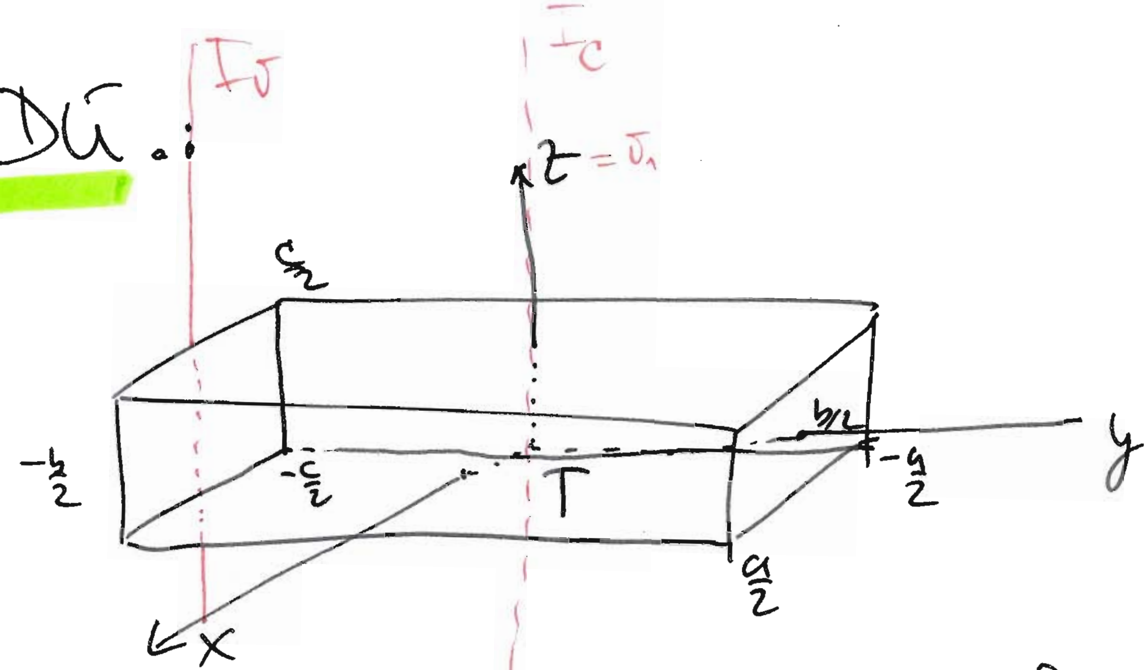
1. 12. 2008

Dobré ráno!

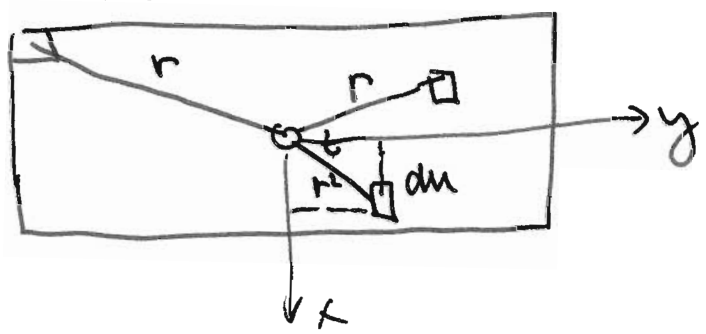
DÚ:



$$I_C = \int_M r^2 \, dm = \int_{-\frac{c}{2}}^{\frac{c}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} r^2 \rho \, dx\, dy\, dz$$

(z) (y) (x)

$$I_O = \int_M r^2 \, dm$$



$$r^2 = x^2 + y^2$$

$$= \rho \int_{-\frac{c}{2}}^{\frac{c}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} (x^2+y^2) \, dx\, dy\, dz = \ldots = \frac{1}{12} m(a^2+b^2)$$

$$I_O = I_C + m\left(\frac{b}{2}\right)^2$$

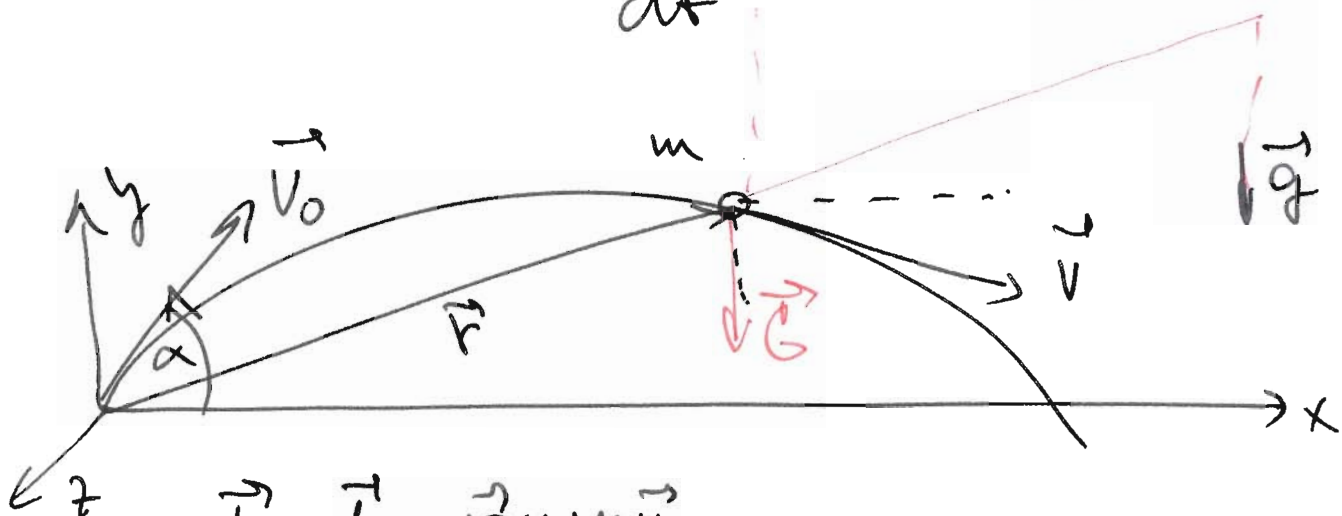
$$\frac{1}{12} m(a^2+b^2) + m\frac{b^2}{4} = \ldots$$

DÚ ①

②/11

$$\frac{d\vec{L}}{dt} = \vec{M}_v^{ext}$$

$$\dot{\vec{l}} = \vec{r}\times\vec{G}$$



$$\vec{L} = \vec{l} = \vec{r}\times m\vec{v}$$

$$\vec{r}(t) = \left((v_0\cos\alpha)t;\ v_0 t\sin\alpha - \tfrac{g}{2}t^2;\ 0\right)$$

$$\dot{\vec{r}}(t) = \vec{v}(t) = \left(v_0\cos\alpha,\ v_0\sin\alpha - gt,\ 0\right)$$

$$\vec{r}\times m\vec{v} = m\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x(t) & y(t) & 0=z(t) \\ \dot{x}(t) & \dot{y}(t) & 0=\dot{z}(t) \end{vmatrix}$$

$\vec{i}\times\vec{j} = \vec{k}$ a cyk

$$\vec{r}\times m\vec{v} = m\left(x\vec{i} + y\vec{j}\right)\times\left(\dot{x}\vec{i} + \dot{y}\vec{j}\right) =$$

$$= m x\dot{y}\vec{k} - m y\dot{x}\vec{k} = \dots$$

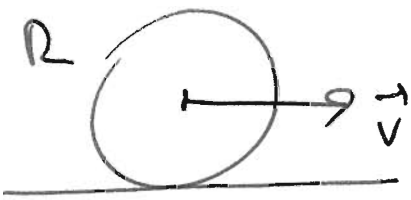
$$\dot{\vec{l}} = \vec{r}\times m\vec{g} = \left(x\vec{i} + y\vec{j}\right)\times mg(-\vec{j})$$

Práce & energie při rotaci

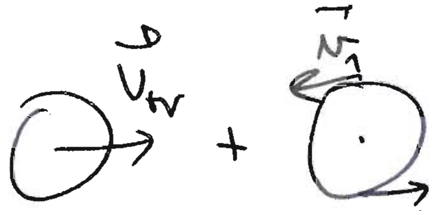
$$\vec{M} = \vec{r} \times \vec{F}$$

$$\frac{1}{2} m v^2 + E_k = \frac{1}{2}\left(\frac{1}{2} m R^2\right)\frac{v^2}{R^2} = \frac{3}{4} m v^2 \qquad \omega = \frac{v}{R}$$

Rotace & valení

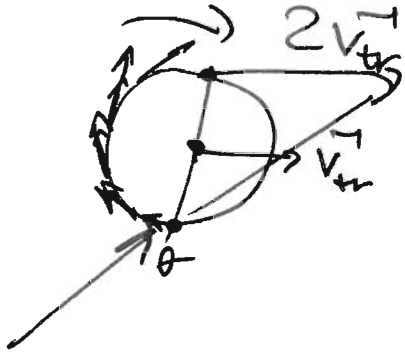


čistá



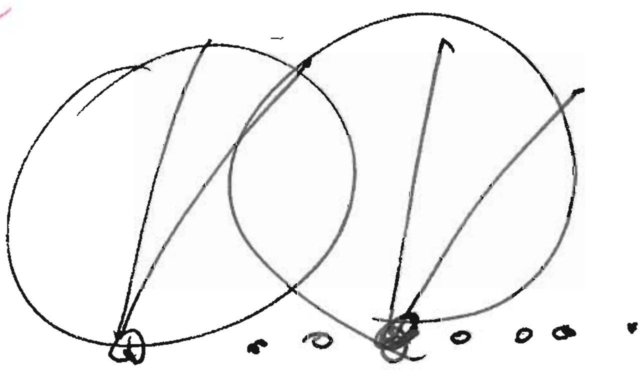
translace $\frac{1}{2} m v^2$ rotace $\frac{1}{2} I_c \omega^2$

valení



VÁLEC

$$I_c = \frac{1}{2} m R^2$$



$$\underline{\omega} = \frac{2v}{2R} = \frac{v}{R} \ !$$

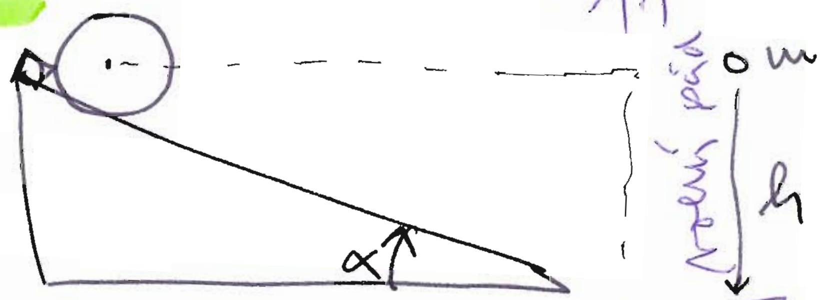
$$E_k = \frac{1}{2} I_0 \omega^2 = \frac{1}{2} \underbrace{\left(\frac{1}{2} m R^2 + m R^2\right)}_{\text{Steiner}} \frac{v^2}{R^2} = \frac{1}{2} \cdot \frac{3}{2} m v^2 = \underline{\frac{3}{4} m v^2}$$

„Závod těles"

Měříme:

Obruč | | koule

MR^2 | $\frac{1}{2}MR^2$ | $\frac{2}{5}MR^2$



$\boxed{v = \sqrt{2gh}}$

zlatá (medaile)

stříbrný

bronzová (medaile)

ZZE:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 =$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I_{1,2,3}\frac{v^2}{R^2}$$

koule:

$$gh = \frac{1}{2}v^2 + \frac{1}{5}v^2 \Rightarrow v = \sqrt{\frac{10gh}{7}}$$

obruč:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\frac{v^2}{R^2}$$

$$gh = v^2 \Rightarrow v = \sqrt{gh}$$

válec

$$mgh = \frac{3}{4}mv^2 \Rightarrow v = \sqrt{\frac{4gh}{3}}$$

! V nezávisí na R, m !

Příklady setrvné

(cvičení)

① B. š. n. (ir)

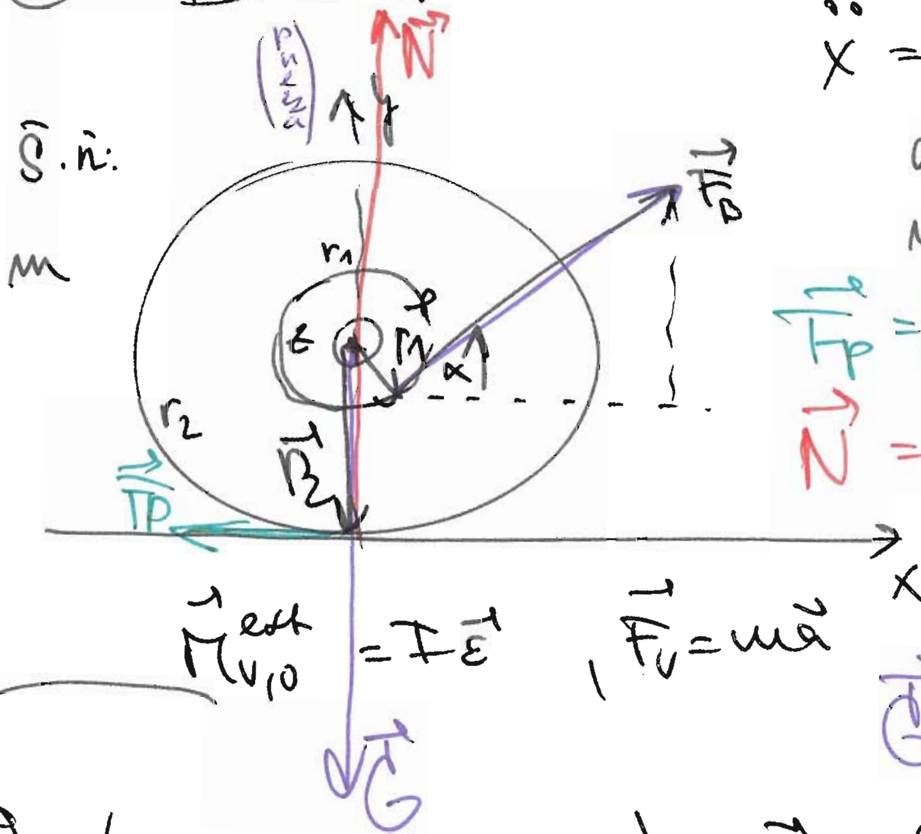
$$\ddot{x} = f(\alpha)$$

dáno $m, \vec{g}, r_1, r_2$

neznámé

š. n.:

m



$$\vec{F_p} = (-F_p, 0, 0)$$

$$\vec{N} = (0, N, 0)$$

$$\vec{F_B} = (F_B \cos\alpha, F_B \sin\alpha, 0)$$

$$\vec{G} = (0, -mg, 0)$$

$$\vec{M}^{ext}_{v,0} = I\vec{\varepsilon}, \quad \vec{F_v} = m\vec{a}$$

$$\vec{r_1} = (r_1 \sin\alpha, -r_1 \cos\alpha, 0) \qquad \vec{r_2} = (0, -r_2, 0)$$

$$\vec{r_1} \times \vec{F_B} + \vec{r_2} \times \vec{F_p} + \vec{0} \times \vec{N} + \vec{0} \times m\vec{g} = I\vec{\varepsilon}$$

$$\vec{F_v} = m\vec{a}$$

+ vztah mezi $\vec{\varepsilon}$ a $\vec{a}$ $\boxed{R\varepsilon = -\ddot{x}}$ (ε) $(\ddot{x})$

$$\ddot{x}_a(t) = \frac{\cos\alpha - \frac{r_1}{r_2}}{m + I/r_2^2} F_B$$

Pokud

$$\frac{r_1}{r_2} = \frac{1}{2} \qquad \cos\alpha = \frac{1}{2} \Rightarrow \alpha = 60^\circ$$

(ii) dojo MRW DÚ

Závěr soustav:

STATICKÁ ROVNOVÁHA

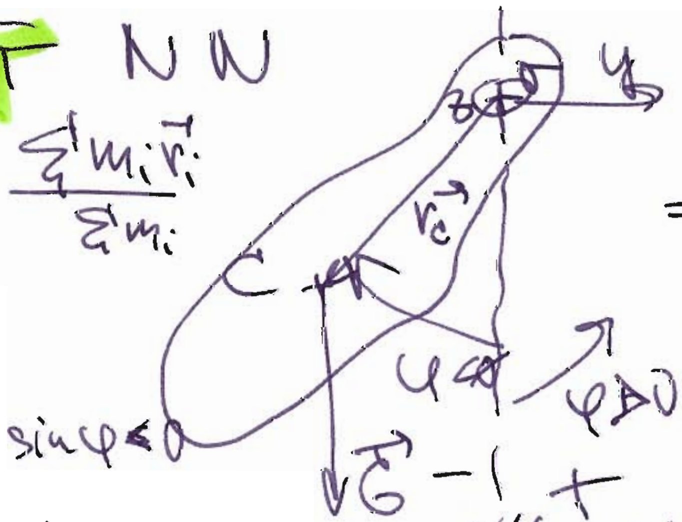
$$\vec{F}_V^{ext} = m\vec{a}_T = \vec{0}$$
$$\vec{M}_{V,0}^{ext} = I\vec{\varepsilon} = \vec{0}$$

$\Leftrightarrow$ $\vec{a}_T = \vec{0}$ $(\vec{v}_0 = \vec{0})$

$\vec{\varepsilon} = \vec{0}$ $(\vec{\omega}_0 = \vec{0})$

Γ NN

$$\vec{r}_T = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$



$$\vec{M}_{V,A}^{ext} = \sum_i \vec{r}_i \times m_i \vec{g}$$
$$= \left(\sum_i \vec{r}_i \times m_i\right) \vec{g}$$
$$= \left(\sum_i m_i \vec{r}_i\right) \times \vec{g}$$
$$\left(\sum_i m_i\right) \vec{r}_T \times \vec{g} = \vec{0}$$
$$\vec{r}_T \times M\vec{g} = \vec{r}_T \times \vec{G}$$

$$\vec{M}_{V,0}^{ext} = I\vec{\varepsilon}$$

$$\boxed{\ddot{\varphi} + \frac{r_T M g}{I} \sin\varphi = 0}$$

$$\vec{r}_T \times M\vec{g} = I\vec{\varepsilon}$$

$$-r_T M g \sin\varphi \, \vec{k} = I \ddot{\varphi} \, \vec{k}$$

Malé úhly: $\sin\varphi \approx \varphi$

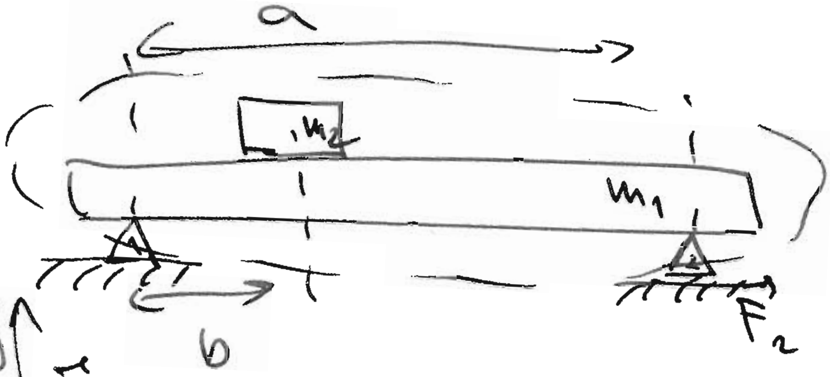
$$\ddot{\varphi} + \frac{r_T M g}{I_C} \varphi = 0$$

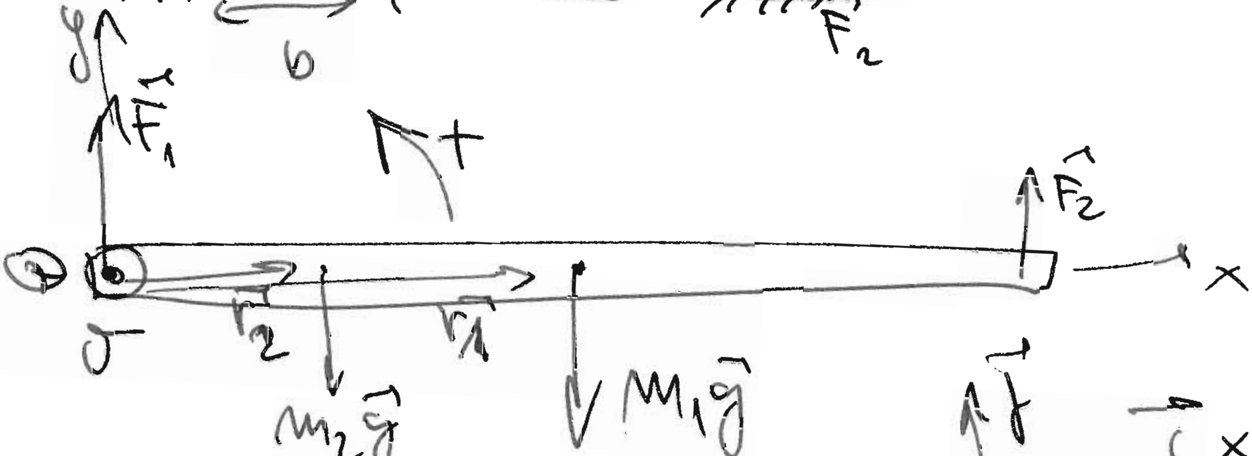
$$\omega^2 = \frac{r_T M g}{I_C} = \frac{4\pi^2}{T^2} \Rightarrow T = 2\pi \sqrt{\frac{I_C}{r_T M g}}$$

⑦ /11

Př. statickou rovnováhu

i)





$\vec{i} \times \vec{j} = \vec{k}$ a cykl.

$\vec{i} \to \vec{j} \to \vec{k} \to \vec{i}$

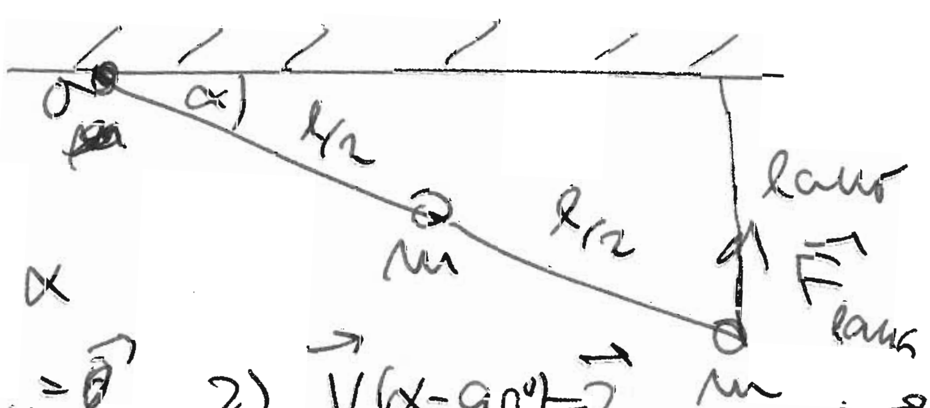
$\hat{F}_V^{ext} = \vec{0}$

$$\boxed{F_1 + F_2 - m_2 g - m_1 g = 0}$$

$$\hat{M}_V^{ext} = \vec{0} = \vec{r}_2 \times m_2\vec{g} + \vec{r}_1 \times m_1\vec{g} + \vec{r}_3 \times \vec{F}_2$$

$$\boxed{r_2 m_2 g + r_1 m_1 g = r_3 F_2}$$

DÚ ①:

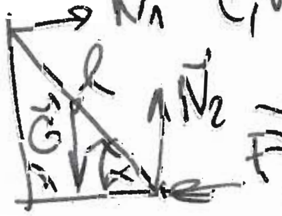


dáno $\vec{g}$

l, m, α

1) $\vec{F}_{lana} = ?$ 2) $\vec{V}(\alpha = 90°) = ?$

DÚ ②:



l, m, α

$\vec{F}_A = ?$

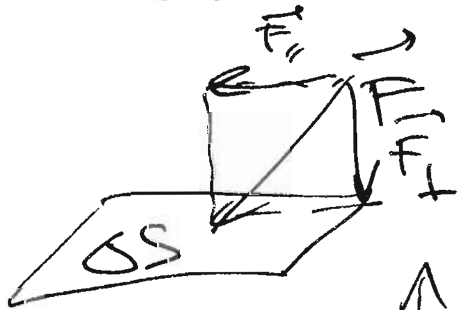
TEKUTINY

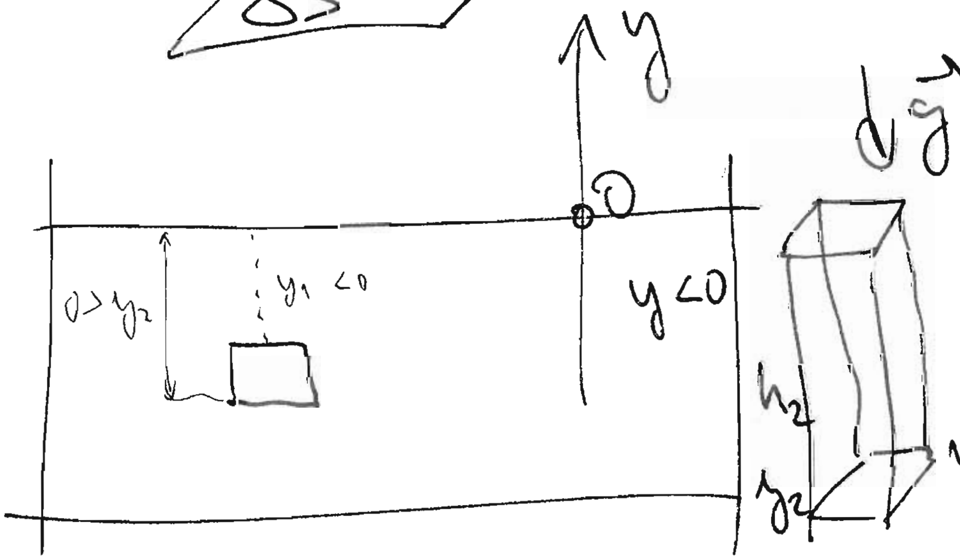
hustota $\rho = \frac{dm}{dV}$; $\frac{\Delta m}{\Delta V}$; $\frac{m}{V}$

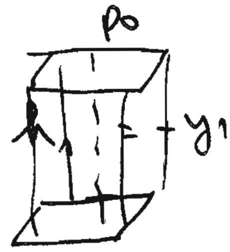
$kg\,m^{-3}$

tlak tekutiny

$$p = \frac{\Delta F}{\Delta S}$$







$$m_1 g = \rho V_1 g$$

F_1

$$p_1 = \frac{F_1}{S_1} = \rho h_1 g$$

$$\rho S_1 h_1 g = \rho S_1 |y_1| g$$

$$\rho S_2 h_2 g = \rho S_1 |y_2| g$$

S_1

$\vec{F_2}$

$$p_2 = \frac{F_2}{S_{2=1}} = h_2 \rho g$$

$$p_1 = p_0 + h_1 \rho g \; (= p_0 - y_1 \rho g)$$

$$p_2 = p_0 + h_2 \rho g \; (= p_0 - y_2 \rho g)$$

$p_0 = p_1 + y_1 \rho g$

$\downarrow$

$p_2 = p_1 + y_1 \rho g - y_2 \rho g =$

$= p_1 + \rho g (y_1 - y_2) = p_0 + h \rho g$

$$\boxed{p = p_0 + h \rho g}$$

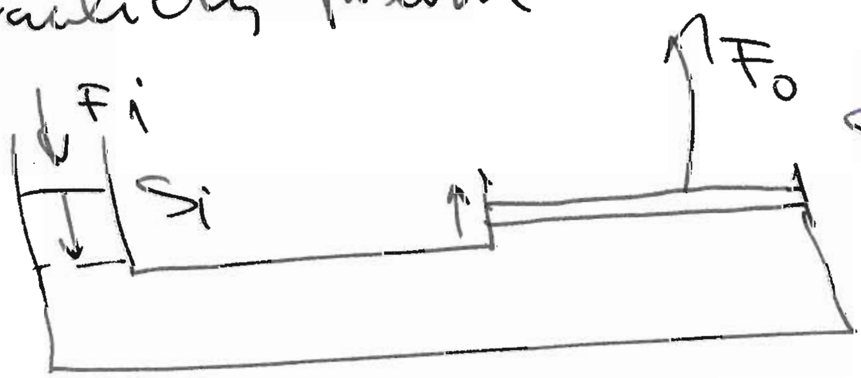
h – hloubka (>0) (pod hladinou)

Pascalův zákon $p = p_0 + h \rho g$

$\downarrow$ p_{ext}

Hydraulický převod

$p = \frac{F_i}{S_i}$



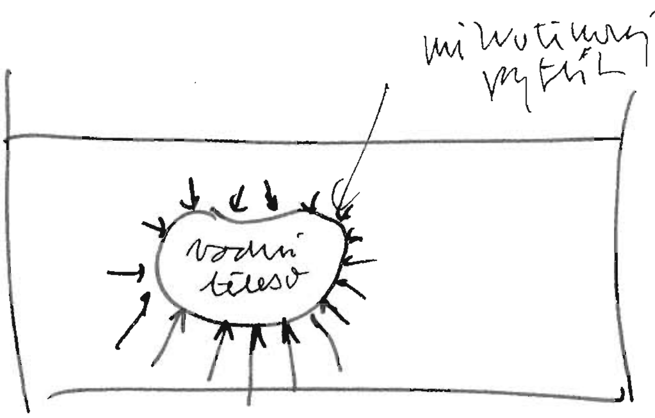
$p = \frac{F_o}{S_o}$

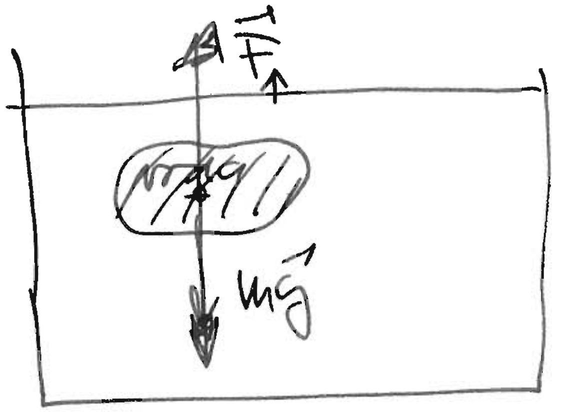
$\frac{F_i}{S_i} = \frac{F_o}{S_o} \Rightarrow F_o = F_i \underbrace{\frac{S_o}{S_i}}_{\gg 1} \Rightarrow F_o \gg F_i$

$F_i \, \Delta x_i = F_o \cdot \Delta x_o$

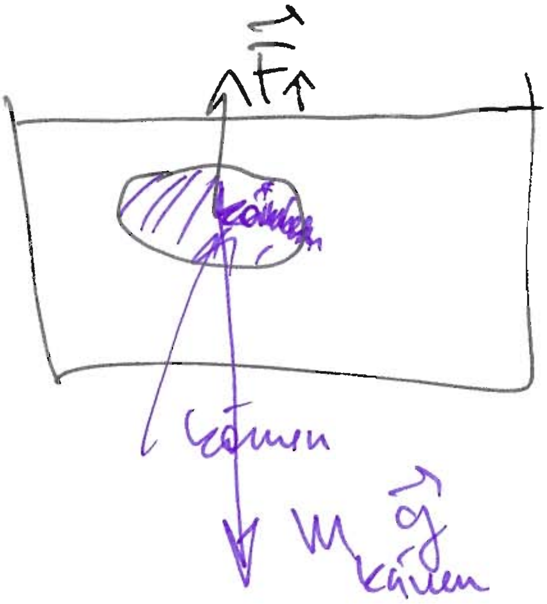
menší velké velké malé

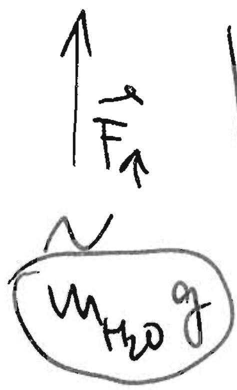
Archimédův zákon

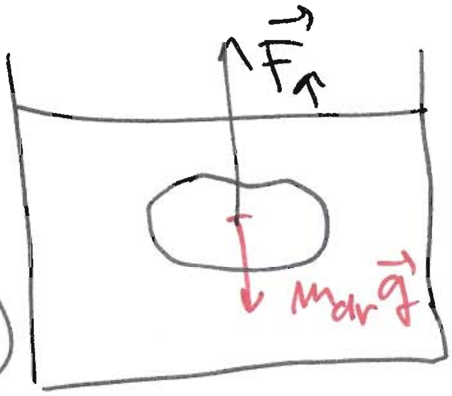


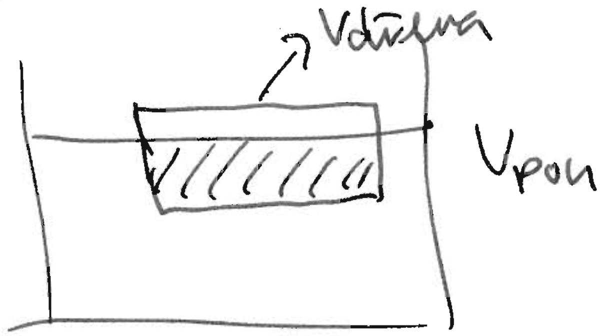


$|\vec{F}_i| = p_i \Delta S_i$ Σ





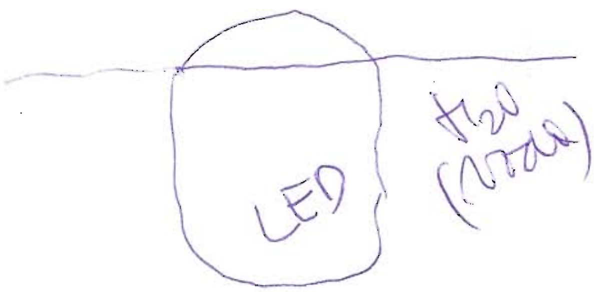




Vodní těleso

DÚ : vodní kra

kolik % je vidět?



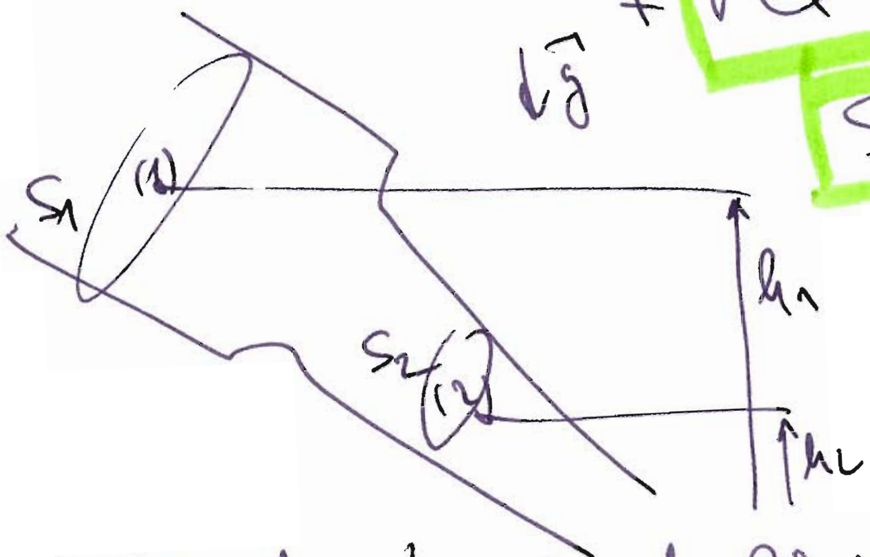
Na příště

11/11

$\boxed{\text{Bernoulliova rce}}$

$\downarrow \vec{g}$ + $\boxed{\text{rce kontinuity}}$

$$\boxed{S_1 v_1 = S_2 v_2}$$



$$p_1 + h_1 \rho g + \frac{1}{2}\rho v_1^2 = p_2 + h_2 \rho g + \frac{1}{2}\rho v_2^2$$

$$\boxed{p + h\rho g + \frac{1}{2}\rho v^2 = \text{konst.}}$$