

# 1. Úpravy algebraických výrazů

① Sečtěte zlomky

a) $\frac{3+2x}{2-x} - \frac{2-3x}{2+x} - \frac{x(16-x)}{4-x^2}$	a) $\frac{1}{x+2}$
b) $\frac{4a^2-3a+5}{a^3-1} - \frac{1-2a}{a^2+a+1} + \frac{6}{1-a}$	H $-\frac{12a}{a^3-1}$
c) $\frac{3}{x+2} - \frac{4}{x-2} + \frac{2x}{x^2+4x+4}$	c) $\frac{x^2-20x-28}{(x-2)(x+2)^2}$

② Násobte:

a) $-\frac{(a+b)^2}{(a-b)^2} \cdot \frac{3(a-b)^2}{4(a+b)^3}$	$-\frac{3}{4(a+b)}$
b) $\frac{ax+ay}{x^2-2xy+y^2} \cdot \frac{2x+2y}{ax^2+2axy+ay^2}$	$\frac{2}{(x-y)^2}$
c) $\frac{5x^2-10xy}{x^2+4y^2} \cdot \frac{x^4-16y^4}{15(x-2y)^2}$	$\frac{x(x+2y)}{3}$

③ Vypočtěte:

a) $\left(\frac{a}{x-a} - \frac{a}{x+a}\right) \cdot \frac{x^2+2ax+a^2}{2a^2}$	$\frac{x+a}{x-a}$
b) $\left(1 + \frac{a}{x} + \frac{a^2}{x^2}\right) \left(1 - \frac{a}{x}\right) \cdot \frac{x^3}{a^3-x^3}$	$-1$

④ Dělte:

a) $\frac{a^4-x^4}{a^3-x^3} : \frac{a^2+x^2}{a^2-x^2}$	$\frac{(a-x)(a+x)^2}{a^2+ax+x^2}$
b) $\frac{am^2-am^2}{m^2+2mm+m^2} : \frac{am^2-2amm+am^2}{3m+3m}$	$\frac{3}{m-m}$

⑤ Proveďte:

a) $\left(\frac{2a}{a+2} + \frac{6a}{6-3a} + \frac{8a}{a^2-4}\right) : \frac{a-4}{a-2}$	$0$
b) $\left(\frac{a^2+b^2}{a} + b\right) : \left[\left(\frac{1}{a^2} + \frac{1}{b^2}\right) \cdot \frac{a^3-b^3}{a^2+b^2}\right]$	$\frac{ab^2}{a-b}$
c) $6a + \left(\frac{a}{a-2} - \frac{a}{a+2}\right) : \frac{4a}{a^4-2a^3+8a-16}$	$\frac{1}{(a+2)^2}$

⑥ Zjednodušte složene zlomky: a)  $\frac{\frac{x-y}{x+y} + \frac{x+y}{x-y}}{\frac{x}{y} + \frac{y}{x}}$   $\left| \begin{array}{l} a) \frac{2xy}{x^2-y^2} \end{array} \right.$

b)  $1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{x}}}$  c)  $\frac{x}{x - \frac{1}{x - \frac{x}{1-x}}}$   $\left| \begin{array}{l} b) \frac{10x+3}{7x+2} \\ c) \frac{x^3}{x^3-x+1} \end{array} \right.$

⑦ Proveďte a určete, kdy mají provedené úpravy smysl:

a)  $\frac{\frac{1}{x} - \frac{1}{y+z}}{\frac{1}{x} + \frac{1}{y+z}} \cdot \frac{\frac{1}{y} + \frac{1}{a+z}}{\frac{1}{y} - \frac{1}{a+z}}$   $\left| \begin{array}{l} \frac{z+y-x}{z-y+x} \quad y \neq 0 \quad x \neq 0 \\ y \neq -z \quad x \neq -z \\ x+z+y \quad x+y+z \neq 0 \end{array} \right.$

b)  $\frac{\left(\frac{1}{a} + \frac{1}{b+c}\right) \left(1 + \frac{b^2+c^2-a^2}{2bc}\right)}{\frac{1}{a} - \frac{1}{b+c}}$   $\left| \begin{array}{l} \frac{(a+b+c)^2}{2bc} \quad a \neq 0 \quad b \neq 0 \quad c \neq 0 \\ b \neq -c \quad a \neq b+c \end{array} \right.$

⑧ Vypočítejte: a)  $\left(\frac{3^2 \cdot 5^3}{2^4 \cdot 11}\right)^6 \cdot \left(\frac{3^5 \cdot 5^9}{2^{11} \cdot 11^3}\right)^{-2}$   $\left| 2,25 \right.$

b)  $\frac{5^{-5} \cdot 0,1^{-4} + \left(\frac{1}{7}\right)^0 \cdot 5^{-1}}{(-2)^{-2} \cdot \left(-\frac{1}{2}\right)^{-4} + \left(-\frac{1}{2}\right)^{-1}}$   $\left| 2 \right.$

⑨ Upravte: a)  $\left[\left(1 + 2m^{-1}\right)^{-1} - 1\right]^{-1}$   $\left| -\frac{1}{2}m - 1 \right.$

b)  $\frac{(x+x^{-1})^{-3} + (x-x^{-1})^{-3}}{(x^2-x^{-2})^{-3}}$   $\left| 2\left(x^3 + \frac{3}{x}\right) \right.$

⑩ Vypočítejte a)  $a(\sqrt{a} + \sqrt{b}) - \sqrt{a}(a-b) - a\sqrt{b} + b\sqrt{a}$   $\left| 2b\sqrt{a} \quad \begin{array}{l} a \geq 0 \\ b \geq 0 \end{array} \right.$

b)  $\sqrt[4]{16^2} + \sqrt[6]{2^{12}} + \sqrt[9]{3^8} + \sqrt[3]{8 \cdot 3^3}$   $\left| 23 \right.$

c)  $(x^3 \cdot \sqrt[4]{x})^4$   $\left| x^{13} \right.$

$$d) \sqrt{\sqrt{x}} + \sqrt[3]{\sqrt{x}} - \sqrt[3]{\sqrt[4]{x^3}} \quad | \sqrt[6]{x} \quad x \geq 0$$

$$e) 10 \sqrt[3]{5} - 7 \sqrt[3]{40} + 5 \sqrt[3]{135} - 4 \sqrt[3]{320} + 2 \sqrt[3]{625} \quad | 5 \sqrt[3]{5}$$

$$f) \frac{2}{3} a \sqrt{9a} + 6a \sqrt{a} + 3 \sqrt{a^3} - 5a^2 \sqrt{\frac{1}{a}} + \frac{1}{a^2} \sqrt{a^7} \quad | 7a \sqrt{a} \quad a > 0$$

11) Proveďte: a)  $\frac{1-\sqrt{x}}{1+\sqrt{x}} - \frac{3\sqrt{x}}{\sqrt{x}-1} - \frac{3+\sqrt{x}}{1-x} \quad | \frac{4x-2}{1-x}$

b)  $\frac{\sqrt{6}}{2\sqrt{2}-\sqrt{5}} - \frac{\sqrt{10}+3}{\sqrt{3}} \quad | \frac{\sqrt{3}}{3}$

c)  $\frac{\sqrt{2}-1}{2} - \frac{2}{\sqrt{2}+1} \quad | -\frac{1}{2}$   
 $\frac{\sqrt{2}-1 + \frac{2}{\sqrt{2}+1}}{2}$

12) Upravte výrazy;

a)  $\left( \frac{\sqrt{1+a}}{\sqrt{1+a}-\sqrt{1-a}} + \frac{1-a}{\sqrt{1-a^2}-1+a} \right) \cdot \left( \sqrt{\frac{1}{a^2}-1} - \frac{1}{a} \right) \quad | -1$

b)  $\left( \frac{\sqrt[4]{a^3} - \sqrt[4]{b^3}}{\sqrt{a}-\sqrt{b}} - \sqrt[4]{a} - \sqrt[4]{b} \right) \left( \sqrt[4]{\frac{a}{b}} + 1 \right) \quad | -\sqrt[4]{a}$

c)  $\left( \sqrt{a(1-a)} + \frac{\sqrt{a^3}}{\sqrt{1-a}} \right) : \left( \frac{1}{1+\sqrt{a}} + \frac{\sqrt{a}}{1-a} \right) \quad \text{pro } 0 < a < 1 \quad | \sqrt{a(1-a)}$

d)  $\left( \frac{(\sqrt{x}+1)^2 - \frac{a-\sqrt{ax}}{\sqrt{a}-\sqrt{x}}}{(\sqrt{x}+1)^3 - a\sqrt{a} + 2} \right)^{-3} \quad \left| \begin{array}{l} 2\neq, a \geq 0, x \geq 0, a \neq x \\ (\sqrt{x}+1)^3 - a\sqrt{a} + 2 \neq 0 \end{array} \right.$

e)  $\left( \frac{\sqrt{a^3}-\sqrt{b^3}}{\sqrt{a}-\sqrt{b}} + \sqrt{ab} \right) \cdot \frac{\sqrt{a}-\sqrt{b}}{a-b} \quad \left| \begin{array}{l} \sqrt{a}+\sqrt{b} \quad a \geq 0 \\ b \geq 0 \\ a \neq b \end{array} \right.$

f)  $\frac{\sqrt[3]{a^{-2}} \sqrt{a^3}}{\sqrt[3]{\sqrt{a^4}} \cdot \sqrt{a^{-3}}} \quad | \sqrt[3]{a^3} \quad a > 0$

13) Zjednoczcie wyrazy a uroczcie, dla ktore' hodnoty promennych  
maji smysl:

$$a) \frac{\frac{1-x}{1-x+x^2} + \frac{1+x}{1+x+x^2}}{\frac{1+x}{1+x+x^2} - \frac{1-x}{1-x+x^2}} \quad \left| \frac{1}{x^3}, x \neq 0 \right.$$

$$b) \left( \frac{4x^3}{x^3-y^3} : \frac{2x^3}{x^2-2xy+y^2} \right) \cdot \frac{x^2+xy+y^2}{x^2-y^2} \quad \left| \frac{2}{x+y}, x \neq 0, x \neq \pm y \right.$$

$$c) \left[ \left( 1 - \frac{2}{1-3x} \right) \left( 1 - \frac{9x-9x^2}{3x+1} \right) \right] : (1-9x^2) \quad \left| -\frac{1}{3x+1}, x \neq \pm \frac{1}{3} \right.$$

$$d) \frac{1-\sqrt{3}}{1+|2-\sqrt{3}|+2|1-\sqrt{3}|} \quad \left| \sqrt{3}-2 \right.$$

$$e) \left[ \left( \frac{1}{2} \cdot \frac{1}{3} \right)^{\frac{1}{2}} \right]^{\frac{1}{3}} : \left[ \left( \frac{1}{2} \cdot 3^2 \right)^{\frac{1}{3}} \right]^{\frac{1}{2}} \quad \left| \left( \frac{1}{3} \right)^{\frac{1}{2}} \right.$$

$$f) \left( \frac{a^{-\frac{2}{3}}}{b^{-1}} - \frac{b^{-1}}{a^{-\frac{2}{3}}} \right) : \left( \frac{a^{-\frac{1}{3}}}{b^{-\frac{1}{2}}} - \frac{b^{-\frac{1}{2}}}{a^{-\frac{1}{3}}} \right) - a^{\frac{1}{3}} \cdot b^{-\frac{1}{2}} \quad \left| \frac{b^{\frac{1}{2}}}{a^{\frac{1}{3}}} \right. \begin{array}{l} a > 0 \\ a \neq 0 \\ b > 0 \\ b \neq \pm \sqrt[3]{a} \end{array}$$

$$g) \left( \frac{a^x+a^{-x}}{b^y+b^{-y}} \right)^{-1} \cdot \left( \frac{a^x-a^{-x}}{b^y-b^{-y}} \right) \cdot \left( \frac{a^{2x}-1}{b^{2y}-1} \right)^{-2} : \left( \frac{a+1}{b^{2y}+1} \right)^{-1} \quad \left| \frac{b^y}{a^x} \right. \begin{array}{l} a > 0, x \neq 0 \\ b > 0, y \neq 0 \\ a \neq 1 \\ b \neq 1 \end{array}$$

$$h) \left( (-x)^{-2n} : (-x)^{-2n-1} \right)^{-2} \cdot \left( (-x)^{2n+1} \cdot (-x)^{-2n+1} \right)^{-3} \quad \left| \frac{1}{x^8}, x \neq 0 \right.$$