8. přednáška (9. týden)

Dobrý den!

$$\vec{F}_V = \vec{F}_k + \vec{F}_j$$

elektrická
pružiny
tíhová

$\vec{F}_C = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{1,2}^2} \vec{r}^{\,0}_{1,2}$

$\vec{F}_p = -k\vec{r}$ $\quad (\vec{F}_p = -kx\vec{i})$

$\vec{F}_G = \vec{G} = m\vec{g}$

$\vec{F}_g = -\varkappa \frac{M_Z m}{r^2} \vec{r}^{\,0}$

$\vec{F}_K$ ... gravitační

$$W_{\vec{F}_K, I \to II} = \int_{I, C_1}^{II} \vec{F}_k \cdot d\vec{r} = \int_{I, C_2}^{II} \vec{F}_k \cdot d\vec{r}$$

$$\oint \vec{F}_K \cdot d\vec{r} = \vec{0}$$

$$\mathrm{rot}\, \vec{F}_K = \vec{0} \quad (\vec{\nabla} \times \vec{F}_K = \vec{0}) \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \vec{0}$$

$$\int_{I, C}^{II} \vec{F}_V \cdot d\vec{r} = \int_{I, C}^{II} \vec{F}_K \cdot d\vec{r} + \int_{I, C}^{II} \vec{F}_j \cdot d\vec{r}$$

$$\Delta E_K = -\Delta E_p + W_{\vec{F}_j, I \to II}$$

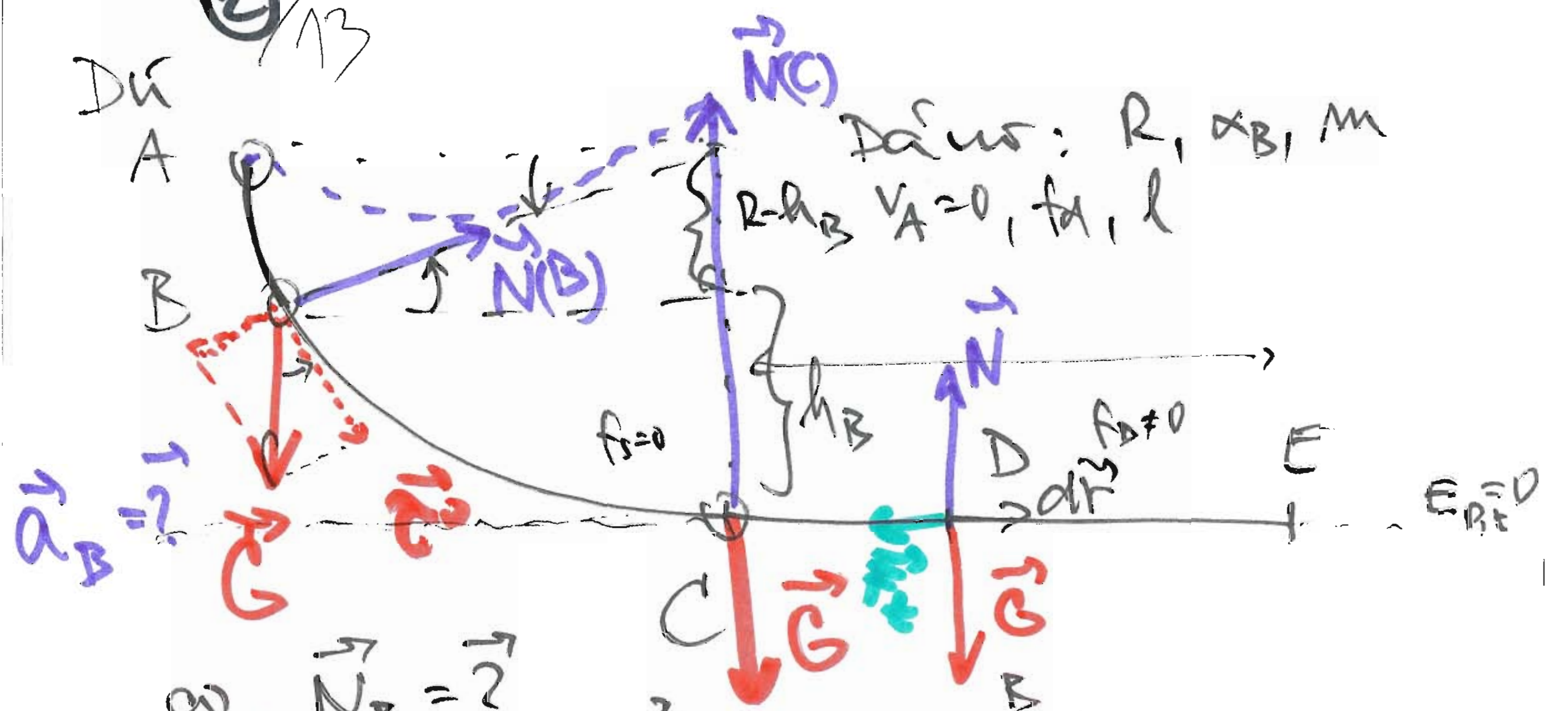
$$E_{k,II} - E_{k,I} + E_{p,II} - E_{p,I} = W_{\vec{F}_j, I \to II} \quad ; E_{M_1} = E_{k_1} + E_{p_1}$$

$$E_{M,II} - E_{M,I} = W_{\vec{F}_j, I \to II} \quad \Rightarrow \text{ZZME}: \; ? \; \boxed{\Delta E_M = 0}$$

Dú

Dáno: R, α_B, m

$v_A = 0, f_d, l$



a) $\vec{N}_B = ?$

$$\int_A^B \vec{F}_{V_{A\to B}} \cdot d\vec{r} = \int_A^B \vec{G} \cdot d\vec{r} + \int_A^B \vec{N} \cdot d\vec{r}$$

$$E_k(B) - E_k(A) = E_p(A) - E_p(B) + \cancel{0}$$

$$a_n(B) = \frac{v^2(B)}{R} \qquad m a_n(B) = F_{v,n} = N - mg \sin\alpha_B$$

$$m \frac{v_B^2}{R}$$

$$\frac{1}{2} m v_B^2 - 0 = mgR - mgh_B = mg(R - h_B)$$

$h_B = ?$ $\quad R - h_B = R \cdot \sin\alpha_B \Rightarrow h_B = R(1 - \sin\alpha_B)$

$$\frac{1}{2} \cancel{m} v_B^2 = \cancel{m} g R \sin\alpha_B \Rightarrow \underline{\underline{v_B^2 = 2gR\sin\alpha_B}}$$

$$m \frac{v_B^2}{R} = N(B) - mg\sin\alpha_B \Rightarrow$$

$$N(B) = m\left(\frac{v_B^2}{R} + \cancel{mg}\, g\sin\alpha_B\right) = m\, 3g \sin\alpha$$

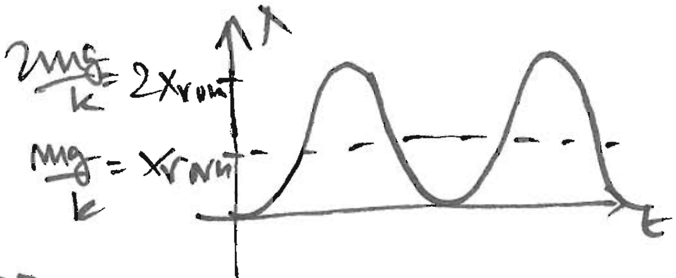
$$\Delta E_M = E_M(E) - E_M(C) = \int_C^E \vec{F}_t \cdot d\vec{r} = \boxed{-mgf\,d\,l}$$

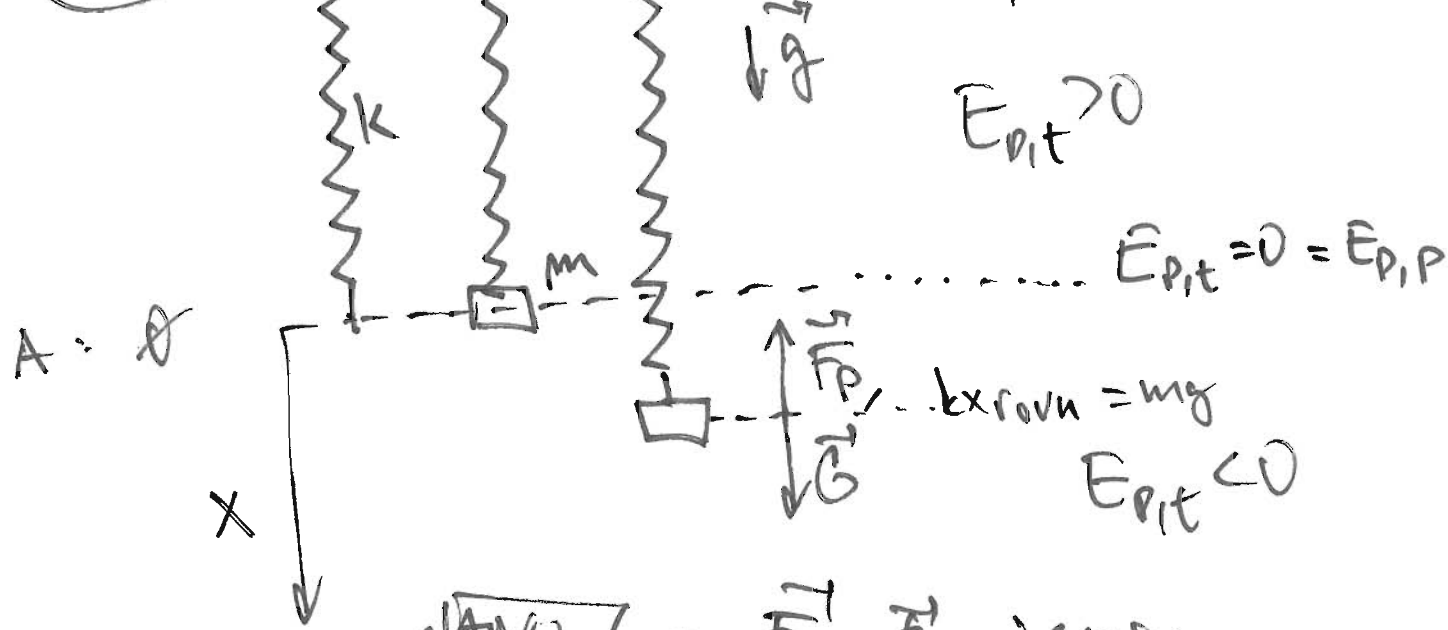
$$\boxed{\frac{1}{2}mv_E^2 - \frac{1}{2}mv_C^2}$$

K_1

⋮

Pr.



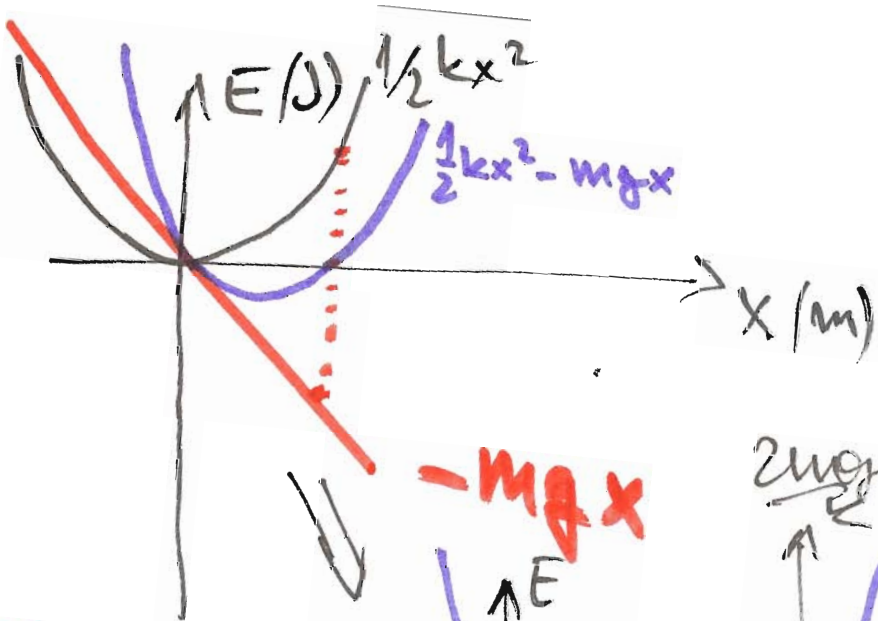


ZEME ... $\nearrow$ $\boxed{\text{ANO}}$ $\to$ $\vec{F}_p, \vec{G}$ jenom

$\searrow$ NE

$$E_{M,I} = E_{M,II} = \text{konst} = \ldots$$

$$E_M(A) = \frac{1}{2}mv_A^2 + E_{p,p}(A) + E_{p,t}(A) = 0 + 0 + 0 = 0$$



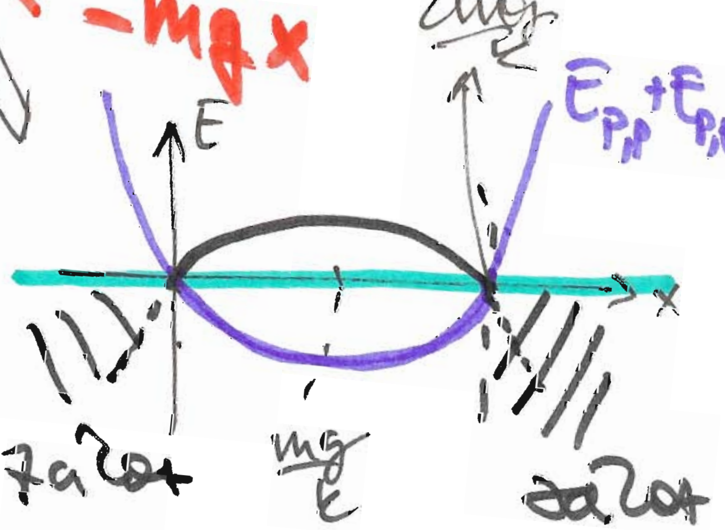
$E_{p,p}(x) = \frac{1}{2}kx^2$

$E_{p,t}(x) = -mgx$

$E_M = 0$

$E_K + E_p = 0$

$\frac{1}{2}mv^2 \geq 0$



Body obratu:

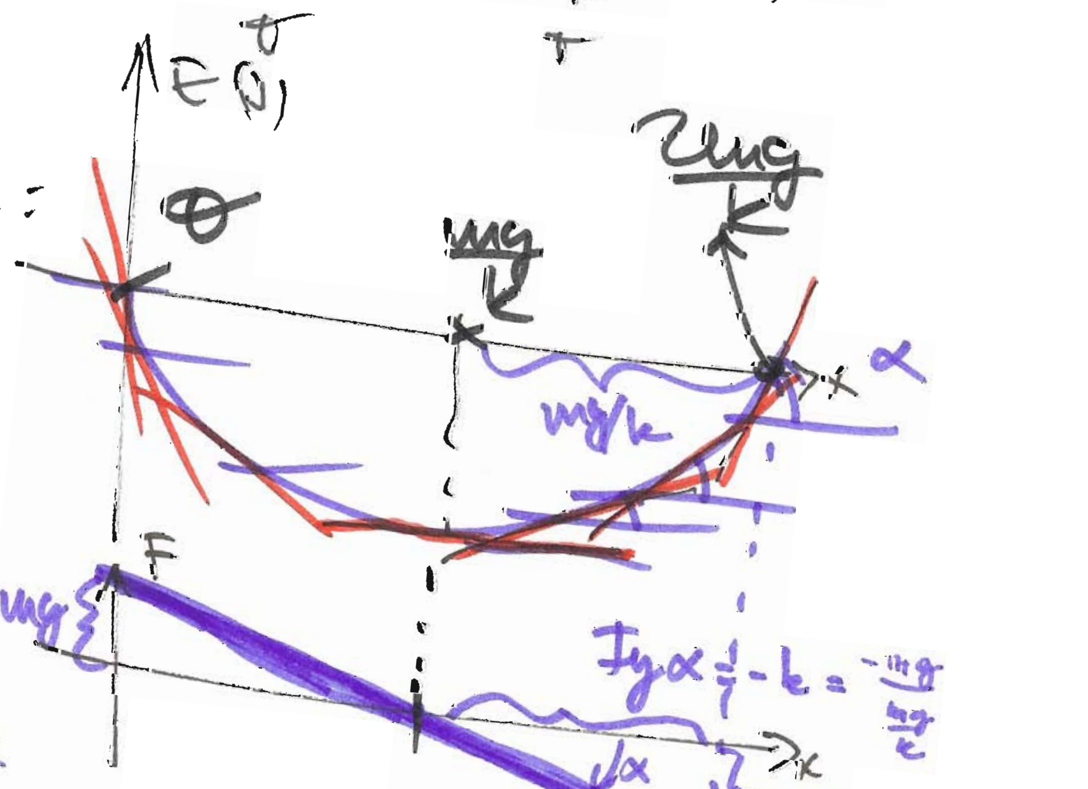
$E_k = 0$: $\frac{1}{2}kx_{B,0}^2 - mgx_{B,0} = 0$

$x_{B,0}\left(\frac{1}{2}kx_{B,0} - mg\right) = 0$

$x_{B,0}(1) = 0$

$x_{B,0}(2) = \frac{2mg}{k}$

Výsledná síla:



$F = -\mathrm{grad}\, E_p$

$\frac{d}{dx}\left(\frac{1}{2}kx^2 - mgx\right) =$

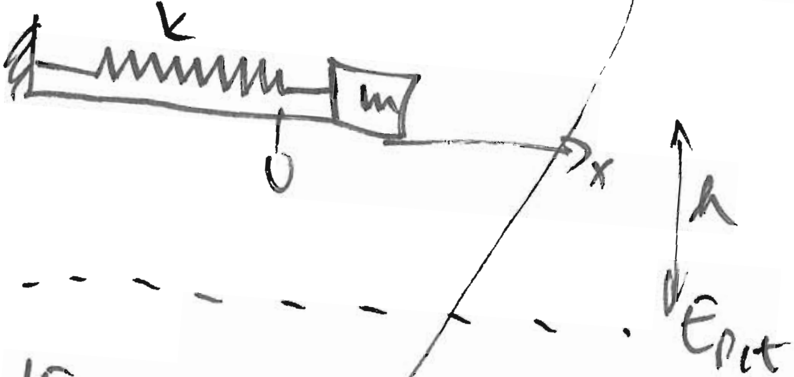
$= -(kx - mg) =$

$F_{x,V} = mg - kx$

5 ΓNN

/13

$$\frac{dE_M(x,\dot{x})}{dt} = \frac{\partial E_M}{\partial x}\frac{dx}{dt} + \frac{\partial E_M}{\partial \dot{x}}\frac{d\dot{x}}{dt} = \frac{d(\text{konst.})}{dt} = 0$$



$$E_M = \frac{1}{2}m\dot{x}^2 + mgh + \frac{1}{2}kx^2 = \underline{\text{konst.}}$$

$$\frac{dE_M}{dt} = kx\dot{x} + m\dot{x}\ddot{x} = 0$$

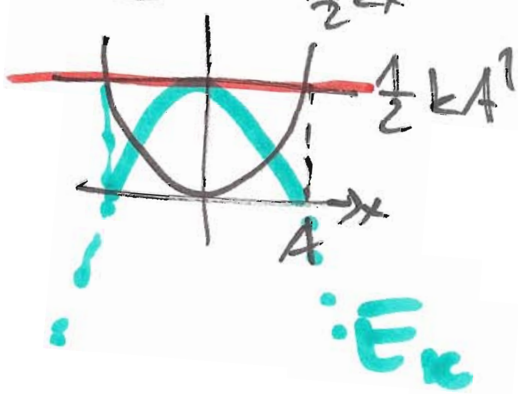
$$\dot{x}(kx + m\ddot{x}) = 0$$

$$\boxed{\dot{x} = 0} \qquad kx + m\ddot{x} = 0 \Rightarrow \boxed{\ddot{x} + \frac{k}{m}x = 0}$$

$$E_M = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$x(t) = A\sin(\omega t + \alpha)$$

DÚ



DÚ <u>37ME VOLNÝ PÁD</u>

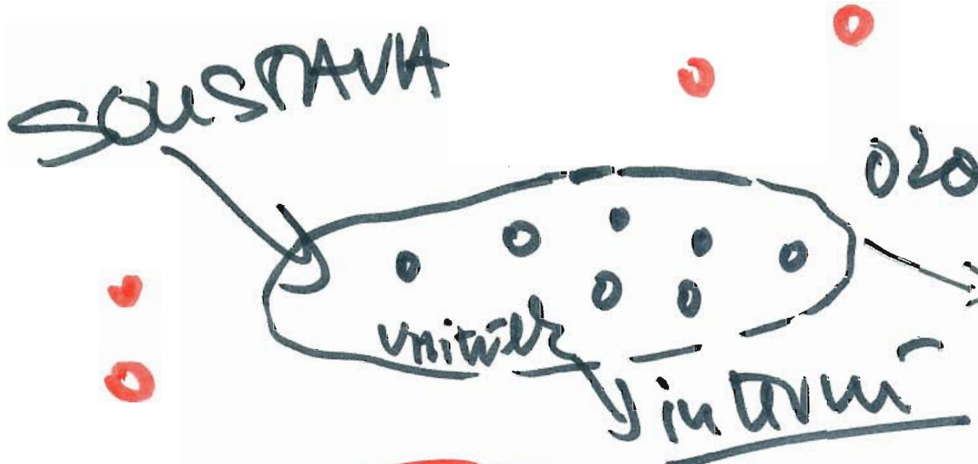
MRW: VÝKON

$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

okamžitý

střední $\bar{P} = \frac{\Delta W}{\Delta t}$

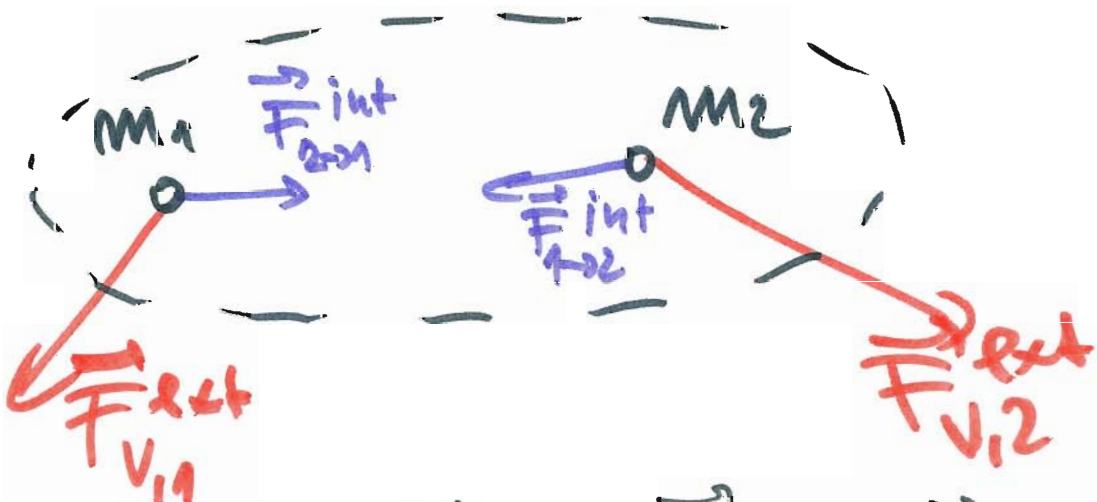
SOUSTAVY ČÁSTIC



okolí (externí)

soustava složená z N ... diskrétních (7) hmotných bodů

spojitě rozložená hmotnost



m_1: $\vec{F}_{V\to 1} = m_1\vec{a}_1$ $\qquad \vec{F}_{V\to 2} = m_2\vec{a}_2$

II. NPZ: $m_1\vec{a}_1 = \vec{F}^{ext}_{V,1} + \vec{F}^{int}_{2\to 1}$

$$m_2\vec{a}_2 = \vec{F}^{ext}_{V,2} + \vec{F}^{int}_{1\to 2}$$

+ III. NPZ $\Rightarrow 0$

$$m_1\vec{a}_1 + m_2\vec{a}_2 = \vec{F}^{ext}_V \left(+ \underbrace{\vec{F}^{int}_{2\to 1} + \vec{F}^{int}_{1\to 2}}\right)$$

7/13 $m_1\vec{a}_1 + m_2\vec{a}_2 = \vec{F}_V^{ext}$

$$\vec{a}_1 = \frac{d\vec{v}_1}{dt} \qquad \vec{a}_2 = \frac{d\vec{v}_2}{dt}$$

$$\frac{d(m_1\vec{v}_1)}{dt} + \frac{d}{dt}(m_2\vec{v}_2) = \vec{F}_V^{ext}$$

Zavedeme $\vec{p} \equiv m\vec{v}$ hybnost $(kg\,m\,s^{-1})$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{F}_V^{ext} = \dot{\vec{p}}_1 + \dot{\vec{p}}_2$$

$$\frac{d}{dt}(\underbrace{\vec{p}_1 + \vec{p}_2}_{\vec{P}}) = \vec{F}^{ext}$$

celková hybnost soustavy

1. věta impulsová:

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}_V^{ext}}$$

? kdy platí ZZH ? $\vec{P} = \overrightarrow{konst} \Rightarrow \vec{F}_V^{ext} = \vec{0}$

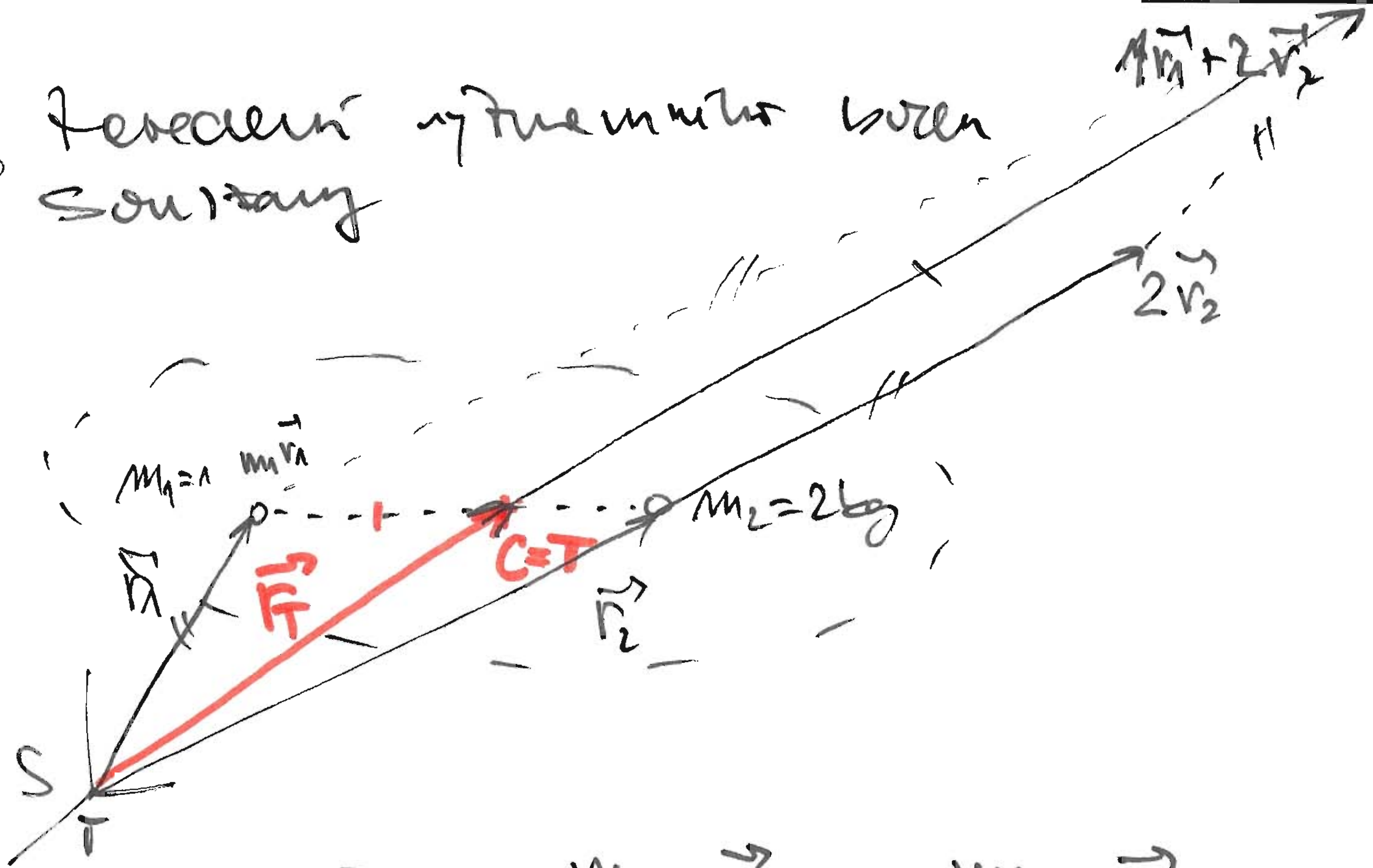
$F_{V,x}^{ext} = 0$

$F_{V,y}^{ext} = 0$

$F_{V,z}^{ext} = 0$

8
13

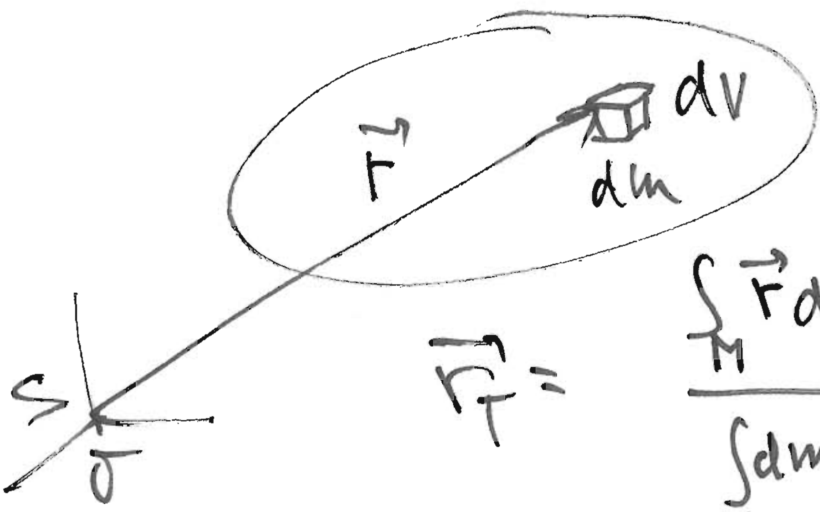
Zavedení významného bodu soustavy



$$\vec{r}_T = \frac{m_1}{m_1+m_2}\vec{r}_1 + \frac{m_2}{m_1+m_2}\vec{r}_2$$

$$= \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N=2} m_i \vec{r}_i}{} = \frac{1}{M}\sum_i m_i \vec{r}_i$$

– spojitě rozl. hmotnost:



$$\vec{r}_T = \frac{\int_M \vec{r}\,dm}{\int_M dm}$$

$$= \frac{\sum_i m_i \vec{r}_i}{\underbrace{\sum_i m_i}_{M}}$$

9/13 $\rho(\vec{r}) = \frac{dm}{dV}$

$$\vec{r}_T = \frac{\int_M \vec{r}\,dm}{\int_M dm} = \frac{\iiint_V \vec{r}\,\rho(\vec{r})\,dV}{\iiint_V \rho(\vec{r})\,dV} =$$

jednodušší případ $\rho(\vec{r}) = \rho = \text{konst.}$

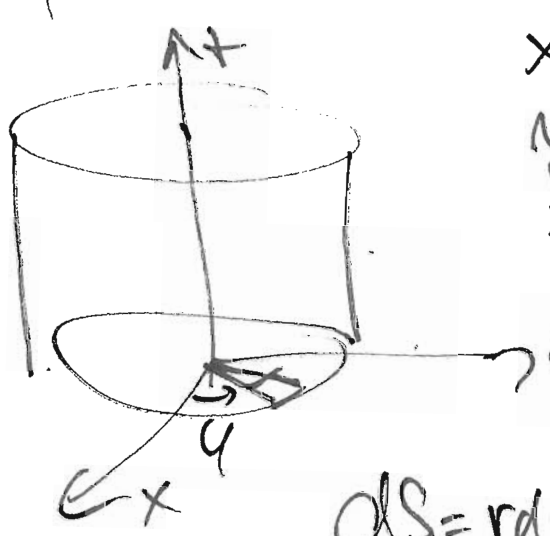
$$= \frac{\cancel{\rho}\iiint_V \vec{r}\,dV}{\cancel{\rho}\iiint_V dV} = \frac{1}{V}\iiint_V \vec{r}\,dV$$

$r_{T,x} = \frac{1}{V}\iiint_V x\,dV$ $\qquad dV \,..\, dx\,dy\,dz$

$r_{T,y} = \frac{1}{V}\iiint_V y\,dV$ $\qquad ..\, r\,dr\,d\varphi\,dz$

$r_{T,z} = \frac{1}{V}\iiint_V z\,dV$ $\qquad ..\, r^2 \sin\varphi\,dr\,d\varphi\,d\vartheta$

VÁLEC, KVÁDR



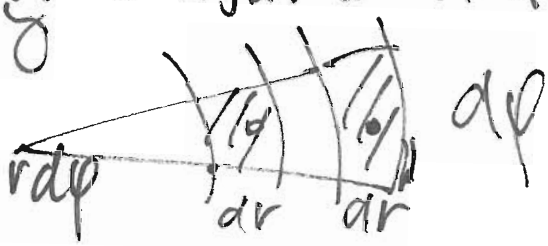
$x = r\cos\varphi$

$y = r\sin\varphi$

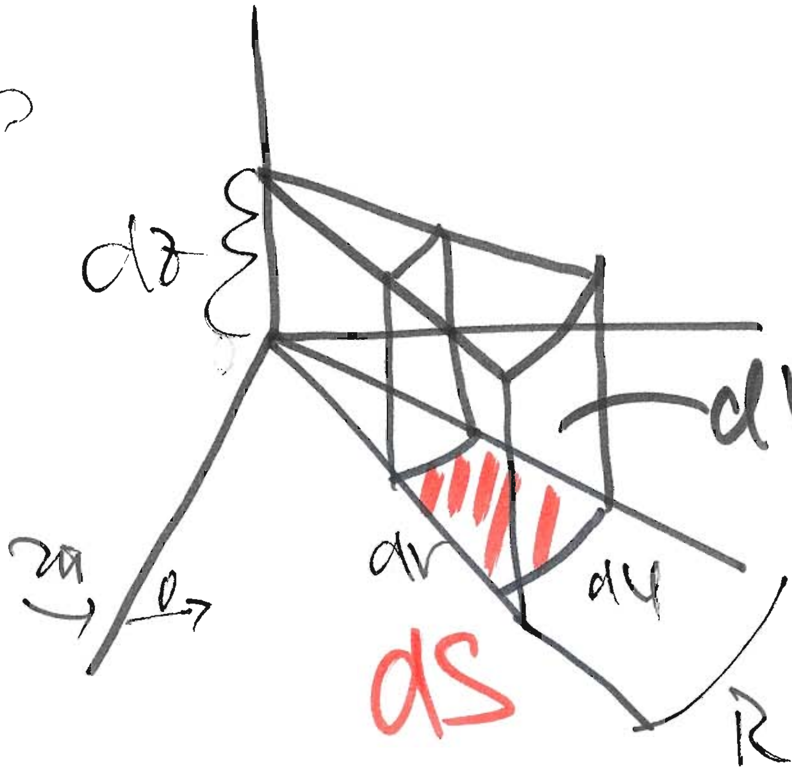
$z = z$

$dV = dx\,dy\,dz = r\,dr\,d\varphi\,dz$

$dS = r\,dr\,d\varphi$



10/13



$dV = dS \cdot dz$

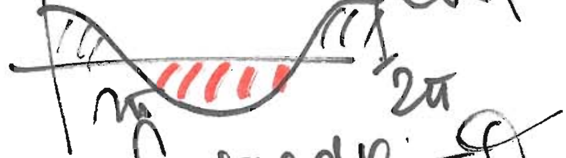
$r\,dr\,d\varphi\,dz$

$$\vec{r}_T = \frac{1}{V}\iiint \vec{r}\,dV \qquad \left[* \frac{1}{\cancel{\pi R^2} \cancel{h}} \frac{\cancel{R^2}}{2} \cancel{2\pi} \frac{\cancel{h^2}}{\cancel{2}} = \frac{h}{2} \right]$$

$$x_T = \frac{1}{V} \int_0^h \int_0^{2\pi} \int_0^R \cancel{x}\, r\,dr\,d\varphi\,dz = \frac{1}{V}\int_0^h \int_0^{2\pi} \int_0^R r^2 \cos\varphi\,dr\,d\varphi\,dz = 0$$

$x \to r\cos\varphi$

(z) (φ) (r)



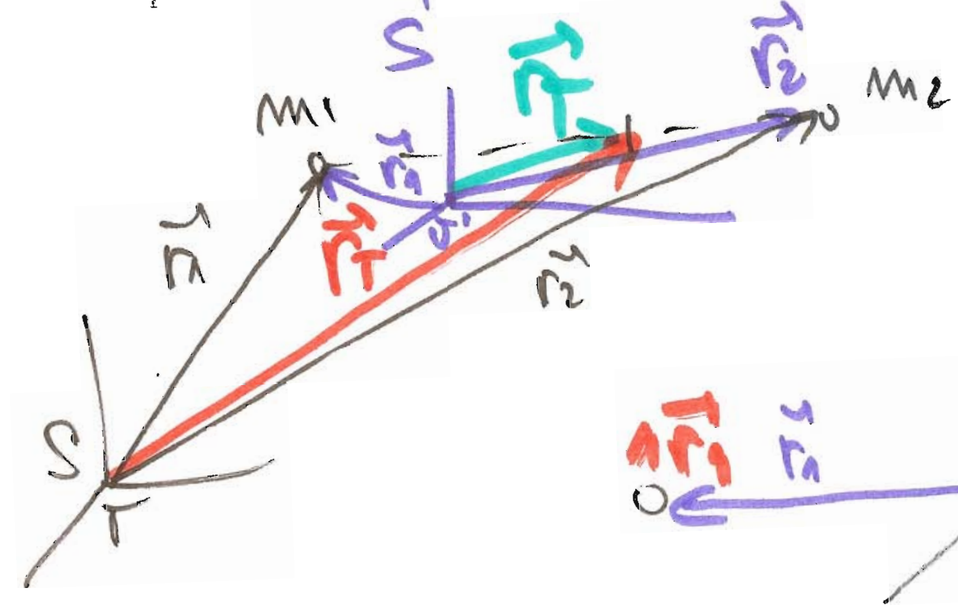
$$\int_0^{2\pi} \cos\varphi\,d\varphi = 0$$

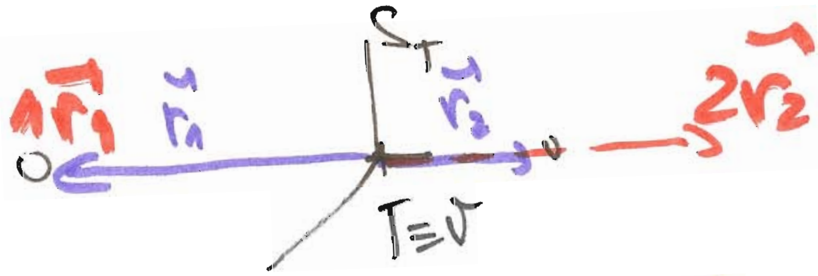
$$\int_0^{2\pi} \sin\varphi\,d\varphi = 0$$

$y_T = 0$

$$z_T = \frac{1}{\pi R^2 \cdot h} \int_0^h \int_0^{2\pi} \int_0^R z\, r\,dr\,d\varphi\,dz = \frac{1}{V} \left[\frac{r^2}{2}\right]_0^R \left[\frac{z^2}{2}\right]_0^h \cdot \left[\varphi\right]_0^{2\pi}$$

*





těžišťová soustava: $\vec r_T = \vec 0 = \frac{\sum_i m_i \vec r_i}{\sum_i m_i}$

$$\boxed{\sum_i m_i \vec r_i = \vec 0}$$

$$\boxed{\vec r_T = \frac{\sum_i m_i \vec r_i}{\sum_i m_i}} \; ; \quad \dot{\vec r}_T = \boxed{\vec v_T = \frac{\sum_i m_i \dot{\vec r}_i}{\sum_i m_i}}$$

$$\frac{d\vec v_T}{dt} = \dot{\vec v}_T = \ddot{\vec r}_T = \boxed{\frac{\sum_i m_i \ddot{\vec r}_i}{\sum_i m_i} = \vec a_T}$$

poloha těžiště

rychlost těžiště

zrychlení těžiště

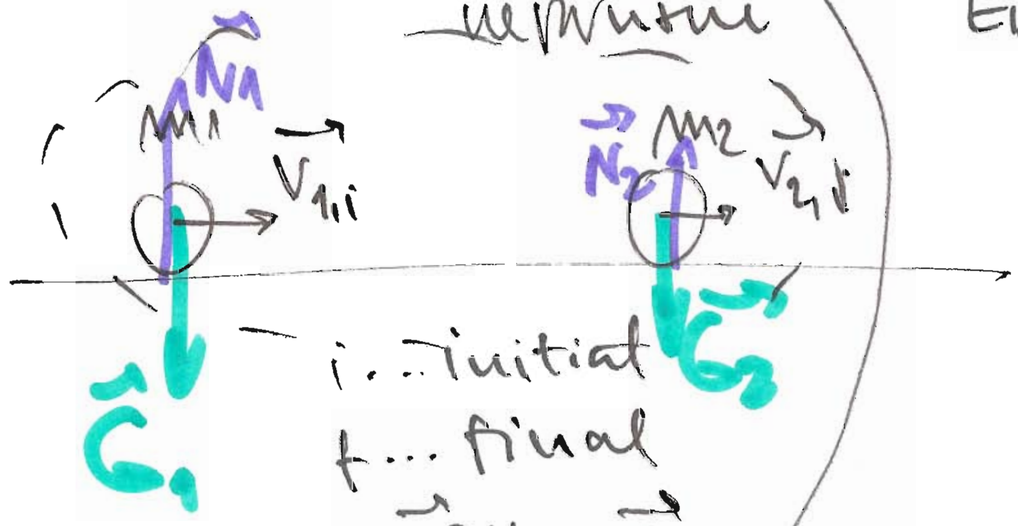
$$m_1 \vec a_1 + m_2 \vec a_2 = \vec F_V^{ext} \qquad \sum_i m_i \ddot{\vec r}_i = \vec a_T \underbrace{\sum_i m_i}_{M}$$

$$\boxed{M \vec a_T = \vec F_V^{ext}}$$ 1. věta impulsová

Srážky

Dokonale pružné ... $E_M = \text{konst}$

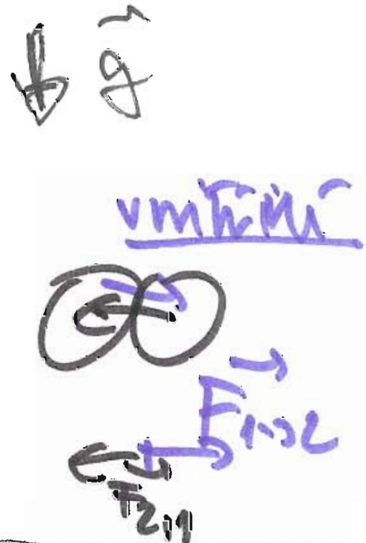
nepružné $E_k \neq \text{konst.}$



i ... initial

f ... final

$\vec{F}_v^{\,ext} = \vec{0} \Rightarrow$



$\boxed{\vec{P} = \text{konst}}$ + $\boxed{E_k \text{ před} = E_k \text{ po srážce}}$

DÚ:
$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$
$$\tfrac{1}{2} m_1 v_{1,i}^2 + \tfrac{1}{2} m_2 v_{2,i}^2 = \tfrac{1}{2} m_1 v_{1,f}^2 + \tfrac{1}{2} m_2 v_{2,f}^2$$

$$\boxed{\begin{aligned} v_{1,f} &= \frac{m_1 - m_2}{m_1 + m_2}\, v_{1,i} \quad \frac{2m_2}{m_1 + m_2}\, v_{2,i} \\ v_{2,f} &= \frac{2m_1}{m_1 + m_2}\, v_{1,i} \quad \frac{m_2 - m_1}{m_1 + m_2}\, v_{2,i} \end{aligned}}$$

$$(2)\quad m_1 (v_{1,i} - v_{1,f}) = -m_2 (v_{2,i} - v_{2,f})$$

$$v_{1,f} = v_{2,f} + v_{2,i} - v_{1,i}$$

$$v_{2,f} = v_{1,i} + v_{1,f} - v_{2,i}$$

$$m_1 (\underbrace{v_{1,i}^2 - v_{1,f}^2}) = -m_2 (v_{2,i}^2 - v_{2,f}^2)$$

$$(1)\quad (v_{1,i} + v_{1,f})(v_{1,i} - v_{1,f}) \qquad (v_{2,i} + v_{2,f})(v_{2,i} - v_{2,f})$$

13

13

Doroucea nipmutuo

$$m_1 \vec{V_1} + m_2 \vec{V_2} = (m_1 + m_2) \vec{V_T}$$