M1130 Seminář z matematiky I

3. zápočtová písemka

jméno: .... 1. prosince 2008

V **R** řešte rovnice a nerovnice:

1. **[2b]** $0{,}25^{2-\sqrt{5x+1}} = 4 \cdot 2^{\sqrt{5x+1}}$
2. **[3b]** $\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}} \leq \left(\frac{512}{125}\right)^{3-x}$
3. **[2b]** $x^{\log x} + 10x^{-\log x} = 11$
4. **[3b]** $\log_x(6-x) > 2$

Pěkné počítání ☺

① $0{,}25^{2-\sqrt{5x+1}} = 4 \cdot 2^{\sqrt{5x+1}}$ $0{,}25 = \frac{1}{4} = 2^{-2}$

$$2^{-2(2-\sqrt{5x+1})} = 2^{2+\sqrt{5x+1}}$$

$$-4 + 2\sqrt{5x+1} = 2 + \sqrt{5x+1}$$

$$\sqrt{5x+1} = 6 \quad |^2$$

$$5x+1 = 36$$

$$x = 7$$

$$K = \{7\}$$

② $\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}} \leqq \left(\frac{512}{125}\right)^{3-x}$

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}} \leqq \left[\left(\frac{8}{5}\right)^3\right]^{3-x}$$

$\left(\frac{5}{8}\right)^x$ ... klesající fce ⇒ otáčíme znaménko!

$$\left(\frac{5}{8}\right)^{\frac{2x+1}{x-1}} \leqq \left(\frac{5}{8}\right)^{-3(3-x)}$$

$$\frac{2x+1}{x-1} \geqq 3x-9$$

$$\frac{2x+1-(3x-9)(x-1)}{x-1} \geqq 0$$

$$\frac{2x+1-3x^2+12x-9}{x-1} \geqq 0 \quad / \cdot(-1)$$

$$\frac{3x^2-14x+8}{x-1} \leqq 0$$

$$x_{1,2} = \frac{14 \pm \sqrt{14^2-4\cdot3\cdot8}}{2\cdot3} = \frac{14\pm10}{6} = \begin{cases} 4 \\ \frac{2}{3} \end{cases}$$

⊖ + ⊖ +

$\frac{2}{3}$ 1 4

$$K = (-\infty; \tfrac{2}{3}\rangle \cup (1, 4\rangle$$

③ $x^{\log x} + 10\, x^{-\log x} = 11$

$x^{\log x} + 10\left(x^{\log x}\right)^{-1} = 11$

subst.: $a = x^{\log x}$

$a + 10\,\frac{1}{a} = 11 \quad / \cdot a$

$a^2 - 11a + 10 = 0$

$a_1 = 1 \quad \vee \quad a_2 = 10$

$\boxed{1} \qquad \boxed{2}$

$\boxed{1}$ $x^{\log x} = 1 \quad / \log$

$\log x^{\log x} = \log 1$

$(\log x)^2 = 0$

$\log x = 0$

$\underline{\underline{x = 1}}$

$\boxed{2}$ $x^{\log x} = 10 \quad / \log$

$\log x^{\log x} = \log 10$

$(\log x)^2 = 1$

$\log x = 1 \qquad \log x = -1$

$x = 10^1 \qquad x = 10^{-1}$

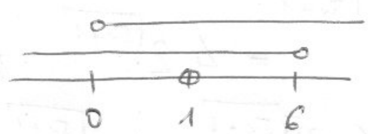
$\underline{\underline{x = 10}} \qquad \underline{\underline{x = \frac{1}{10}}}$

$\underline{\underline{K = \left\{\frac{1}{10};\ 1;\ 10\right\}}}$

④ $\log_x (6-x) > 2$

D_f: $6 - x > 0 \qquad x \neq 1 \qquad x > 0$

$x < 6$



$D_f = (0,6) \setminus \{1\}$

$\boxed{1}$ $x \in (0,1)$ → $\log_x$ je fce klesající ⇒

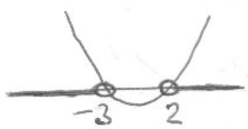
$\log_x (6-x) > 2$ ⇒ otáčíme znaménko!

$\log_x (6-x) > \log_x x^2$

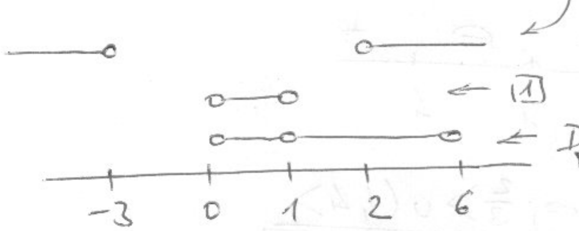
$6 - x < x^2$

$x^2 + x - 6 > 0$

$x_1 = -3 \quad x_2 = 2$



$x \in (-\infty,-3) \cup (2,\infty)$



$K_1 = \emptyset$

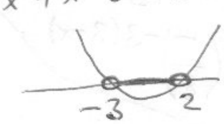
$\boxed{2}$ $x \in (1,\infty)$

$\log_x (6-x) > 2$

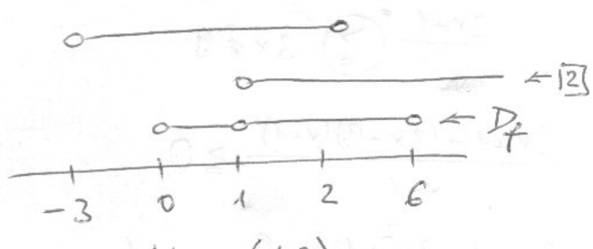
$\log_x (6-x) > \log_x x^2$

$6 - x > x^2$

$x^2 + x - 6 < 0$



$x \in (-3;2)$



$K_2 = (1,2)$

$K = K_1 \cup K_2$

$\underline{\underline{K = (1,2)}}$