M1130 Seminář z matematiky I

2. zápočtová písemka

jméno: .... 3. listopadu 2008

1. **[2b]** V **C** vyřešte rovnici $x^2 + 2(i - 1)x + 3 + 2i = 0$.
2. **[2b]** Vypočítejte $\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{20}$
3. **[2b]** Najděte číslo $z \in \mathbf{C}$ takové, pro které platí: $|z - 2 + i| = |z - 5 + 2i| = |z - 10 - 3i|$.
4. **[1b]** Určete kvadratickou rovnici s reálnými koeficienty, je-li jeden její kořen číslo $5 - 2i$.
5. **[1b]** Vyjádřete $\cos 4x$ pomocí $\cos x$.
6. **[2b]** Řešte kvadratickou rovnici v **R** v závislosti na reálném parametru a: $x^2 - 2(a + 1)x + 4a = 0$.

Pěkné „počítání" ☺

1: $x^2+2(i-1)x+3+2i=0$

$D=[\underbrace{2(i-1)}_{2i-2}]^2-4\cdot 1(3+2i) = -4-8i+4-12-8i = -12-16i$

$\sqrt{-12-16i} = a+bi \;/^2, \; a,b\in\mathbb{R}$

$-12-16i = a^2+2abi-b^2$

$-16 = 2ab \Rightarrow b=-\frac{8}{a}$

$-12 = a^2-b^2$

$-12 = a^2-\left(-\frac{8}{a}\right)^2$

$-12 = a^2-\frac{64}{a^2} \;/a^2$

$-12a^2 = a^4-64$

$a^4+12a^2-64=0$

$t^2+12t-64=0$

$t_1=4 \Rightarrow a=\pm 2$

$t_2=-16 \Rightarrow a\in\emptyset$

$a=2 \Rightarrow b=-4$

$a=-2 \Rightarrow b=4$

$x_1 = \frac{-2(i-1)+2-4i}{2} = 2-3i$

$x_2 = \frac{-2(i-1)-2+4i}{2} = i$

6: $x^2-2(a+1)x+4a=0$

$D = 4(a+1)^2-4\cdot 1\cdot 4a = 4a^2+8a+4-16a =$

$= 4a^2-8a+4 = (2a-2)^2$

$\sqrt{D} = |2a-2| = 2|a-1|$

I. $D=0$ ... $a=1$ $\quad x = \frac{2(a+1)}{2} = a+1 = 2$

II. $D>0$ .... $2|a-1|>0$

$|a-1|>0$

$a\in\mathbb{R}\setminus\{1\}$

$x_{1,2} = \frac{2(a+1)\pm 2|a-1|}{2} = \begin{cases} 2a \\ 2 \end{cases}$

III $D<0$ ... nelze $(2|a-1|<0)$

Závěr:

parametr	neznámá
$a=1$	$x=2$
$a\in\mathbb{R}\setminus\{1\}$	$x_1=2a,\ x_2=2$

② $\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{20} =$

$|z| = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2+\left(\frac{1}{2}\right)^2} = 1$ $\quad \cos\varphi = -\frac{\sqrt{3}}{2}$ $\Rightarrow$ II kvadrant : $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\sin\varphi = \frac{1}{2}$

$= \left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)^{20} = \left(\cos\underbrace{\frac{100\pi}{6}}_{16\pi+\frac{2}{3}\pi} + i\sin\frac{100\pi}{6}\right) = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \underline{\underline{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}}$

③ $A = [2,-1],\ B = [5,-2],\ C = [10,3],\ X = [x,y]$

$|AX| = |BX|$

$\sqrt{(x-2)^2+(y+1)^2} = \sqrt{(x-5)^2+(y+2)^2}\ /^2$

$x^2-4x+4+y^2+2y+1 = x^2-10x+25+y^2+4y+4$

$6x-2y-24=0$

$y = 3x-12$

$|XB| = |XC|$

$\sqrt{(x-5)^2+(y+2)^2} = \sqrt{(x-10)^2+(y-3)^2}$

$x^2-10x+25+y^2+4y+4 = x^2-20x+100+y^2-6y+9$

$10x+10y-80=0$

$y = 8-x$

průsečík :

$3x-12 = 8-x$

$4x = 20$

$\left.x = 5 \Rightarrow y = 3\cdot 5 - 12 = 3\right\} \Rightarrow \underline{\underline{z = 5+3i}}$

④ $z_1 = 5-2i \Rightarrow z_2 = 5+2i$

$a = 1,\ b = -(z_1+z_2) = -(5-2i+5+2i) = -10,\ c = z_1\cdot z_2 = (5-2i)(5+2i) = 25+4 = 29$

$\Rightarrow \underline{\underline{x^2-10x+29=0}}$

⑤ $\cos 4x$ pomocí $\cos x$

$$\left.\begin{aligned}(\cos x+i\sin x)^4 &= \underline{\cos 4x} + i\sin 4x\\(\cos x+i\sin x)^4 &= \underline{\cos^4 x} + 4i\cos^3 x\sin x \underline{\underline{-6\cos^2 x\sin^2 x}} - 4i\cos x\sin^3 x \underline{\underline{+\sin^4 x}}\end{aligned}\right\} \Rightarrow$$

$\Rightarrow \cos 4x = \cos^4 x - 6\cos^2 x\sin^2 x + \sin^4 x = \cos^4 x - 6\cos^2 x(1-\cos^2 x) + (1-\cos^2 x)^2 =$

$= \cos^4 x - 6\cos^2 x + 6\cos^4 x + 1 - 2\cos^2 x + \cos^4 x$

$\underline{\underline{\cos 4x = 8\cos^4 x - 8\cos^2 x + 1}}$