Dobré ráno!

DRÁHOVÝ ÚČINEK SÍLY

Práce & energie (J) skalár

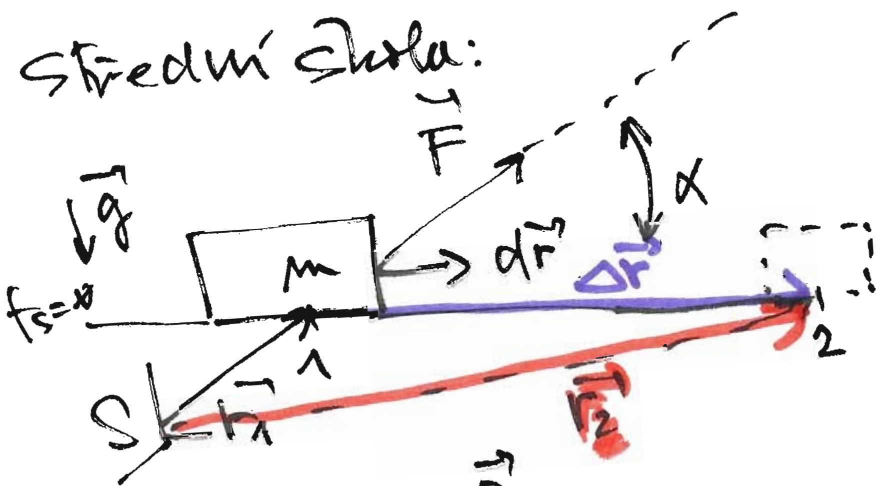
δW

$d\vec{F}$

$$\delta W = \vec{F} \cdot d\vec{r} \begin{cases} >0 \\ =0 \\ <0 \end{cases}$$

$F\, dr \cos\alpha$

Střední škola:



(+) $\alpha \in \langle 0, \pi/2)$

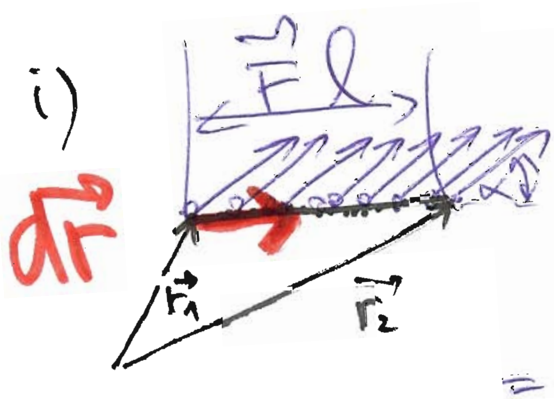
(0) $\alpha = \pi/2$ ($\perp$)

(−) $\alpha \in (\frac{\pi}{2}, \frac{3\pi}{2})$

$$W_{F,1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}$$

?

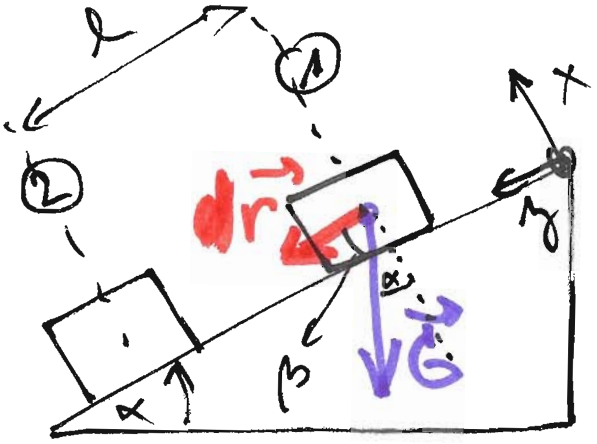
i)



$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{|\vec{r}_1|}^{|\vec{r}_2|} F \cdot dr \cdot \cos\alpha =$$

$$= F \cdot \cos\alpha \int_{r_1}^{r_2} dr = F \cdot l \cdot \cos\alpha$$

Př.:



$\downarrow \vec g$

2/11

$W_{\vec F_t,1\to 2} = ?$

DÚ: $W_{\vec N,1\to 2} = ?$

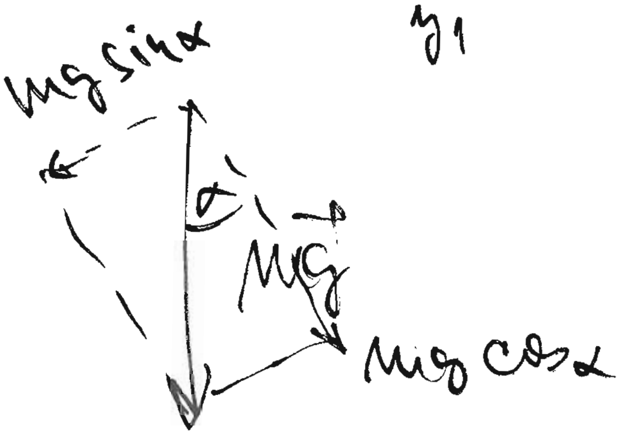
$$W_{\vec G,1\to 2} = \int_{\vec r_1}^{\vec r_2} \vec G \cdot d\vec r = \int_{y_1}^{y} mg\,dy\,\overbrace{\cos\underbrace{\beta}_{(90-\alpha)}}^{\sin\alpha} > 0$$

ΠΓ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$\underbrace{\cos 90 \cos\alpha}_{0} \mp \underbrace{\sin 90}_{1} \sin\alpha$$

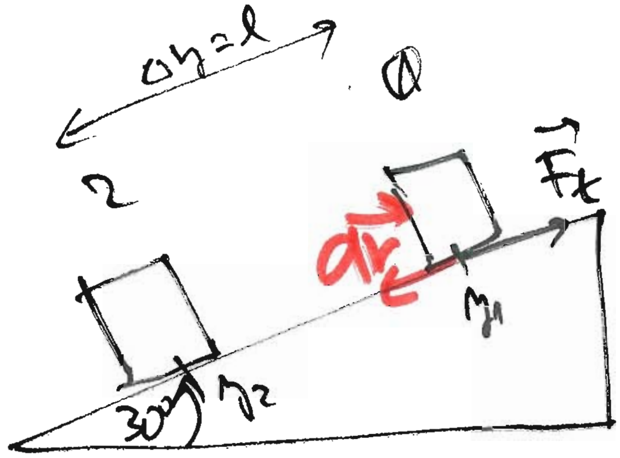
$$= mg\sin\alpha \int_{y_1}^{y_2} dy = mgl\sin\alpha$$

ΓNN



√

DÚ



$$W_{\vec F_t, 1\to 2} = \int_{\vec r_1}^{\vec r_2} \vec F_t \cdot d\vec r = \int_{y_1}^{y_2} |\vec F_t|\,|d\vec r| \cos 180° =$$

$$|\vec F_t| = f_d |\vec N| = f_d\, mg \cos 30°$$

$$= f_d\, mg \cos 30° \cos 180° \int_{y_1}^{y_2} dy = \underbrace{-f_d\, mg \underbrace{\cos 30°}_{\frac{\sqrt{3}}{2} > 0} \cdot l}_{< 0}$$

Potenciální energie

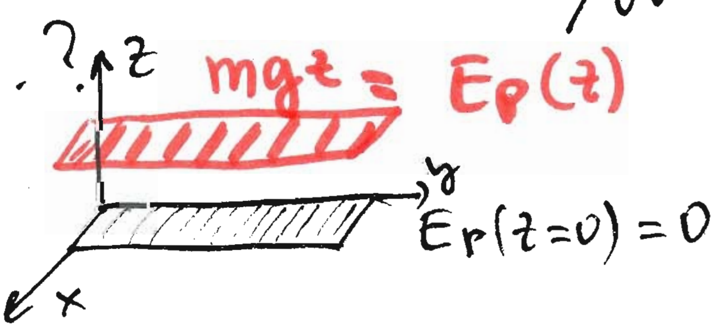
„regál"

i) Volba referenční hladiny ... $E_p = 0$

volba

ii) Výpočet E_p ...?

✓ tíhová G, m

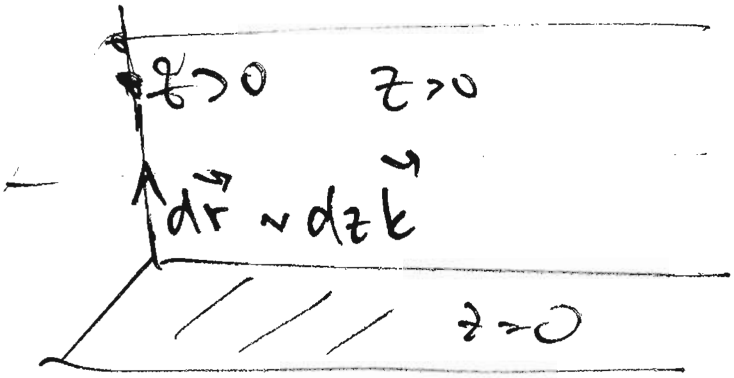


④

$E_p = ?$

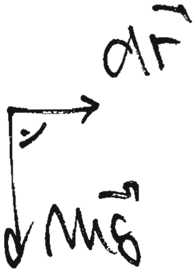
$$W_{\vec G,1\to 2} = \int_{\vec r_1}^{\vec r_2} \vec G \cdot d\vec r = -\int_0^z mg\,dz$$

$\cos 180°$

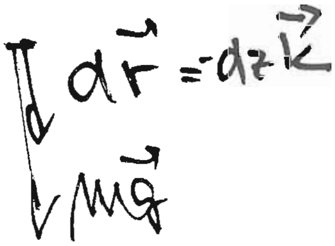


$$\vec G \cdot d\vec r = m\vec g \cdot d\vec r = mg(-\vec k)\,dz\vec k$$
$$= -mg\,dz$$

Γ



$m\vec g \cdot d\vec r = 0$



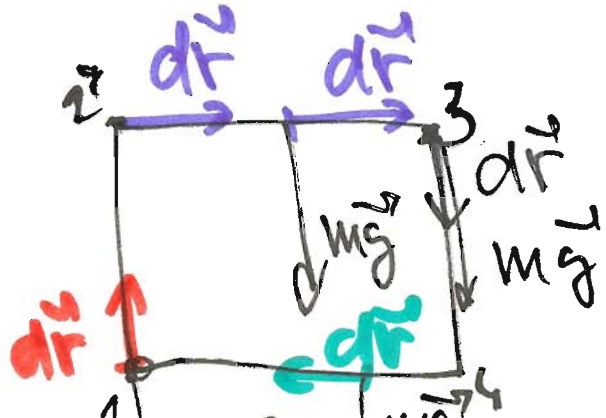
$$m\vec g \cdot d\vec r = mg\,dr = -mg(dz)$$

$$-mg\vec k\,(-dz)\vec k = mg\,dz$$

$dz>0$

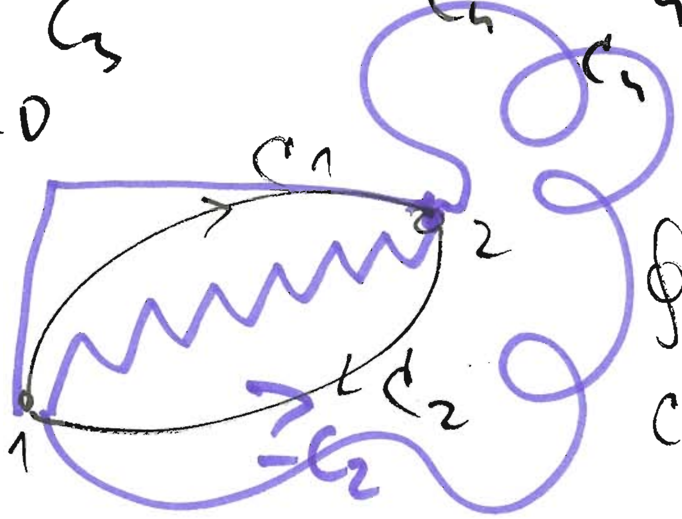
$$\boxed{\Delta E_{p/t} = -W_{\vec G,1\to 2}}$$

$\Delta z > 0$ $\Delta z < 0$



$$\oint_C \vec{G}\cdot d\vec{r} = \int_1^2 \vec{G}\cdot d\vec{r} + \int_2^3 \vec{G}\cdot d\vec{r} + \int_3^4 \vec{G}\cdot d\vec{r} + \int_4^1 \vec{G}\cdot d\vec{r}$$

$$= -\Delta E_{p,t} = 0$$



$$\oint_{C_1+C_2} \vec{G}\cdot d\vec{r} = 0$$

$$0 = \int_{C_1} \vec{G}\cdot d\vec{r} + \int_{C_2} \vec{G}\cdot d\vec{r} \Rightarrow \int_{C_1} \vec{G}\cdot d\vec{r} = -\int_{C_2} \vec{G}\cdot d\vec{r}$$

$$\boxed{\oint \vec{G}\cdot d\vec{r} = 0} \Rightarrow \text{konzervativni pole}$$

Potenciální energie pružiny

$$\boxed{E_{p,p} = \frac{1}{2}kx^2}$$

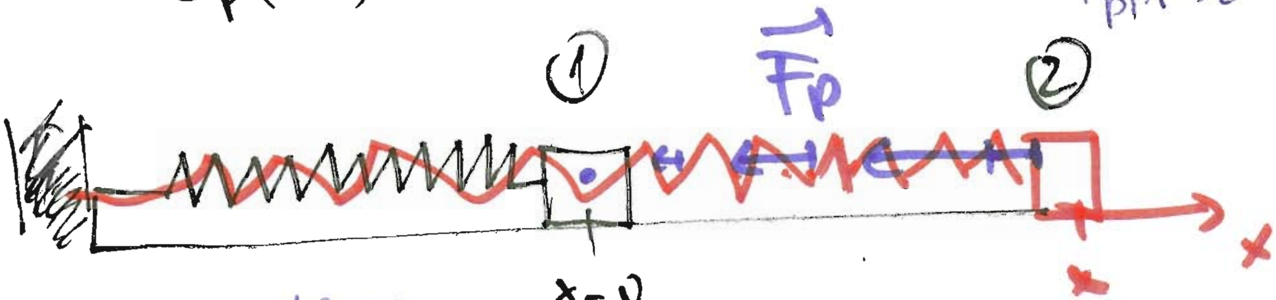
$$E_p(1) - E_p(2) = -\Delta E_p = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_p \cdot d\vec{r}$$

(0) $(x=a)$

$$\vec{F}_p = -kx\vec{i}$$

$$E_p(x=0) - E_p(x>0)$$

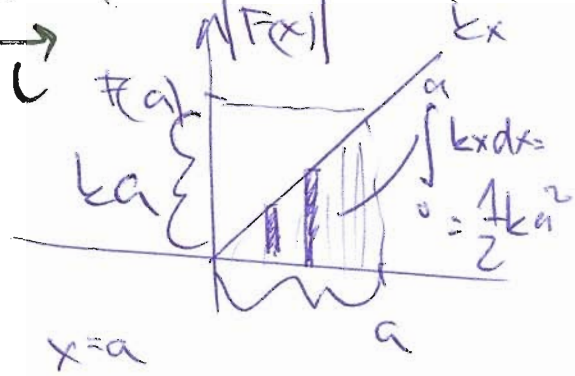
$$W_{\vec{F}_{p,1\to2}} \overset{?}{=} < 0$$



$$dx\,\vec{i}$$

$\cos 180°$

$$W_{F_p\,0\to x>0} = -\int_0^{x>0} \underbrace{kx}_{|\vec{F}_p|}\,dx$$



$$-\Delta E_p = -\int_0^{x>0} kx\,dx = -k\int_0^{x=a} x\,dx = \underbrace{-\frac{1}{2}ka^2}_{<0}$$

$$-(E_p(x=a) - E_p(x=0))$$

$$E_{p,p}(x) = E_{p,p}(x=0) + \frac{1}{2}kx^2$$

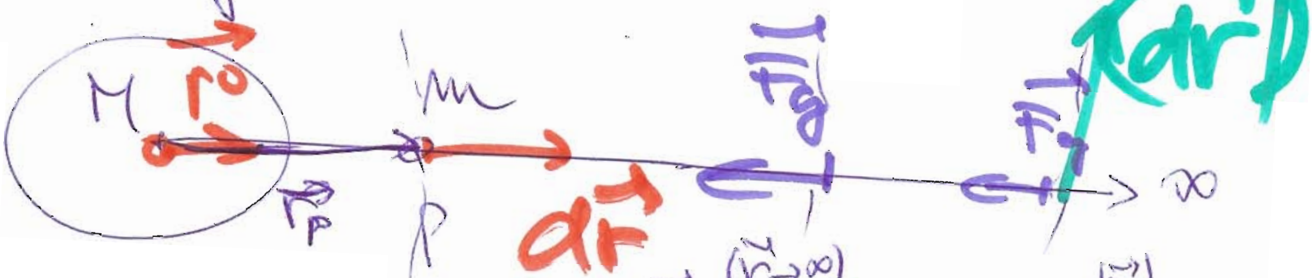
$\Downarrow$ 0 volba

Gravitační pole

$E_{p,g} = ?$

Známe sílu, chci pole
(zákon) (integrál)

$$\vec{F}_g = -\varkappa \frac{Mm}{r^2}\vec{r}^{\,0}$$



$$-\Delta E_{p,g} = \underset{1}{E_{p,g}(P)} - \underset{2}{E_{p,g}(r)} = \int_{\vec{r}_1 = \vec{r}(P)}^{\vec{r}_2\ (\vec{r}\to\infty)} \vec{F}_g \cdot d\vec{r} = \int_{|\vec{r}_P|}^{|\vec{r}|} |\vec{F}_g|\,|d\vec{r}|\cos 180°$$

$$= \varkappa Mm \underbrace{\cos 180°}_{(-1)} \int_{r_p}^{r} \frac{dr}{r^2} = \varkappa Mm \left[\frac{1}{r}\right]_{r_p}^{r}$$

$$= \varkappa Mm\left(\frac{1}{r} - \frac{1}{r_p}\right)$$

$$\int \frac{dx}{x^2} = \left(\frac{-1}{x}\right)'$$

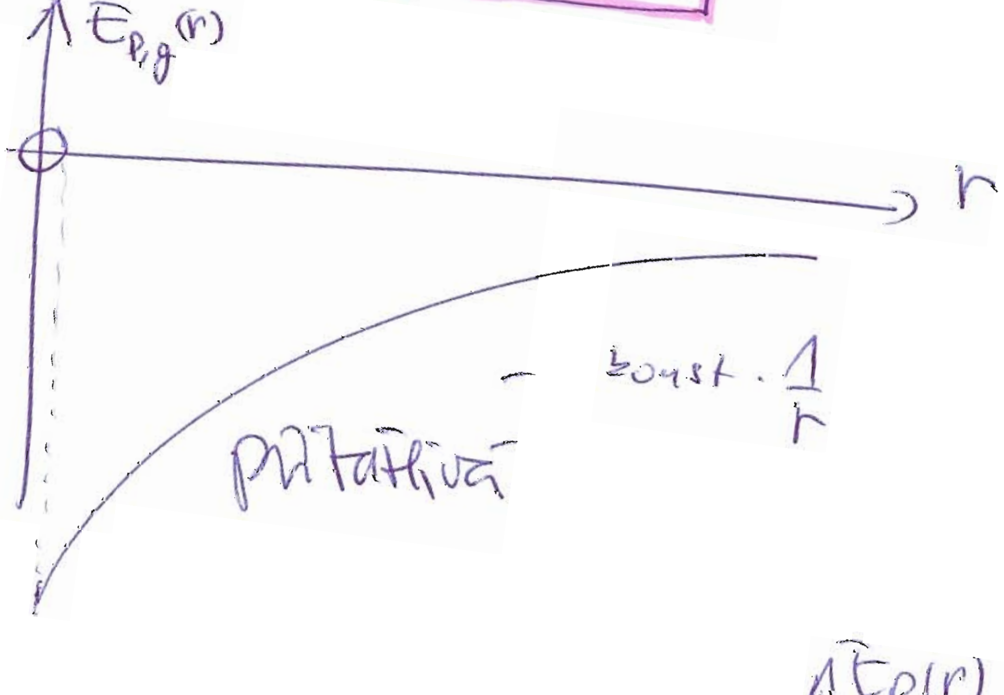
$$r \to \infty \quad \frac{1}{r} \to 0$$

$$E_{p,g}(P) - E_{p,g}(r \to \infty) = \varkappa m M \left(\frac{1}{\infty} - \frac{1}{r_P}\right) =$$ 8/11

$$E_{p,g}(P) - E_{p,g}(\infty) = -\varkappa m M \cdot \frac{1}{r_P}$$

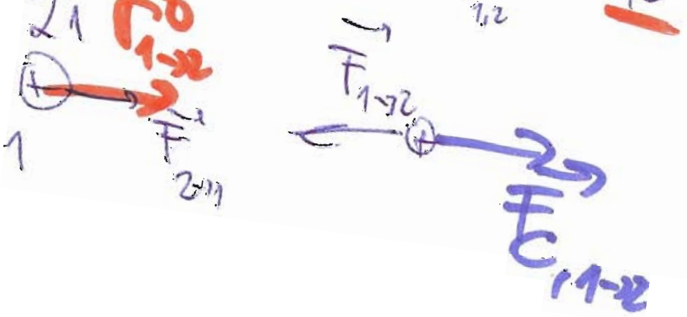
$$E_{p,g}(r_P) = \frac{-\varkappa m M}{r_P} + E_{p,g}(\infty)$$

$$\boxed{E_{p,g}(r) = -\varkappa m M \cdot \frac{1}{r}}$$ volba: 0



Γ NN

$$\vec{F}_C = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \vec{r}^{\,0}_{12}$$



UMíME

☺ SÍLA (skalár) $\Rightarrow$ pole (regál)

Chceme:

pole $\xrightarrow{\text{určit}}$ sílu

konzervativní pole

konzervativní síle

$E_p(\vec{r})$ ... číslo

... $\vec{F}(\vec{r})$

vektor

$\vec{F}(\vec{r}) = \quad E_p(\vec{r}) \qquad \vec{a} = \lambda \vec{b}$

násobení vektorem

vektor

skalár

nelze násobit

gradient $= \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$$\boxed{\vec{F}_k = -\vec{\nabla} E_p} = \left(-\frac{\partial E_p}{\partial x}, -\frac{\partial E_p(x,y,z)}{\partial y}, -\frac{\partial E_p(\ldots)}{\partial z}\right)$$

$$\text{grad} = \vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\text{div} = \vec{\nabla} \cdot \vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\text{rot}\,\vec{A} = \vec{\nabla} \times \vec{A}$$

DÚ

$\vec{F}_p = -k x \vec{i}$

$\vec{G} = m\vec{g}$

$\vec{F}_g = \ldots$

$(\vec{F}_c = \ldots)$

Zákony zachování mechanické energie

$$\vec{F}_v = \vec{F}_k + \vec{F}_j$$

jiné, než konzervativní

konzervativní síly (síla)

$$\int_1^2 \vec{F}_v \cdot d\vec{r} = \int_1^2 \vec{F}_k \cdot d\vec{r} + \int_1^2 \vec{F}_j \cdot d\vec{r}$$

$$\Delta E_k = -\Delta E_p + W_{\vec{F}_j, 1\to 2}$$

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_v \cdot d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} m\vec{a} \cdot d\vec{r} = m\int \frac{d\vec{v}}{dt} \cdot d\vec{r} = m\int_{\vec{v}_1}^{\vec{v}_2} \frac{d\vec{r}}{dt} \cdot d\vec{v}$$

$$= m\int_{\vec{v}_1}^{\vec{v}_2} \underbrace{\vec{v} \cdot d\vec{v}}_{v\,dv} = m\int_{v_1}^{v_2} v\,dv = \frac{1}{2} m \left(v^2\right)_{v_1}^{v_2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$= \Delta E_k$$

$$\Delta E_k + \Delta E_p = W_{\vec{F}_j, 1\to 2}$$

$$\Delta(\underbrace{E_k + E_p}_{E_M}) = W_{\vec{F}_j, 1\to 2}$$

$$\boxed{\Delta E_M = W_{\vec{F}_j, 1\to 2}}$$!

změna mechanické energie

? kdy platí ZZME ?

$\Delta E_M = 0 \Rightarrow W_{\vec{F}_j, 1\to 2} = 0$

→ $\vec{F}_j$ nejsou

→ $\vec{F}_j \perp d\vec{r}$