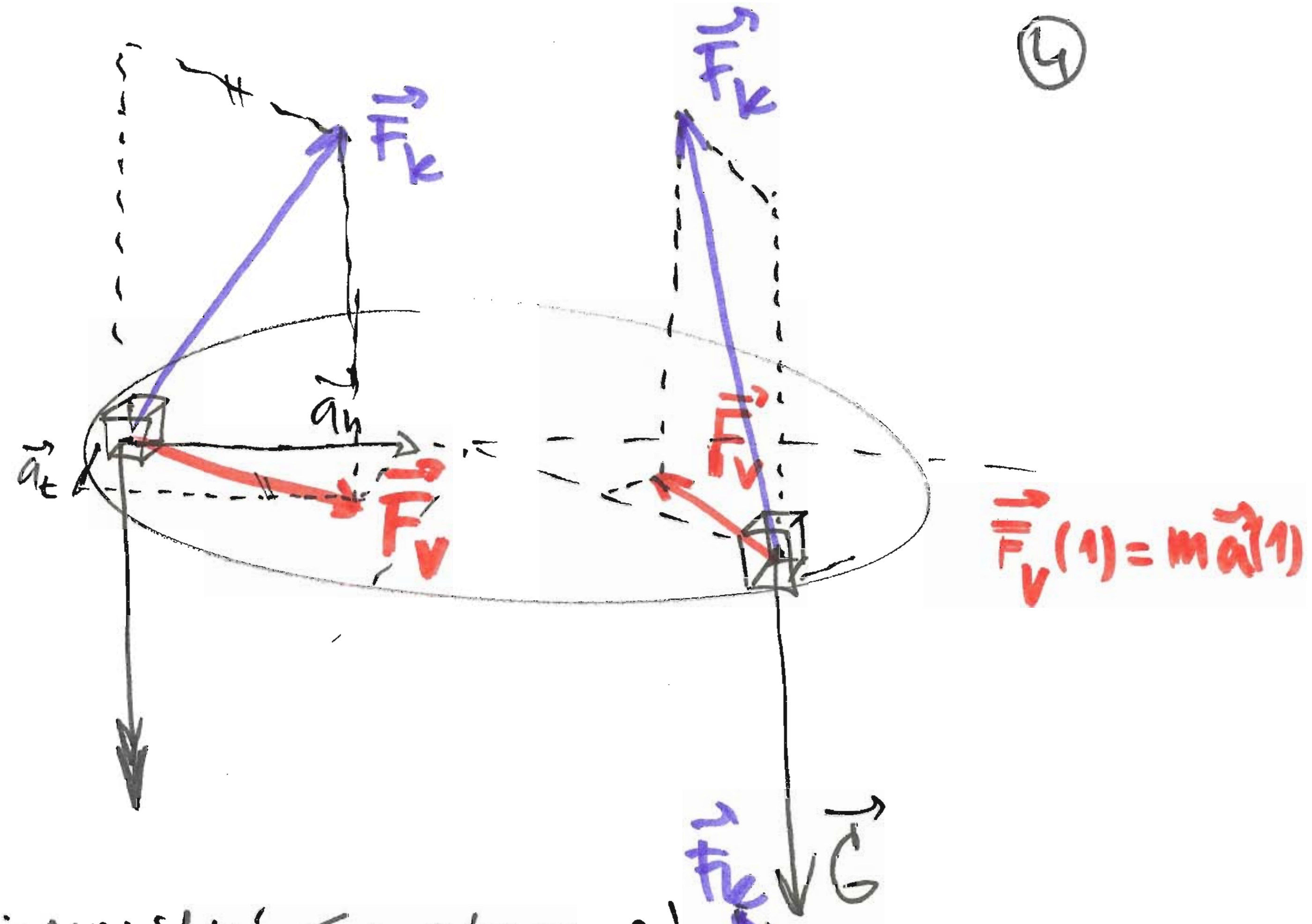
$\varphi(t) = \varphi_0 + \frac{1}{2}\varepsilon t^2$

$\vec{a}(1) = R\varepsilon\,\vec{\tau}(1) + \omega^2(1)\,R\,\vec{n}(1)$

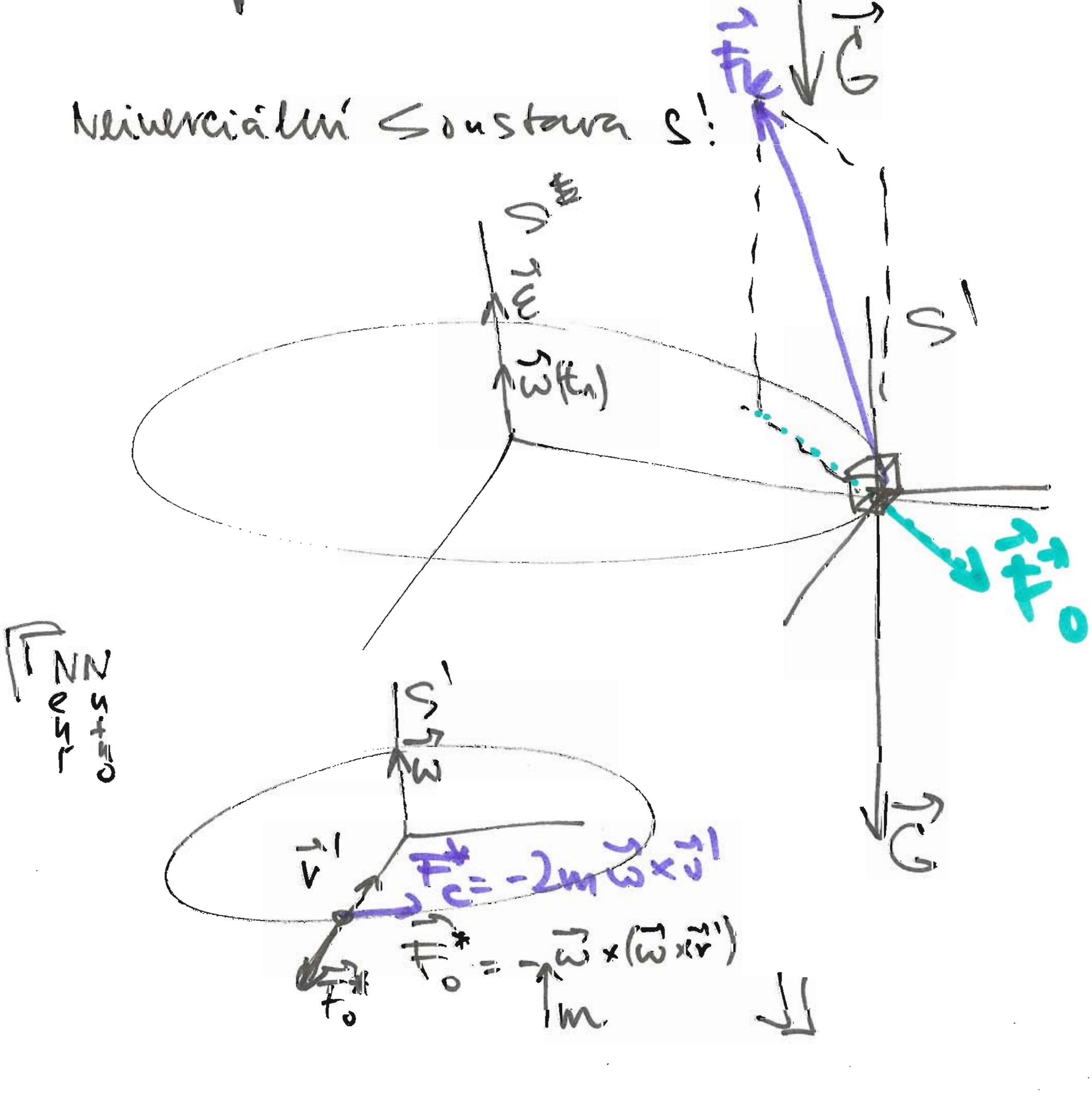
$|\vec{a}(1)| = \sqrt{(R\varepsilon)^2 + (\omega^2(1)R)^2}$

$\Big[\ \vec{v} = \vec{\omega} \times \vec{r}$

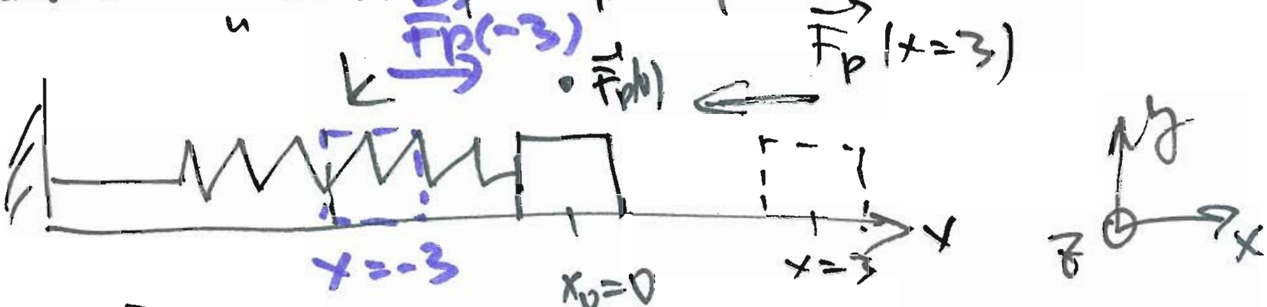
$\vec{a} = \dot{\vec{v}} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \vec{v} = ..\ \Big]$



Neinerciální soustava S'!



Řešení „složitější" pohybové rovnice



$$\vec{F}_p = -kx\vec{i}$$

[r nedeformované pružiny: $x_0=0$]

II. NPZ: $\vec{F}_V = m\vec{a}$

$\vec{G}+\vec{N}+\vec{F}_p = m\vec{a}$

x: $\boxed{m\ddot{x} = -kx}$

y: $m\ddot{y} = N - mg = 0 \Rightarrow N = mg$

x: $\boxed{m\ddot{x} + kx = 0} \rightarrow \boxed{\ddot{x}(t) + \frac{k}{m}x(t) = 0}$

! $\boxed{\ddot{x}(t) + \omega^2 x(t) = 0}$!

$\omega^2 > 0$

LHDR

$\lambda^2 + \omega^2\lambda^0 = 0$

$\lambda_{1,2} = \pm i\omega$

$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

$\dot{x}(t) = A\omega\cos(\omega t + \alpha)$

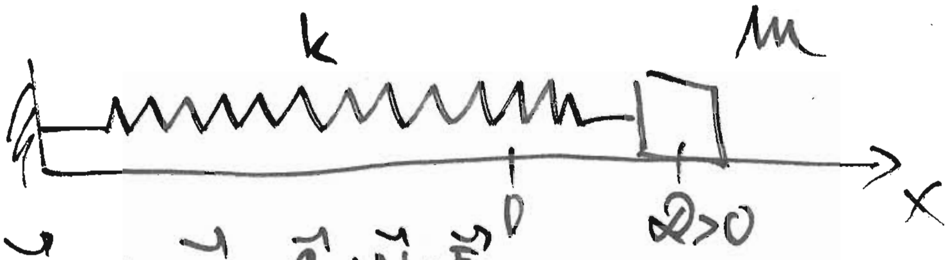
$\ddot{x}(t) = -A\omega^2\sin(\omega t + \alpha)$

$+$ $\leftarrow$ $\omega^2 x(t) = A\omega^2\sin(\omega t + \alpha)$

$= 0$

$A, \alpha \in \mathbb{R}$ konstanty určíme z P.P.

$$x(t) = A\sin(\omega t + \alpha), \quad \omega = \sqrt{\frac{k}{m}}$$



$$\vec{F}_V = m\vec{a} = \vec{G} + \vec{N} + \vec{F}_p$$

$$\vec{r}_0 = (\mathcal{D}, 0, 0) \; ; \; \vec{v}_0 = (0, 0, 0)$$

$$m\ddot{x} = -kx$$

$$\ddot{x} + \frac{k}{m}x = 0$$

$$x(t) = A\sin\left(\sqrt{\tfrac{k}{m}}\, t + \alpha\right)$$

$$\dot{x}(t) = A\omega\cos(\omega t + \alpha)$$

$$\ddot{x}(t) = -A\omega^2\sin\left(\sqrt{\tfrac{k}{m}}\, t + \alpha\right) = -\omega^2 x(t)$$

$$\underline{x(0) = \mathcal{D} = A\sin\alpha}$$

$$\underline{\dot{x}(0) = 0 = A\omega\cos\alpha}$$

$$\alpha = \pi/2$$

$$\Rightarrow A = \mathcal{D}$$

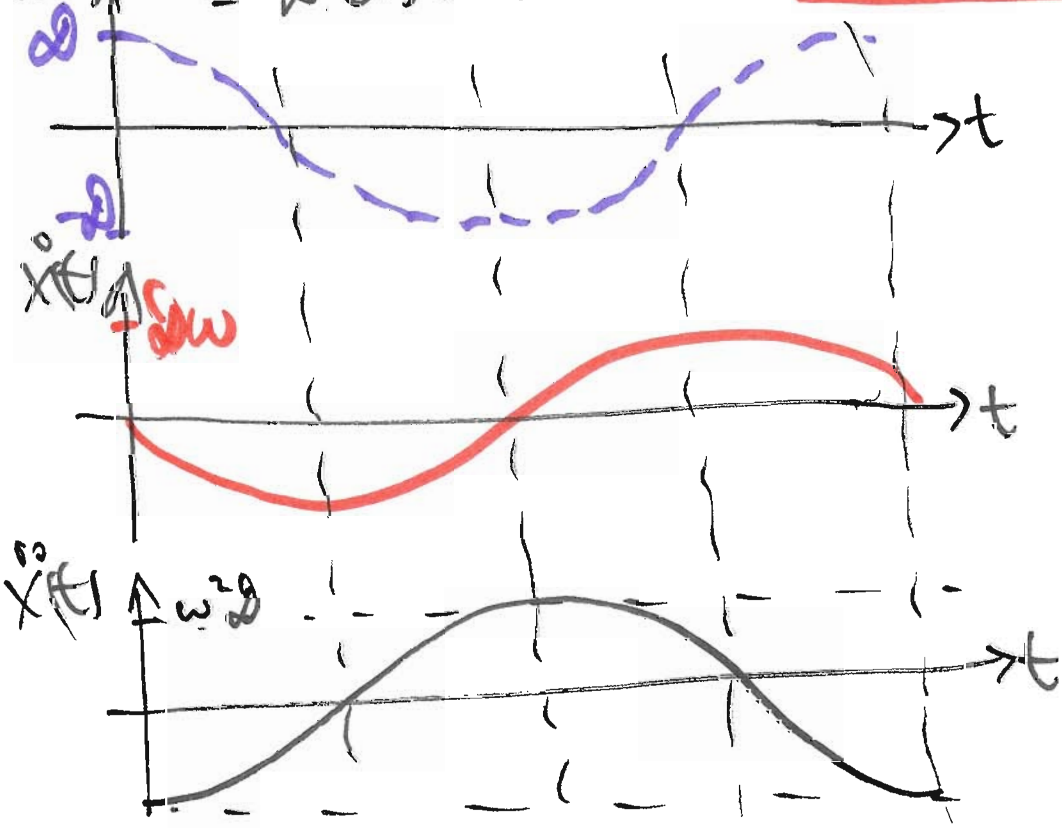
$$x(t) = \mathcal{D}\sin(\omega t + \pi/2) =$$

$$= \mathcal{D}\cos\omega t$$

$$\dot{x}(t) = -\mathcal{D}\omega\sin\omega t$$

$$\ddot{x} = -\mathcal{D}\omega^2\cos\omega t$$

$$= -\omega^2 x(t)$$



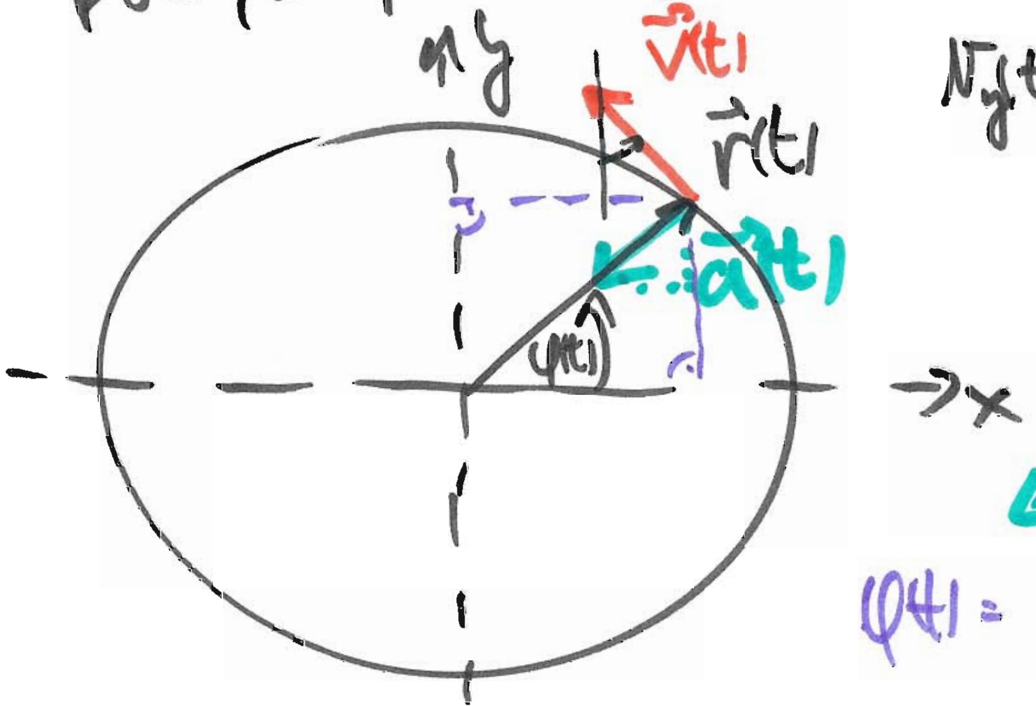
$$x(t) = x_{max}\cos\omega t \qquad \dot{x}(t) = -\dot{x}_{max}\sin\omega t \qquad \ddot{x}(t) = -\ddot{x}_{max}\cos\omega t$$

$$x_{max} = \mathcal{D} \qquad \dot{x}_{max} = \mathcal{D}\omega \qquad \ddot{x}_{max} = \mathcal{D}\omega^2$$

Harmonické kmitání

[a LT

pohyb po kružnici



ωR

$v_y(t) = v_{max} \cos\varphi(t)$

$\varphi(t) = \omega t + \varphi_0$

0

$y(t) = y_m \sin\varphi(t) = y_m \sin\omega t$

R

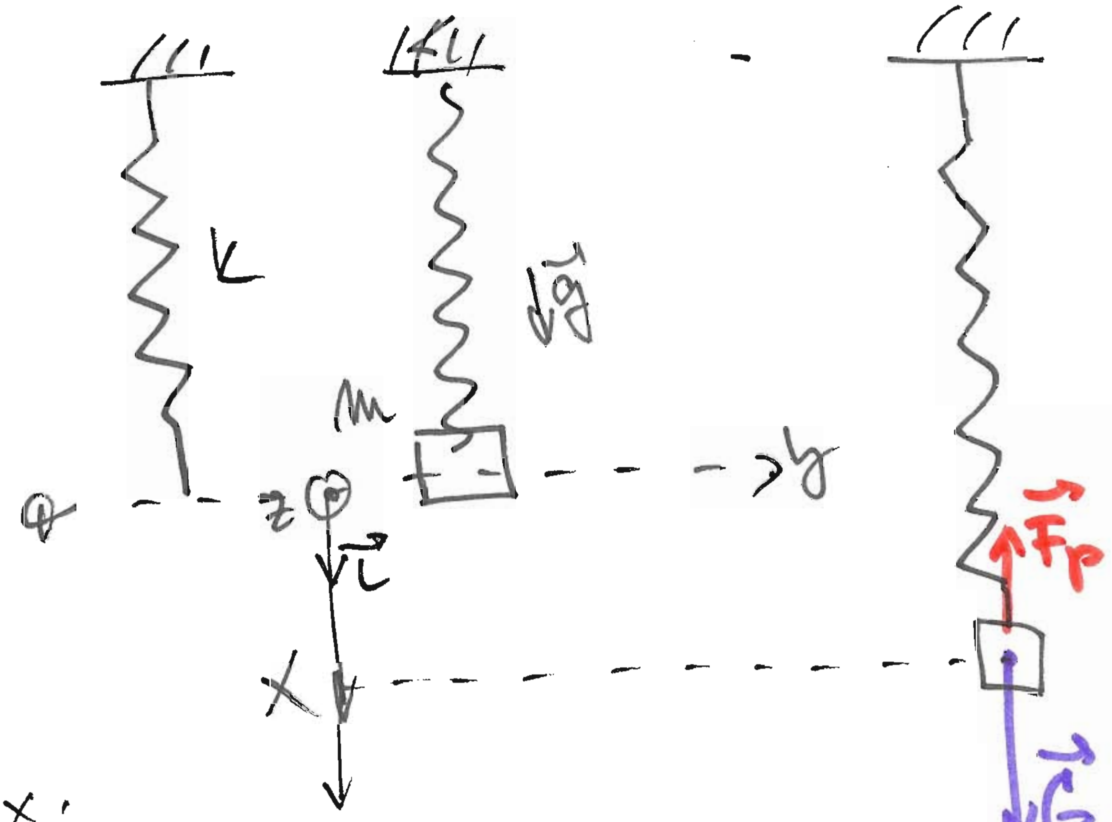
$v_y(t) = v_{max} \cos\omega t$

$y_{max} = R$

$v_{max} = \omega R$

$a_{max} = \omega^2 R$

$a_y(t) = - a_{max} \sin\omega t$



$\vec{F_v} = m\vec{a}$

$\vec{F_p} + \vec{G} = m\vec{a}$

$k x_r = mg$

$x_r = \frac{mg}{k}$

x:

$$m\ddot{x} = mg - kx$$

$$\boxed{\ddot{x} + \frac{k}{m}x = g} \Rightarrow \ddot{x}_0(t) + \omega^2 (x_0(t) + K) = g$$

$$\underbrace{\ddot{x}_0(t) + \omega^2 x_0(t)}_{0} + \omega^2 K = g$$

$$\ddot{x}_0(t) + \omega^2 x_0(t) = 0$$

$$x_0(t) = A\sin(\omega t + \alpha)$$

$$X(t) = x_0(t) + K$$

$$\boxed{K = \frac{g}{\omega^2}}$$

$$\dot{x}(t) = \dot{x}_0(t)$$

$$\ddot{x}(t) = \ddot{x}_0(t)$$

$$X(t) = A\sin(\omega t + \alpha) + \frac{g}{\omega^2}$$

$$X(0) = \underbrace{0 = A\sin\alpha + \frac{g}{\omega^2}}$$

$$\dot{x}(0) = \underbrace{A\omega\cos\alpha = 0}$$

$$\alpha = \frac{\pi}{2}, -\frac{\pi}{2}$$

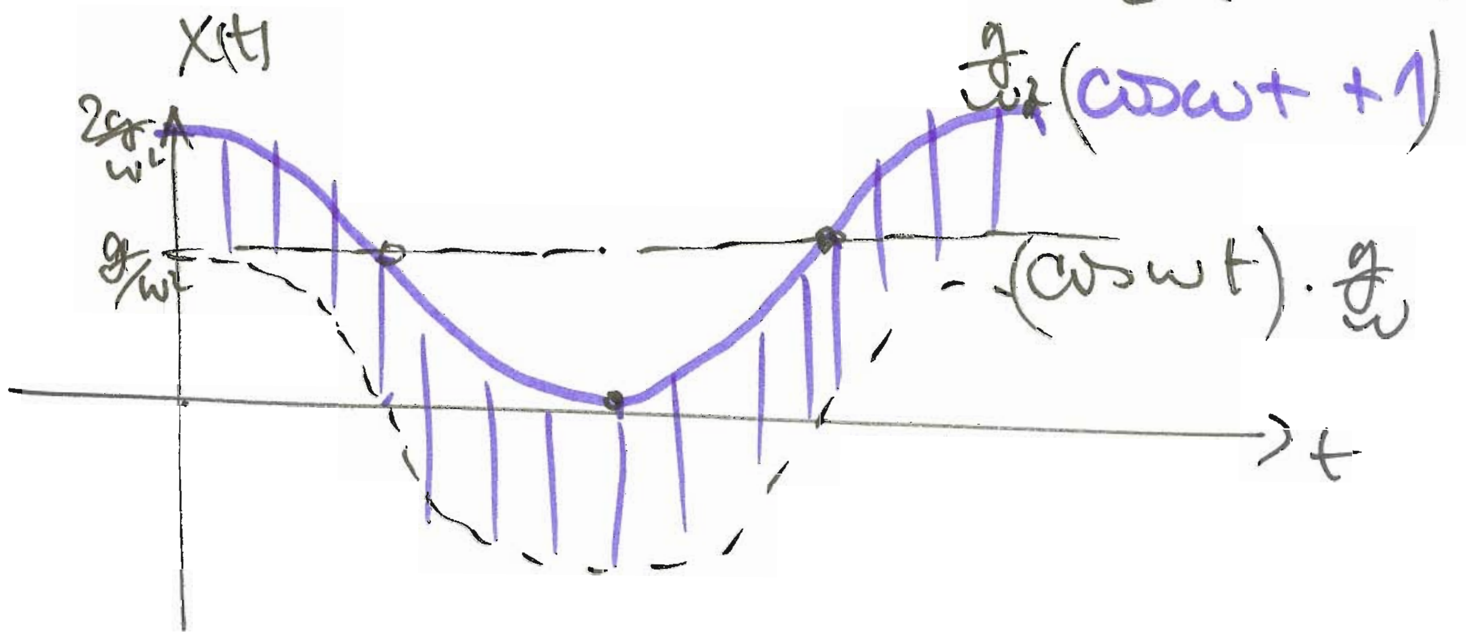
$$A = \frac{g}{\omega^2}$$

$$x(t) = \frac{g}{\omega^2}\sin\left(\omega t + \frac{\pi}{2}\right) + \frac{g}{\omega^2}$$

$$X(t) = \frac{g}{\omega^2}\cos\omega t + \frac{g}{\omega^2} \Rightarrow$$

$$\underline{\underline{X(t) = \frac{g}{\omega^2}(\cos\omega t + 1)}} = \frac{g}{k/m}(\ldots)$$

$$= \frac{mg}{k}(\cos\omega t + 1)$$



ČASOVÝ INTEGRÁL SÍLY (impuls)

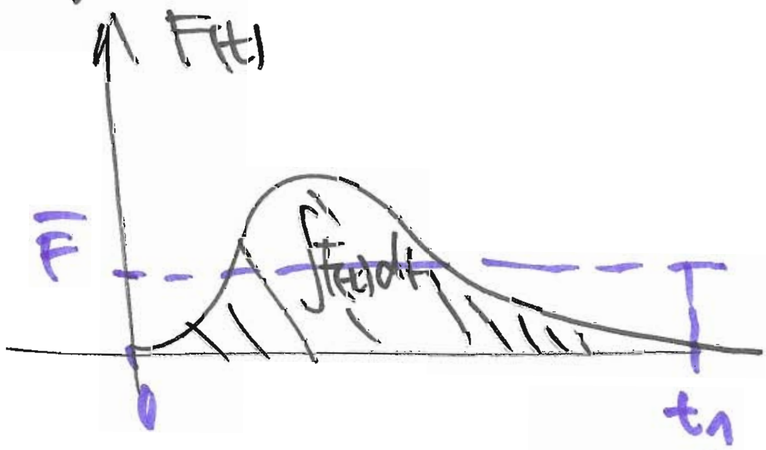
DRÁHOVÝ INTEGRÁL SÍLY (práce, energie)

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{F}(t)\,dt = d\vec{p}$$

$$\underbrace{\int \vec{F}(t)\,dt}_{\vec{I}} = \int d\vec{p}$$

$\vec{I}$ impuls síly

$$|\vec{I}| = \int_0^t F(t)\,dt$$



$$\bar{F}(t_1 - t_0) = \boxed{} = \int_{t_0}^{t_1} F(t)\,dt \Rightarrow \bar{F} = \frac{1}{t_1 - t_0} \underbrace{\int_{t_0}^{t_1} F(t)\,dt}_{p(t_1) - p(t_0)}$$

$$\bar{F} = \frac{1}{\Delta t}\,\Delta p$$