1/11

15.12.08

VESELÉ VÁNOCE!

I. VT:

Pokračování:

TERMODYNAMIKA

$$\boxed{\delta Q = C_V\,dT + p\,dV}$$

Stavová rce (id. plynu)

$pV = n_m RT$ ← univerzální plynová k.

$n_m = \frac{N}{N_A}$ — počet částic; počet molů; Avogadrova k.

Boltzmann.

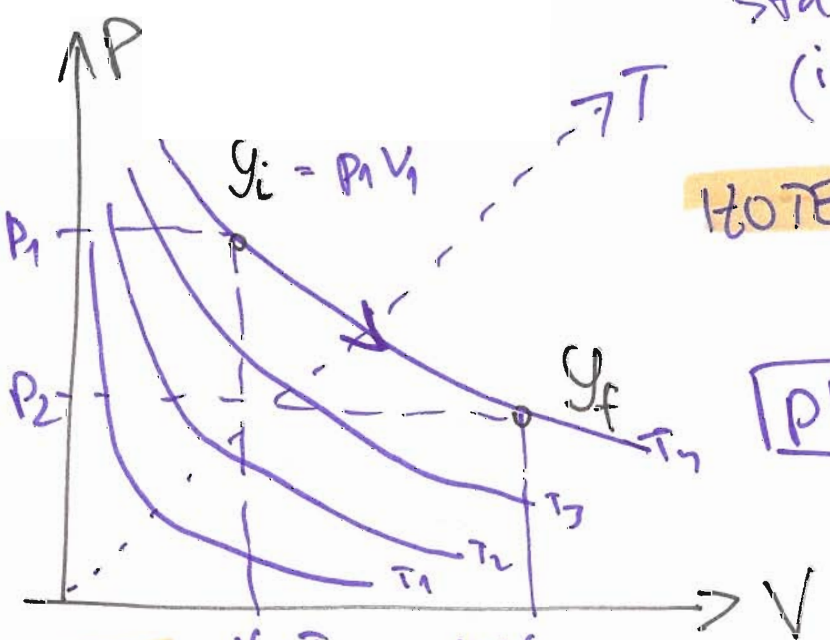
ΓNN $pV = n_m RT$

$$p = \frac{N}{N_A}\cdot\frac{1}{V}RT = \frac{N}{V}\frac{R}{N_A}T = nkT$$

koncentrace; as. energie

DĚJE & STAVY:

Stav S_i (initial) $\Rightarrow \boxed{p_1V_1 = n_m R T_1}$



IZOTERMICKÝ DĚJ

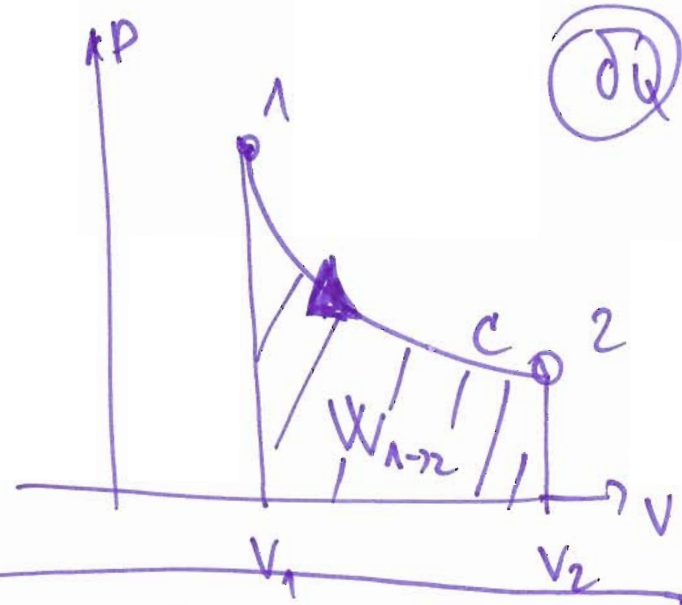
$T = \text{konst}$

$\boxed{pV = \text{konst}}$ $p_1V_1 = p_2V_2 = \ldots = n_m R T_1$

$p = \frac{\text{konst}}{V} \Rightarrow$

$$Q_{1\to2} = c_V\int_{T_1}^{T_2} dT + \int_{V_1}^{V_2} p\,dV = 0 + n_m RT\int_{V_1}^{V_2}\frac{dV}{V} = n_m RT\ln\frac{V_2}{V_1}$$

Diskuse izotermického děje



$$\delta Q = C_V \underset{\substack{\downarrow\\ 0\\ dU=0}}{dT} + \underset{\substack{\downarrow\\ \delta W}}{p\,dV}$$

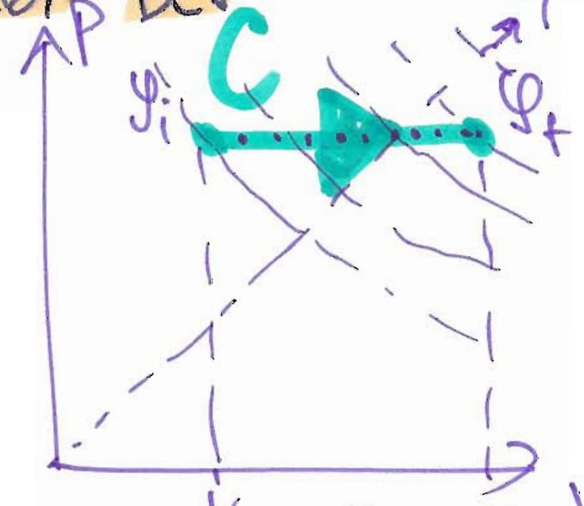
$$\int_C \delta Q = \Delta W_C = Q_{1\to2}$$

$$= \int_{V_1\,C}^{V_2} p\,dV$$

$$\boxed{W_{1\to2} = Q_{1\to2} = n_m RT \ln \frac{V_2}{V_1}}$$

IZOBARICKÝ DĚJ

③



$$p_1 V_1 = n_m R T_1$$

$$p_2 V_2 = n_m R T_2$$

$p_2 = p_1$

$$pV = n_m RT$$

p = konst

$$p(V) = \text{konst.}$$

1 mol:

$$Q_{1\to2} = \int_{y_i\,C}^{y_f} \delta Q = C_V \int_{T_i\,C}^{T_f} dT + \int_{V_i\,C}^{V_f} p\,dV =$$

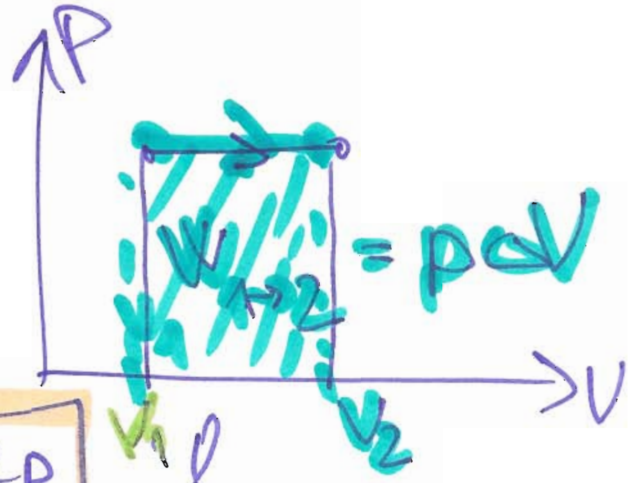
$$Q_{1\to2} = C_V(T_2 - T_1) + p\int_{V_1}^{V_2} dV = C_V(T_2-T_1) + p(V_2-V_1)$$

n molů:

$$Q_{1\to2} = n_m C_V (T_2 - T_1) + n_m(\overset{p_1=p_2}{p_2V_2 - p_1V_1})$$

ze stavové rovnice

$$Q_{1\to 2} = n_m c_V (T_2 - T_1) + n_m R (T_2 - T_1)$$



$$c_V = \frac{\delta Q}{n_m\, dT} \quad \Downarrow$$

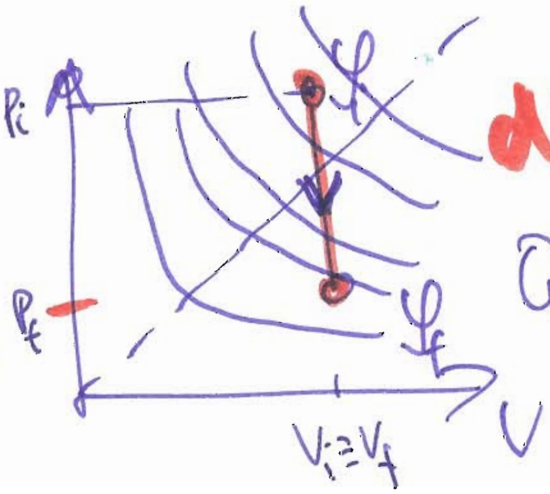
$$\frac{Q_{1\to 2}}{n_m \Delta T} = \boxed{c_V + R = c_p}$$

$$Q_{1\to 2} = c_V (T_2 - T_1) + p (V_2 - V_1)$$

IZOCHORICKÝ DĚJ

$\delta W = 0$

$W_{1\to 2} = 0$

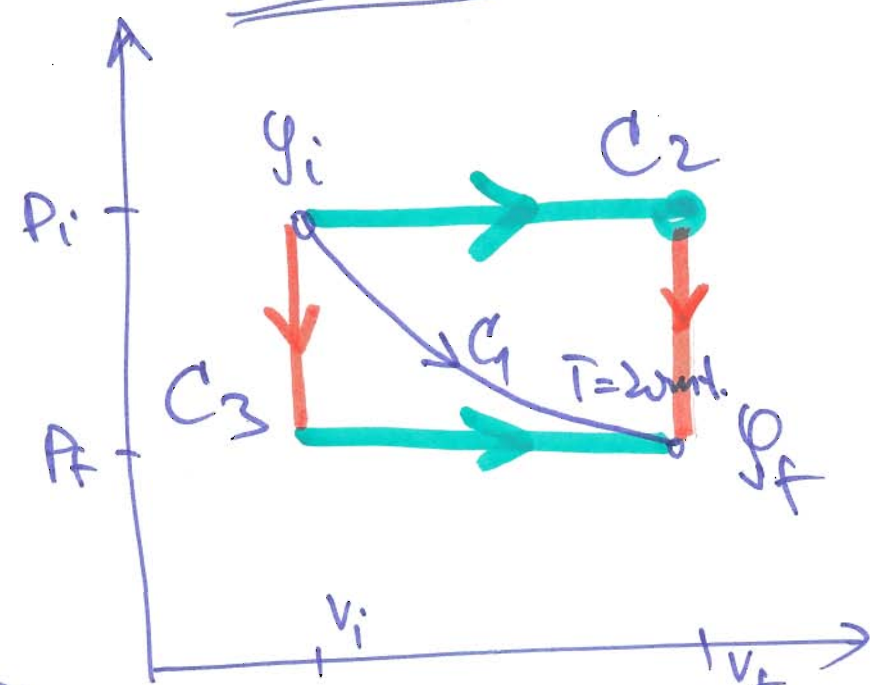


$dV = 0$

$$Q_{1\to 2} = \int_{\varphi_i}^{\varphi_f} \delta Q = c_V (T_f - T_i)$$

C

Příklad:



$V_f > V_i$

$$W_{1\to 2}^{C_2} = p_i(V_f - V_i) \;>\; W_{1\to 2}^{C_1} = RT \ln \frac{V_f}{V_i} \;>\; W_{1\to 2}^{C_3} = p_f(V_f - V_i)$$

protože: $\Delta U = 0$ $\left(\int dU \Leftarrow \int_{C_1} = \int_{C_2} = \int_{C_3} \right)$

⑤ adiabatický děj

/11

$\Delta Q = 0$ (izolovaný systém)

$\boxed{\delta Q = 0}$

1. VT: $\delta Q = dU + \delta W$

$\delta Q = \boxed{0 = c_V\, dT + p\, dV}$

n_m

1. VT:

stavová rce:

$\boxed{pV = n_m RT}$

p, V, T se mění

diference:

$$d(pV) = n_m R\, dT$$

$$dp \cdot V + p \cdot dV = n_m R\, dT$$

$n_m dT = \frac{-p\,dV}{c_V}$

$$V\,dp + p\,dV = R\left(-\frac{p\,dV}{c_V}\right)$$

$$c_V V\,dp + c_V p\,dV = -R p\,dV$$

$$c_V V\,dp + p\,dV(c_V + R) = 0 \quad / : c_V$$

$$V\,dp + p\,dV \varkappa = 0 \quad / : pV$$

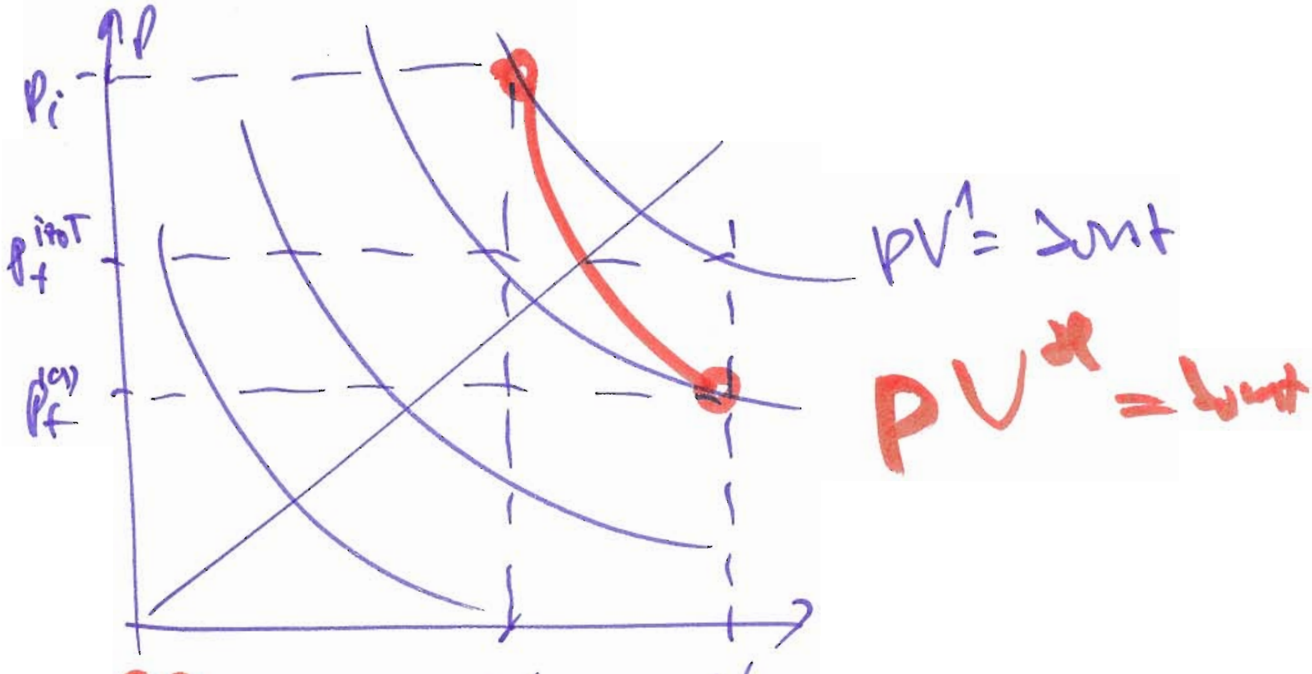
$$\boxed{\int \frac{dp}{p} = -\varkappa \int \frac{dV}{V}}$$

$$\left(pV^{\varkappa} = konst. \quad \varkappa = \frac{c_p}{c_V} = \frac{c_V + R}{c_V} \right)$$

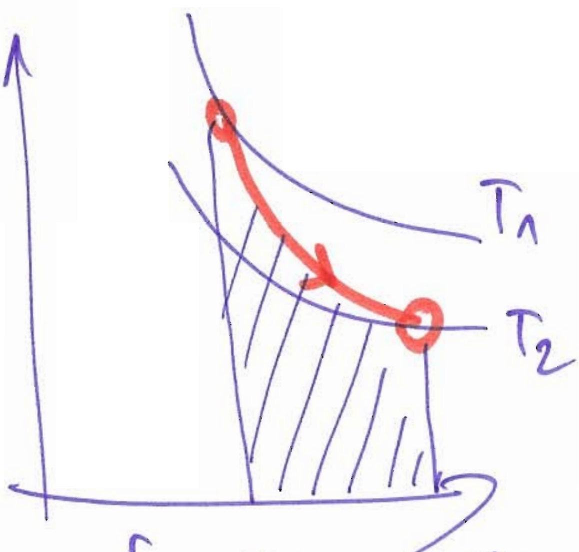
$$\ln p = -\varkappa \ln V + \ln K$$

$$\ln p + \ln V^{\varkappa} = \ln konst$$

$$\ln (pV^{\varkappa}) = \ln konst \Rightarrow \boxed{pV^{\varkappa} = konst}$$



$$pV^{\frac{c_p}{c_v}}$$



$$\delta Q = c_v\,dT + p\,dV$$
$$0 = c_v\,dT + p\,dV$$
$$p\,dV = -c_v\,dT$$
$$\int_{V_i}^{V_f} p\,dV = -c_v \int_{T_i}^{T_f} dT$$

$$\delta Q = 0 \Rightarrow \int p\,dV = -C_v \Delta T = C_v(T_1 - T_2) > 0$$

$\boxed{\text{1atomární } c_v = ? \text{ versus 2atomární}}$

$$\boxed{\varkappa} = \frac{C_p}{C_v} = \frac{c_v + R}{c_v} \qquad c_v = \frac{5}{2}R \ \text{(2at. plyn)}$$

1at

$\cancel{\Gamma}$ NN $\langle E_{kin} \rangle = \frac{3}{2}kT$ (ekvipartiční teorém)

$$N_A \cdot \langle E_{kin} \rangle = \frac{3}{2} N_A \cdot kT = \frac{3}{2}RT = U$$

(1atomární plyn)

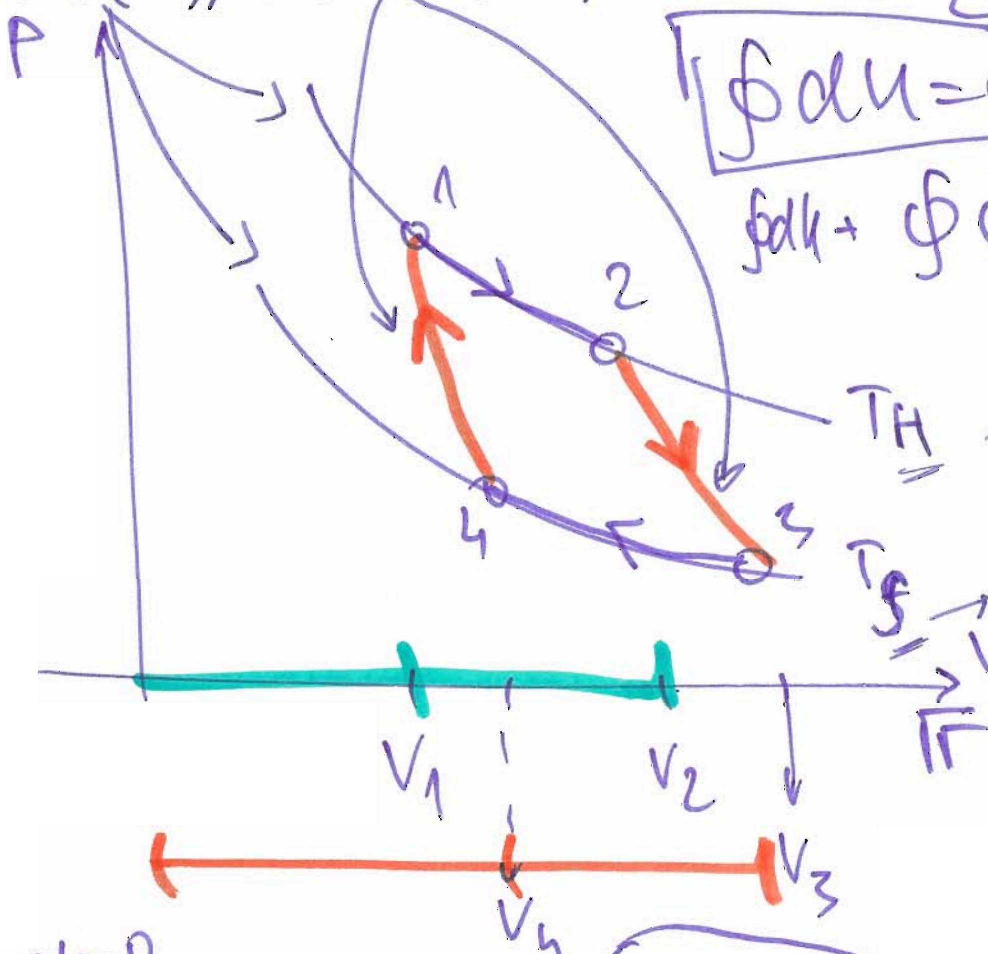
$$c_v = \frac{3}{2}R$$
$$\frac{3}{2}R\,dT = dU = c_v\,dT \Rightarrow c_v = \frac{3}{2}R$$

$\boxed{\text{Kruhový děj}}$

$$\mathcal{S}_i = \mathcal{S}_f$$
$$\Downarrow$$
$$\Delta U = \int_{S_i}^{S_i} dU = 0$$

Carnotův děj (cyklus):

izoterma (2x), adiabata (2x)



$$\boxed{\oint dU = 0}$$

$$\oint dU + \oint \delta W = \oint \delta Q$$

T_H → horký (hot)

T_S → studený (cold)

$$\frac{V_3}{V_4} \stackrel{?}{=} \frac{V_2}{V_1}$$

$1 \to 2$ $\quad \Delta U = 0$ $\quad T_H = \text{konst}$ ... → $p_1 V_1 = p_2 V_2$

$$Q_{1\to2} = n_m R T_H \ln \frac{V_2}{V_1} = W_{1\to2} > 0$$

$2 \to 3$ $\quad \Delta Q = 0$

$$Q_{2\to3} = 0 \;;\; W_{2\to3} = -\Delta U = n_m C_V (T_H - T_S) > 0 \qquad pV^{\varkappa} = \text{konst}$$

$$p_2 V_2^{\varkappa} = p_3 V_3^{\varkappa}$$

$3 \to 4$ $\quad \Delta U = 0,\ T_S = \text{konst}$

$$Q_{3\to4} = n_m R T_S \ln \frac{V_4}{V_3} = W_{3\to4} < 0 \qquad p_3 V_3 = p_4 V_4$$

$4 \to 1$ $\quad \Delta Q = 0$

$$Q_{4\to1} = 0 \Rightarrow W_{4\to1} = -\Delta U = -n_m C_V (T_1 - T_4) < 0$$

($T_1 = T_H$, $T_4 = T_S$)

$$p_4 V_4^{\varkappa} = p_1 V_1^{\varkappa}$$

práce vykonaná plynem

účinnost $\eta = \dfrac{\text{co chceme}}{\text{co za to platíme (čím)}} = \dfrac{W}{Q}$

teplo plynu dodané

$$W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1} = W_{1\to2\to3\to4\to1} =$$

$$= n_m R T_H \ln\frac{V_2}{V_1} + n_m c_V (T_H - T_S) +$$

$$+ n_m R T_S \ln\frac{V_4}{V_3} + n_m c_V (T_S - T_H) =$$

$$= n_m R T_H \ln\frac{V_2}{V_1} - n_m R T_S \frac{V_3}{V_4} = *$$

NN: nějaký vztah mezi $\frac{V_2}{V_1}$ a $\frac{V_3}{V_4}$?

$$p_1 V_1 = p_2 V_2 = n_m R T_H \implies p_1 V_1 V_1^{\varkappa-1} = n_m R T_H V_1^{\varkappa-1}$$

$$\implies p_2 V_2 \frac{V_2^{\varkappa}}{V_2} = n_m R T_H V_2^{\varkappa-1}$$

$$p_2 V_2^{\varkappa} = p_3 V_3^{\varkappa}$$

$$p_3 V_3 = p_4 V_4 = n_m R T_S$$

$$p_3 V_3 \frac{V_3^{\varkappa}}{V_3} = n_m R T_S V_3^{\varkappa-1} \Longleftarrow p_3 V_3$$

$$p_4 V_4^{\varkappa} = p_1 V_1^{\varkappa}$$

$$p_4 V_4^{\varkappa} = n_m R T_S V_4^{\varkappa-1}$$

$$n_m R T_S V_4^{\varkappa-1} = n_m R T_H V_1^{\varkappa-1}$$

$$n_m R T_H V_2^{\varkappa-1} = n_m R T_S V_3^{\varkappa-1}$$

podíl: $\left(\frac{V_4}{V_3}\right)^{\varkappa-1} = \left(\frac{V_1}{V_2}\right)^{\varkappa-1}$

$$\implies \boxed{\frac{V_2}{V_1} = \frac{V_3}{V_4}}$$

$$* \; n_m R T_H \ln\frac{V_2}{V_1} - n_m R T_S \ln\frac{V_2}{V_1}$$

$Q_{1\to2(\to3\to4\to1)} = Q_{1\to2} = n_m R T_H \ln\frac{V_2}{V_1}$

NE ?

η ← dodané (pouze)

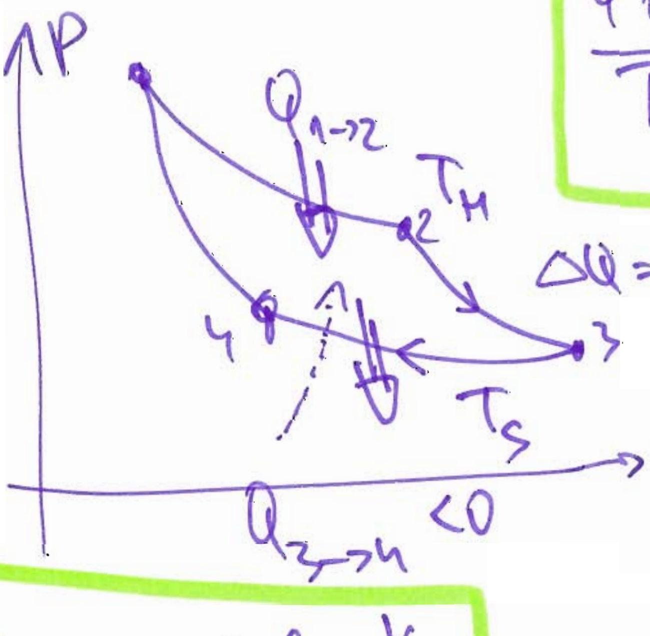
$$\boxed{\eta = \frac{n_m R \ln\frac{V_2}{V_1}(T_H - T_S)}{n_m R T_H \ln\frac{V_2}{V_1}} = \frac{T_H - T_S}{T_H}}$$

účinnost

$$= 1 - \frac{T_S}{T_H} < 1$$

Povšimněte: čemu je rovno?:

$$\boxed{\frac{Q_{1\to2}}{T_H} = n_m R \ln\frac{V_2}{V_1}}$$



$\oint dU = 0$

↑ úplný

$$\boxed{\frac{Q_{3\to4}}{T_S} = n_m R \ln\frac{V_4}{V_3}} = -n_m R \ln\frac{V_3}{V_4} = -n_m R \ln\frac{V_2}{V_1}$$

$$\frac{\delta Q_{1\to2}}{T_H} + 0 + \frac{\delta Q_{3\to4}}{T_S} + 0 = 0$$

$$\boxed{\oint \frac{\delta Q}{T} = 0}$$

NĚCO NOVÉHO

$dS = ?$ → $\boxed{dS = \frac{\delta Q}{T}}$

Na entropii S: „z druhé strany":

I. VT:

$$\delta Q = n_m c_V \, dT + p \, dV$$

↑ jak se zbavíme p, které závisí na V?

Stavová rce:

$$pV = n_m R T$$

Integrační faktor: $\frac{1}{T}$ → $\frac{p}{T} = n_m R \cdot \frac{1}{V}$

Tedy:

$$\frac{\delta Q}{T} = \underbrace{n_m c_V}_{\text{konst.}} \frac{dT}{T} + \underbrace{n_m R}_{\text{konst.}} \frac{dV}{V}$$

a integrály již (výsledky) **nezávisejí** na int. cestě:

$$\int dS = \int \frac{\delta Q}{T} = n_m c_V \int \frac{dT}{T} + n_m R \int \frac{dV}{V}$$

$$\boxed{\Delta S = \int_{S_i}^{S_f} dS = \int_{S_i}^{S_f} \frac{\delta Q}{T} = S_2 - S_1}$$

po libov. křivce: $\boxed{\oint dS = 0}$

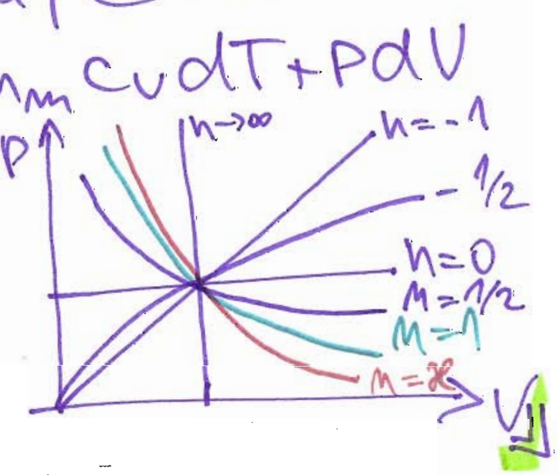
DÚ (N.N.) Polytropický děj $C = \text{konst.}$:

$$\delta Q = n_m c_V \, dT + p \, dV \Rightarrow n_m C \, dT = n_m c_V \, dT + p \, dV$$

$$\vdots$$

$$\underline{pV^n = \text{konst}} \; ; \; n = \frac{C - C_p}{C - C_V}$$

$(C \neq C_V)$



II. zákon termodynamiky

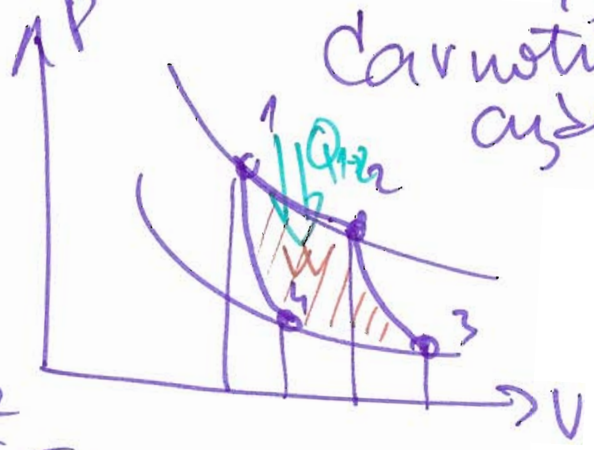
11/11

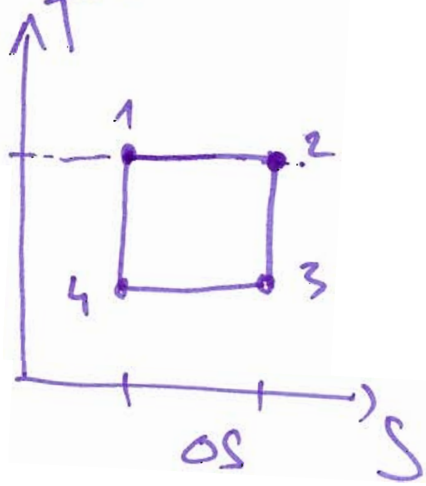
nevratný proces

$$\boxed{\Delta S \geq 0}$$

vratný proces

Carnotův cyklus:





$$\Delta S = \int_1^2 \frac{\delta Q}{T} = n_m R \ln \frac{V_2}{V_1}$$

Hodně štěstí & zdraví
v novém roce 2009!