24.11.2008

Dobrý den!

opak.:

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} + \frac{2m_2}{m_1 + m_2} v_{2,i}$$

⇓ (1→2) ⇓ (1→2)

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2,i}$$

čistě dokonale pružná srážka

$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f} \qquad E_{k,1} + E_{k,2} = \text{konst.}$$

Dokonale nepružné (středové)

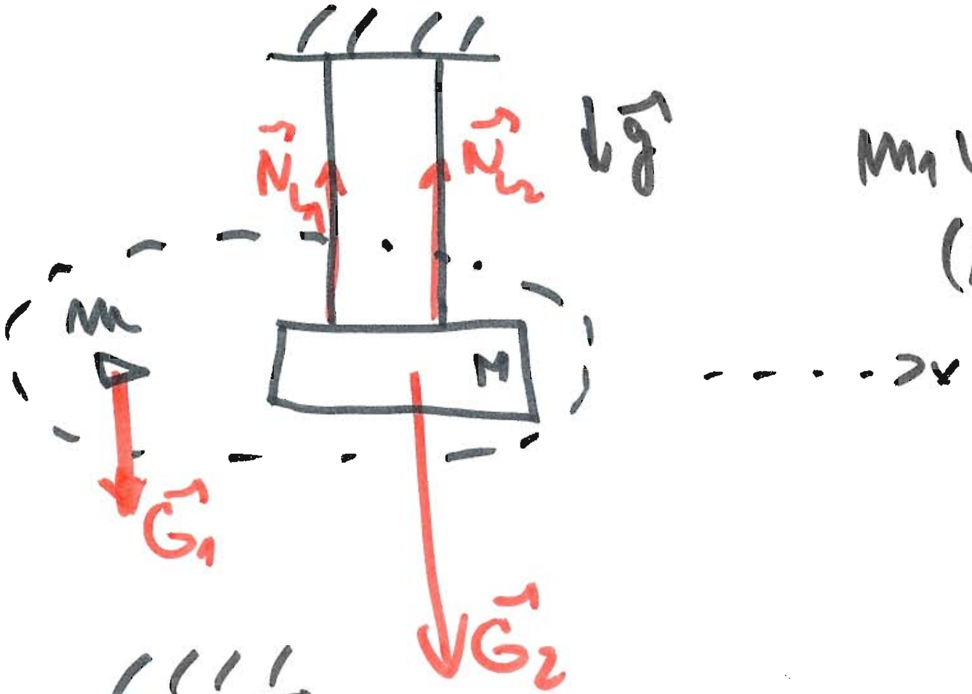
$$m_1 v_{1,i} + m_2 v_{2,i} = (m_1 + m_2) v$$

Př.

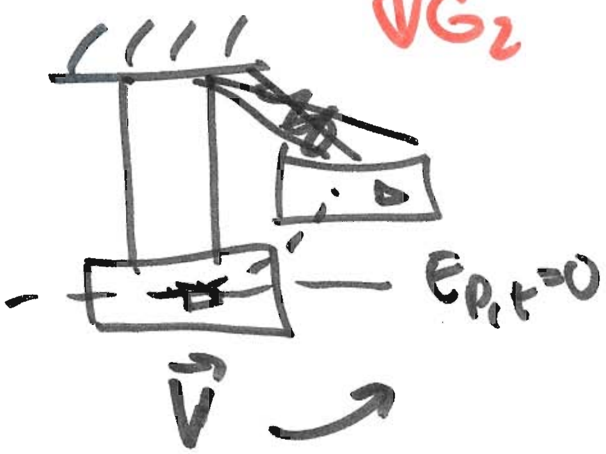
$$\lim_{m_2 \to \infty} \frac{\frac{m_1}{m_2} - \frac{m_2}{m_2}}{\frac{m_1}{m_2} + \frac{m_2}{m_2}} = \lim_{m_2 \to \infty} \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1} = -1$$

↓

Př.: Balistické kyvadlo:



$$m_1 v_{1,i} + 0 = (m_1 + m_2) V$$



ZZE

$$\frac{1}{2}(m+M)V^2 = (m+M)gh$$

⋮

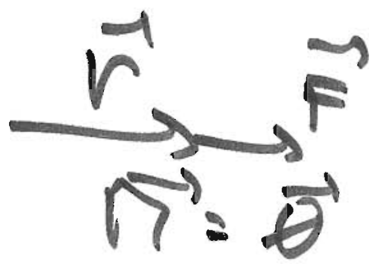
HRW

③

New:

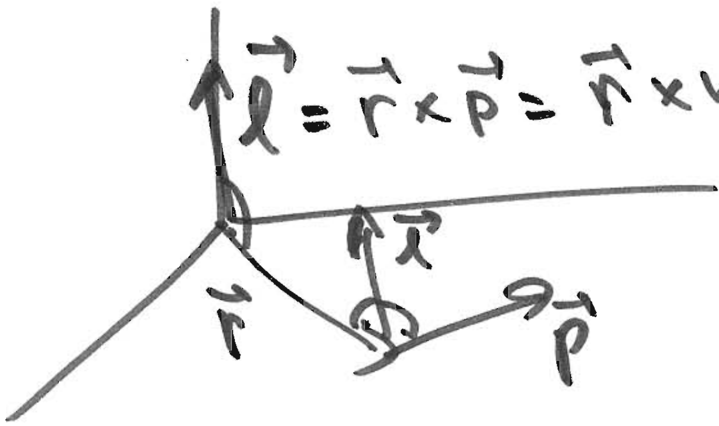
i) moment síly $\vec{M} = \vec{r} \times \vec{F}$

$$|\vec{M}| = |\vec{r}|\,|\vec{F}| \cdot \underline{\underline{\sin\angle(\vec{r},\vec{F})}}$$



ii) moment hybnosti $\vec{l} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ m\dot{x} & m\dot{y} & m\dot{z} \end{vmatrix}$

$\vec{l} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$



iii) moment setrvačnosti

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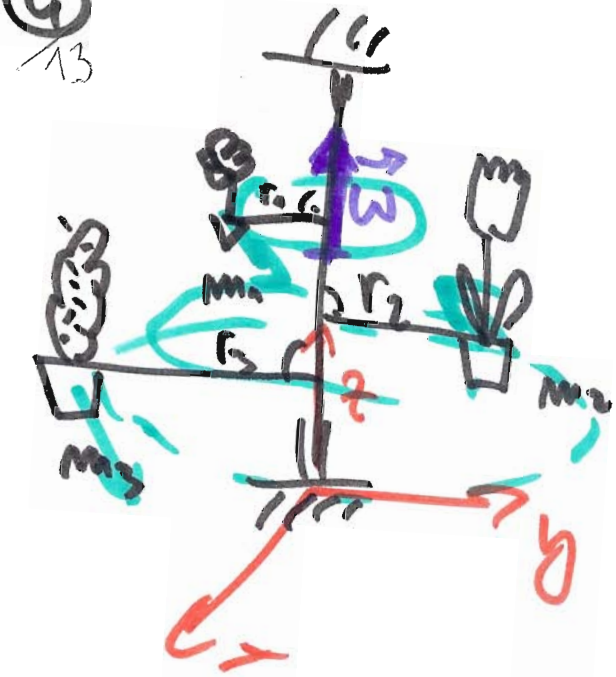


$$E_k = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 = \sum_{i=1}^{3} \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i \omega^2 r_i^2 = \frac{1}{2} \left(\sum m_i r_i^2\right) \cdot \omega^2 = \frac{1}{2} I \omega^2$$

moment setrvačnosti

$$\vec{v}_i = \vec{\omega} \times \vec{r}_i$$

$$|\vec{v}_i| = \omega \cdot r_i$$

$$\boxed{I = \sum_{i=1}^{N} r_i^2 m_i}$$

$$\sigma = \frac{dm}{dS} \qquad \rho = \frac{dm}{dV}$$

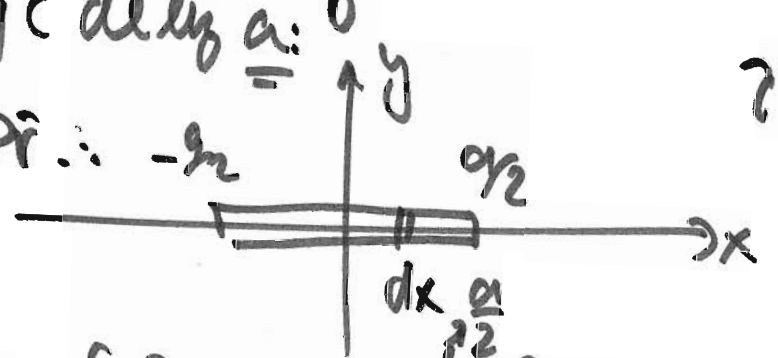
Těleso se spojitě rozloženou hmotností:

$$I = \int_M r^2 \, dm = \iiint_V r^2 \rho \, dV$$

$$\tau = \frac{dm}{dx}$$

5 Tyč délky $\underline{a}$: σ

Př.: i)



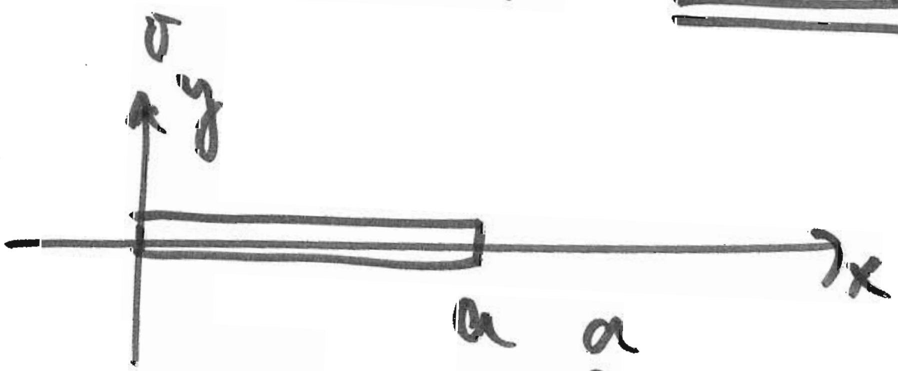
$\lambda = \frac{dm}{dx} = \frac{m}{a} = \text{konst}$

$$I_\sigma = \int_m r^2 dm = \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 \lambda \, dx = \lambda \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx =$$

$$= \frac{m}{a}\left[\frac{x^3}{3}\right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{m}{a}\frac{1}{3}\left(\left(\frac{a}{2}\right)^3 - \left(-\frac{a}{2}\right)^3\right) =$$

$$= \frac{1}{3}\frac{m}{\cancel{a}}\,\cancel{2}\,\frac{a^{\cancel{3}\,2}}{\cancel{8}_{4}} = \underline{\underline{\frac{1}{12} m a^2}} = I_c$$

ii):



$$\int_m r^2 dm = \int_0^a x^2 \lambda \, dx = \frac{m}{a}\int x^2 dx =$$

$$= \frac{m}{a}\left[\frac{x^3}{3}\right]_0^a = \frac{1}{3}\frac{m}{\cancel{a}} a^{\cancel{3}\,2} = \underline{\frac{1}{3} m a^2} = I_\sigma$$

$$I_\sigma = I_c + \boxed{md^2}$$

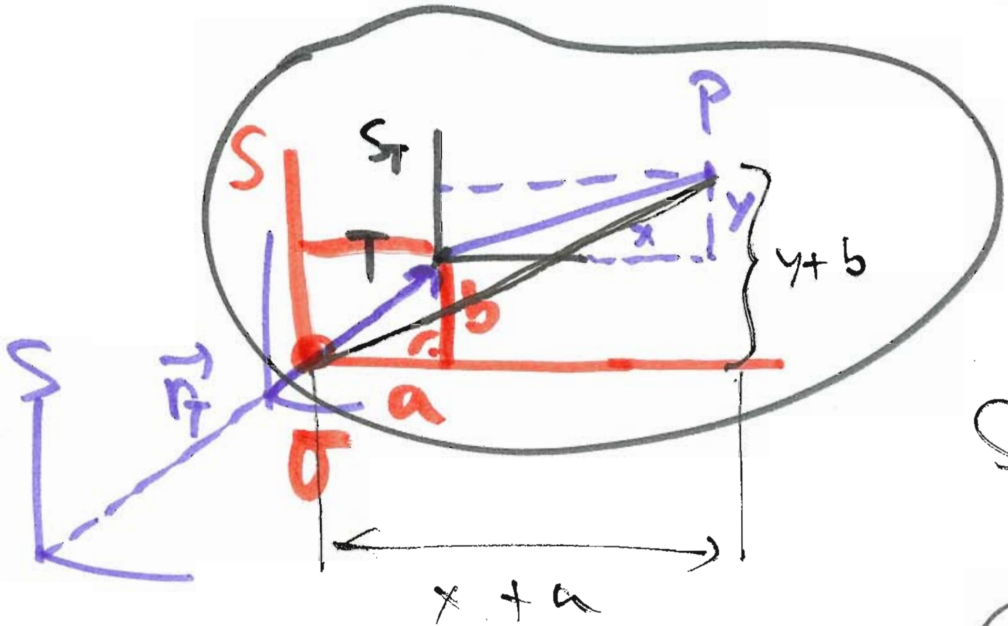
$$\frac{1}{3} m a^2 = \frac{1}{12} m a^2 + m\left(\frac{a}{2}\right)^2$$

Steinerova md^2 věta

✓ OK

Steinerova veta

$$\vec{r}_T = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\iiint_V \rho \vec{r}\, dV}{\iiint_V \rho\, dV}$$



S_T:

$\iiint x\, dV$

$0 = \int_M x\, dm = ?$

$0 = \int_M y\, dm = ?$

$0 = \int_M z\, dm = ?$

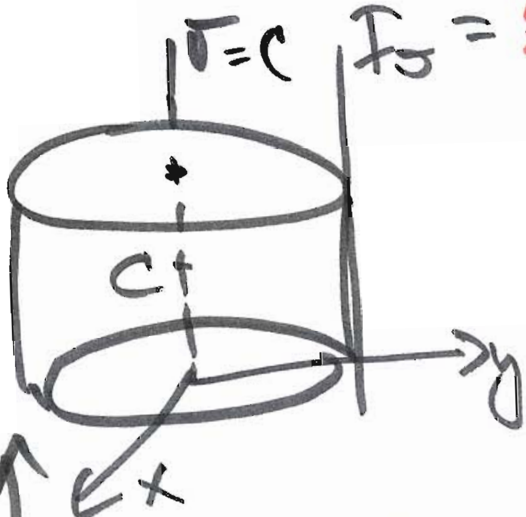
$$I_O = \int_M r^2\, dm = \int_M \left((x+a)^2 + (y+b)^2\right) dm$$

$$= \int \left(x^2 + 2xa + a^2 + y^2 + 2yb + b^2\right) dm$$

$$= \underbrace{\int_M (x^2+y^2)\, dm}_{I_C} + \underbrace{\int_M (a^2+b^2)\, dm}_{md^2} + 2a\int_M x\, dm + 2b\int_M y\, dm$$

$$I_O = I_C + md^2$$

$$I_z = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

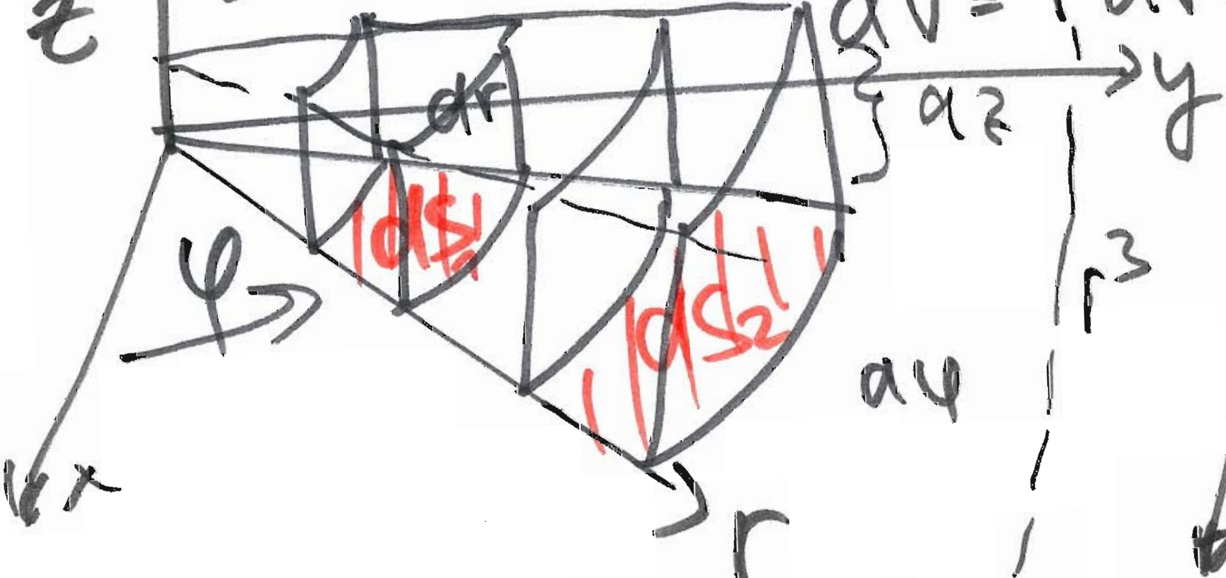


$x = r\cos\varphi$

$y = r\sin\varphi$

$t = z$

$dV = r\,dr\,d\varphi\,dz$



$$I_z = \int_M r^2\,dm = \iiint_V r^2 \rho\, dV$$

$$= \rho \int_0^h \int_0^{2\pi} \int_0^R r^3\,dr\,d\varphi\,dt = \rho\,[z]_0^h\,[\varphi]_0^{2\pi}\left[\frac{r^4}{4}\right]_0^R =$$

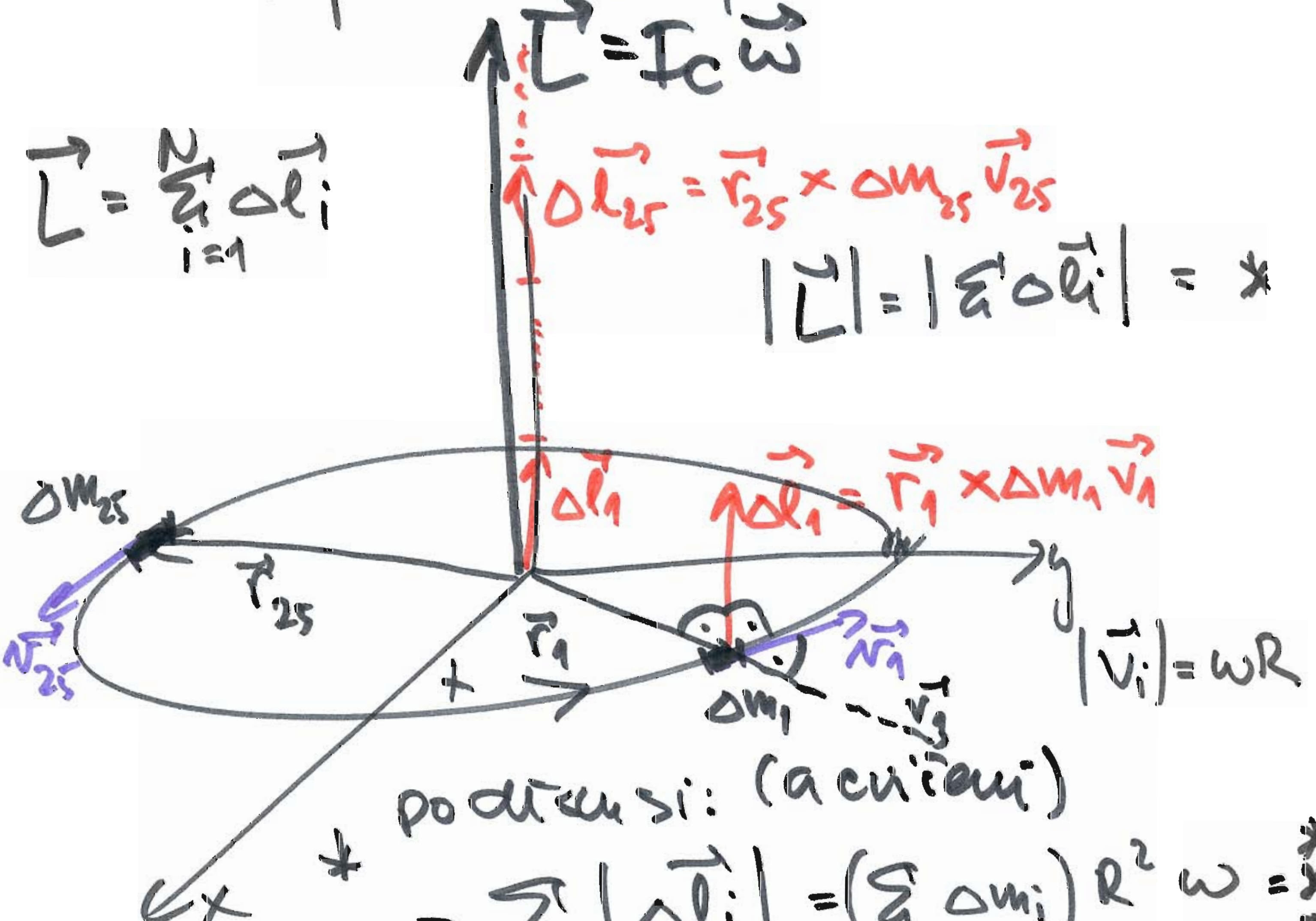
(z)(φ)(r)

$$= \frac{m}{\pi \cancel{R^2} \cdot \cancel{h}} \cdot \cancel{h} \cdot \cancel{2\pi} \cdot \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} = \frac{mR^2}{2}$$

$$I_z = \frac{1}{2}mR^2$$

DÚ : I_c: obruče
I_c: kvádr (desky)

Př.: Odvození celkového momentu $\vec{L}$ hybnosti bicyklového kola:



$$\vec{L} = \sum_{i=1}^{N} \Delta \vec{l}_i$$

$$|\vec{L}| = |\sum_i \Delta \vec{l}_i| = *$$

$$|\vec{v}_i| = \omega R$$

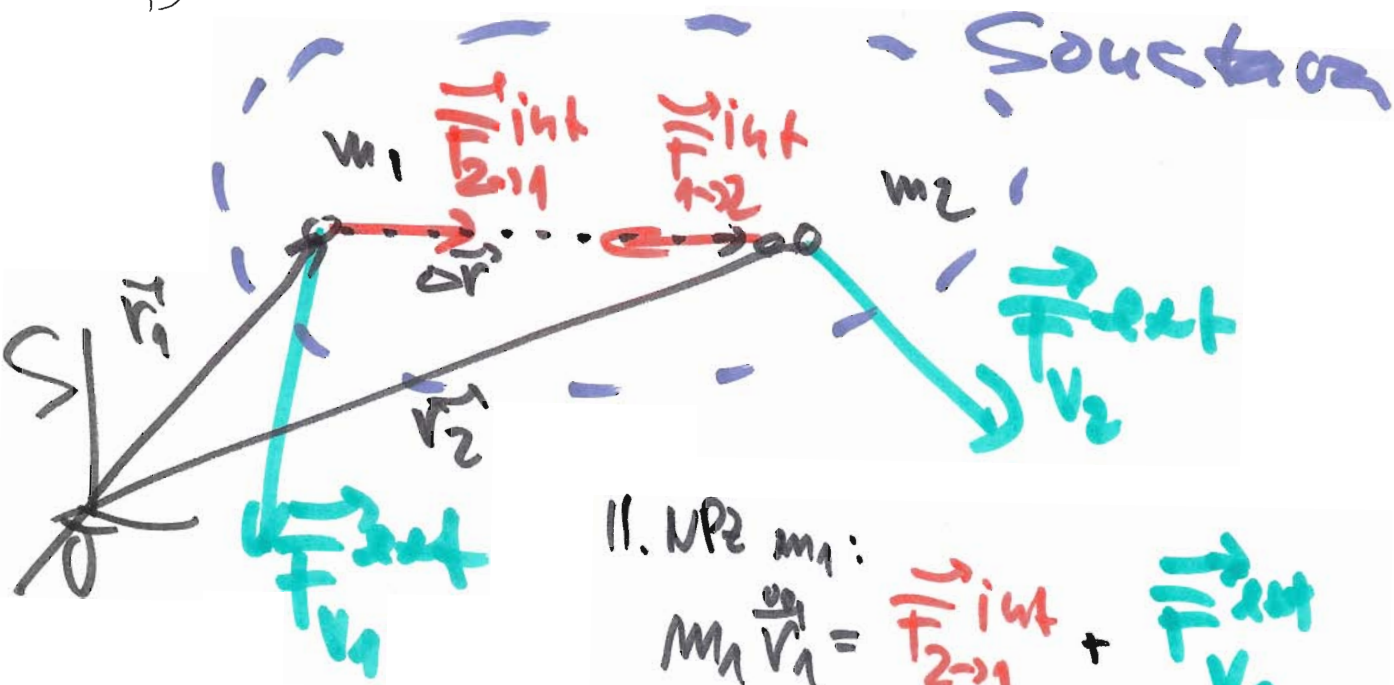
podívám si: (a cvičení)

$$* = \sum_i |\Delta \vec{l}_i| = \left(\sum_i \Delta m_i\right) R^2 \omega = ***$$

$$|\Delta \vec{l}_i| = |\vec{r}_i \times \Delta m_i \vec{v}_i| = |\vec{r}_i|\,|\Delta m_i \vec{v}_i| \underbrace{\sin \sphericalangle(\vec{r}_i, \vec{v}_i)}_{1} =$$
$$= R \Delta m_i \omega R$$

$$*** = m R^2 \omega = \underline{I_c \omega} \Rightarrow \boxed{\vec{L} = I_c \vec{\omega}} \;!$$

II. VI : (Druhá věta impulsová)



II. NPZ m_1:

$$m_1 \ddot{\vec v}_1 = \vec F^{int}_{2\to1} + \vec F^{ext}_{V_1}$$

m_2:

$$m_2 \ddot{\vec v}_2 = \vec F^{int}_{1\to2} + \vec F^{ext}_{V_2}$$

(1) $\vec r_1 \times /$ $\qquad \vec r_1 \times m_1 \ddot{\vec r}_1 = \vec r_1 \times \vec F^{int}_{2\to1} + \vec r_1 \times \vec F^{ext}_{V_1}$

(2) $\vec r_2 \times /$ $\qquad \vec r_2 \times m_2 \ddot{\vec v}_2 = \vec r_2 \times \vec F^{int}_{1\to2} + \vec r_2 \times \vec F^{ext}_{V_2}$

$$\vec l_1 = \vec r_1 \times m_1 \dot{\vec r}_1 \qquad ? \qquad \vec r_1 \times m_1 \underbrace{\ddot{\vec r}_1}_{\dot{\vec v}_1} \overset{?}{=} \frac{d}{dt}(\vec r_1 \times m_1 \vec v_1) =$$

$$= \underbrace{\dot{\vec r}_1}_{\vec v_1} \times m_1 \vec v_1 + \vec r_1 \times m_1 \dot{\vec v}_1 \qquad \emptyset$$

$$\frac{d}{dt}(\vec r_1 \times m_1 \vec v_1) = \vec r_1 \times \vec F^{int}_{2\to1} + \vec r_1 \times \vec F^{ext}_{V_1}$$

$$\frac{d}{dt}(\vec r_2 \times m_2 \vec v_2) = \vec r_2 \times \vec F^{int}_{1\to2} + \vec r_2 \times \vec F^{ext}_{V_2}$$

$$\emptyset \quad \emptyset \quad \emptyset$$

$$\frac{d\vec l_1}{dt} + \frac{d\vec l_2}{dt} = \vec r_1 \times \vec F^{ext}_{V_1} + \vec r_2 \times \vec F^{ext}_{V_2} + \cancel{(\vec r_2 - \vec r_1) \times \vec F^{int}_{1\to2}} \quad \emptyset$$

$$\frac{d}{dt}(\vec{l_1}+\vec{l_2}) = \sum \vec{M}^{ext}$$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{M}_V^{ext}} \quad \text{II. VI}$$

časová změna celkového momentu hybnosti soustavy = výsledný moment vnějších sil

? ZZMH ? $\vec{L} = \overrightarrow{\text{konst.}}$

$$\boxed{\vec{M}_V^{ext} = \emptyset}$$

piruetka
exp. na stolci
kolo na hřídeli

? ! Podmínka ? O_{pev}

O' vůči O_{pev} rovnoměrně nebo

$O' \equiv C = T$

Tuhé těleso kol pevné osy

$\vec{L} = I_0 \vec{\omega}$ & II. VI:

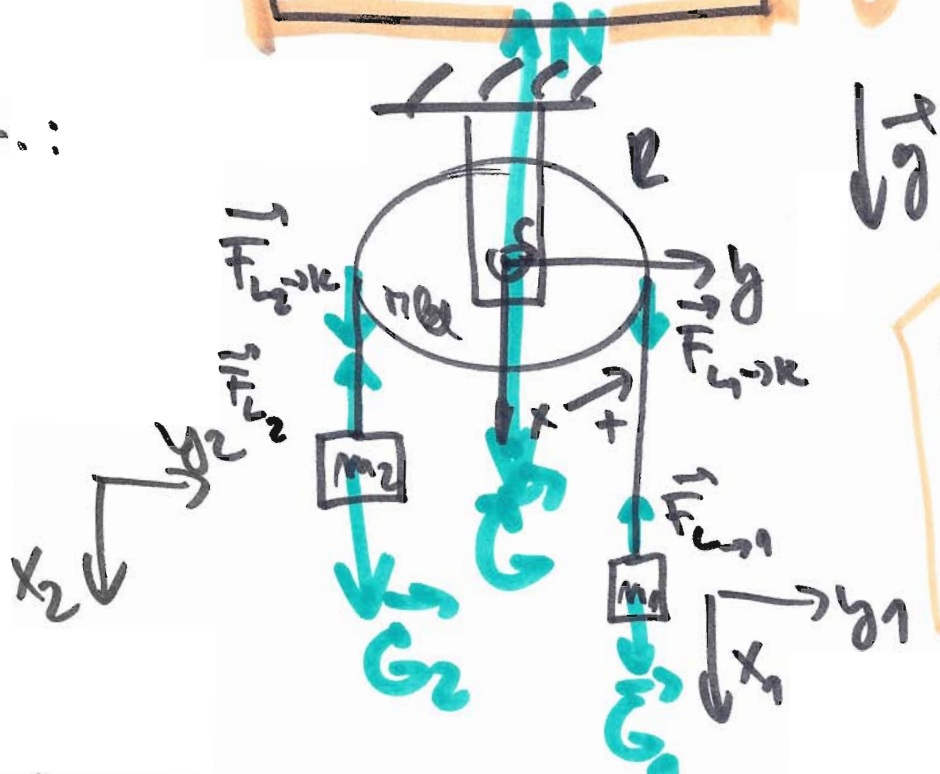
$$\dot{\vec{L}} = I_0 \dot{\vec{\omega}} = \vec{M}^{ext}_{V,0}$$

$\Downarrow$

$$\boxed{\vec{M}^{ext}_{V,0} = I\vec{\varepsilon}} \quad !$$

Pr.:

$m_2 > m_1$



odhad:

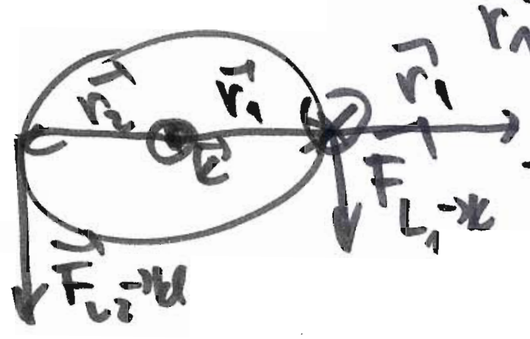
$$\boxed{\ddot{x}_2 = \frac{m_2 - m_1}{m_2 + m_1 + I_T^{kl}/R^2}\, g}$$

m_1: $m_1 \ddot{x}_1 = m_1 g - F_{L \to 1}$

m_2: $m_2 \ddot{x}_2 = m_2 g - F_{L \to 2}$

kl: $\vec{M}^{ext}_{V,0} = I\vec{\varepsilon}$:

$$\vec{r}_1 \times \vec{F}_{L_1 \to kl} + \vec{r}_2 \times \vec{F}_{L_2 \to kl} = I\vec{\varepsilon}$$



$$-R F_{L_1 \to kl} \vec{k} + R F_{L_2 \to kl} \vec{k} = I \varepsilon \vec{k}$$

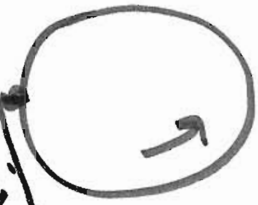
$$m_1\ddot{x}_1 = m_1 g - F_{L\to 1} \qquad -RF_{L_1\to k} + RF_{L_2\to k} = I\varepsilon$$

$$m_2\ddot{x}_2 = m_2 g - F_{L\to 2} \qquad \boxed{F_{L\to 2} = F_{L_2\to k}}\ F_{L2}$$

$$\ddot{x} = \ddot{x}_2 = -\ddot{x}_1$$

$$\downarrow \Delta x_2 = -\Delta x_1 \uparrow$$

$$\boxed{R\varepsilon = a_t = \ddot{x}_2 = \ddot{x}}$$



$$\cancel{\ddot{x}_2} = \ddot{x}\ ?$$

$$\boxed{F_{L\to 1} = F_{L_1\to k}} = F_{L1}$$

$$m_1(-\ddot{x}) = +m_1 g - F_{L1} \qquad /\cdot(-1)\ \to +$$

$$m_2\ddot{x} = m_2 g - F_{L2}$$

$$I\frac{\ddot{x}}{R} = R(F_{L2} - F_{L1}) \Rightarrow F_{L2} - F_{L1} = I\frac{\ddot{x}}{R^2}$$

$$-m_2\ddot{x} - m_1\ddot{x} = -m_2 g + m_1 g + F_{L2} - F_{L1}$$

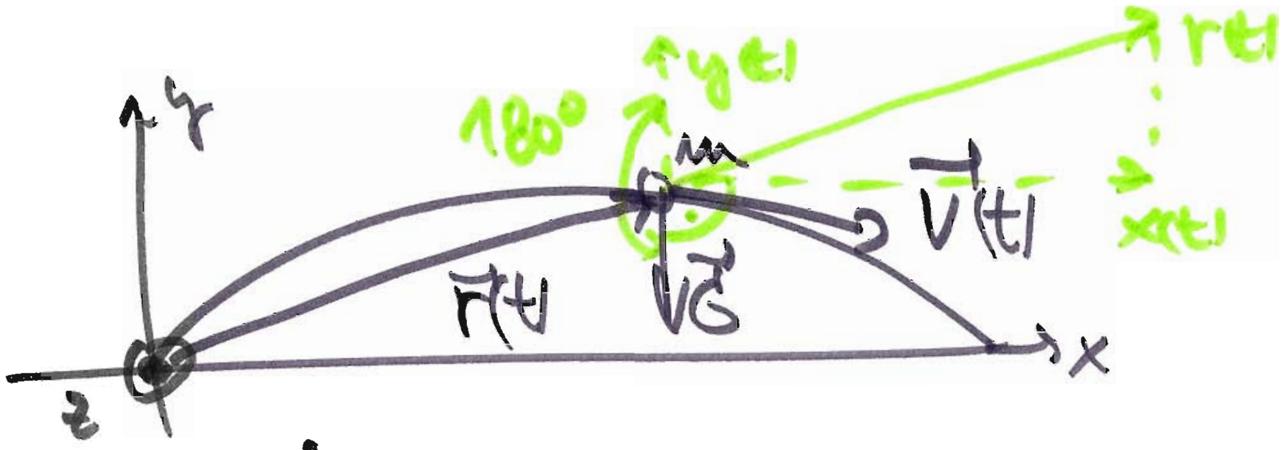
$$-\ddot{x}(m_1+m_2) = g(m_1-m_2) + I\frac{\ddot{x}}{R^2}$$

$$-\ddot{x}\left(m_1+m_2+I/R^2\right) = (m_1-m_2)\,g$$

$$\boxed{\ddot{x} = \frac{m_2-m_1}{m_1+m_2+I/R^2}\, g}\ ✓$$

(13)/13

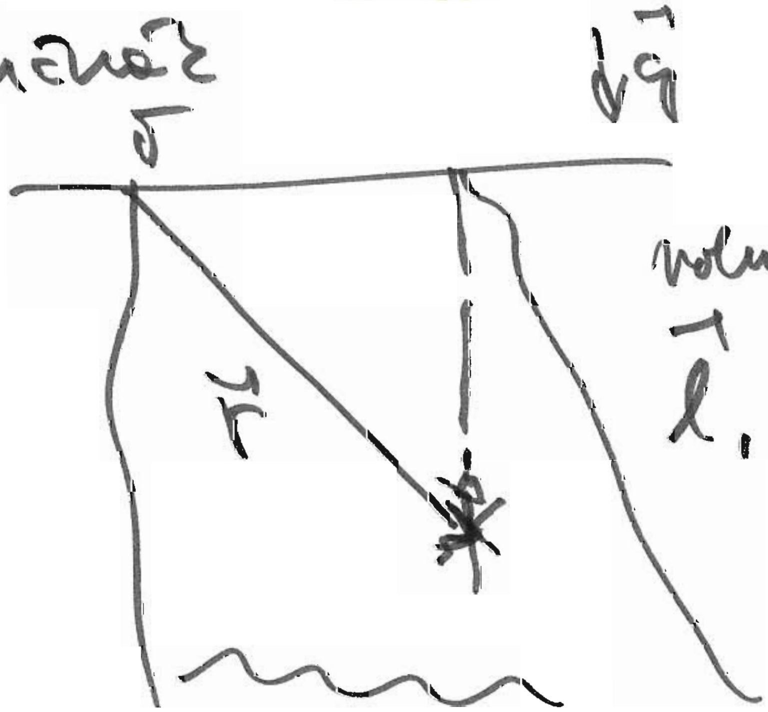
①



$\vec{l}$; $\dot{\vec{l}}$; $\vec{r} \times \vec{G}$

$(x(t)\,\vec{i} + y(t)\,\vec{j}) \times \vec{G}$

② Macocha



volný pád

$\vec{l}$, $\dot{\vec{l}}$, $\vec{r} \times \vec{G}$

HRW

PŘÍŠTÍ přednáška

① VALENT

② TEKUTINY