9. cvičení

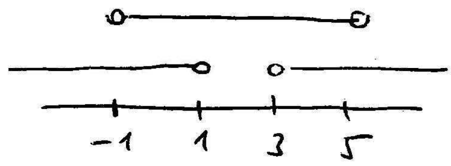
14. listopadu 2008

1) $\log_8 (x^2 - 4x + 3) < 1$

D_f: $x^2 - 4x + 3 > 0 \quad \Rightarrow \quad D_f = (-\infty, 1) \cup (3, \infty)$

$x_1 = 1, \quad x_2 = 3$

$$\log_8 (x^2 - 4x + 3) < \log_8 8$$
$$x^2 - 4x + 3 < 8$$
$$x^2 - 4x - 5 < 0$$
$$x_1 = -1, \quad x_2 = 5 \quad \Rightarrow x \in (-1, 5)$$



$\underline{K = (-1, 1) \cup (3, 5)}$

2) $\log_{0,7} \frac{x^2 - 4x + 6}{x} < 0$

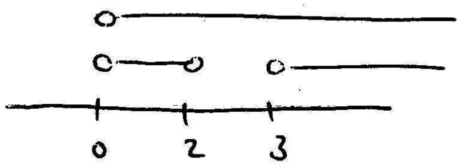
D_f: $\frac{x^2 - 4x + 6}{x} > 0$

$x_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 16}}{2} \Rightarrow x^2 - 4x + 6$ vždy kladný $\Rightarrow D_f = (0, \infty)$

$$\log_{0,7} \frac{x^2 - 4x + 6}{x} < \log_{0,7} 1$$
$$\frac{x^2 - 4x + 6}{x} > 1$$
$$\frac{x^2 - 4x + 6}{x} - 1 > 0$$
$$\frac{x^2 - 4x + 6 - x}{x} > 0$$
$$\frac{x^2 - 5x + 6}{x} > 0$$
$$\frac{(x-2)(x-3)}{x} > 0$$

	$(-\infty,0)$	$(0,2)$	$(2,3)$	$(3,\infty)$
$x-2$	−	−	+	+
$x-3$	−	−	−	+
x	−	+	+	+
	−	⊕	−	⊕

$\Rightarrow x \in (0,2) \cup (3, \infty)$



$\underline{K = (0, 2) \cup (3, \infty)}$

pozn. 1: při řešení (logaritmických) **ne**rovnic prvně určíme definiční obor!

pozn. 2: při (od)logaritmování nerovnice logaritmem o základu $\underline{a}$ se znaménko nerovnosti:
- otáčí, pokud $a \in (0, 1)$
- ponechá, pokud $a \in (1, \infty)$

3) $\log_x(x+2) > 2$

$D_f: \; x+2>0 \Rightarrow D_f = (-2,\infty)$

$\boxed{a}$ $x\in(0,1) \Rightarrow$ klesající fce $\Rightarrow$ obrací se znak nerovnosti při odlogaritmování

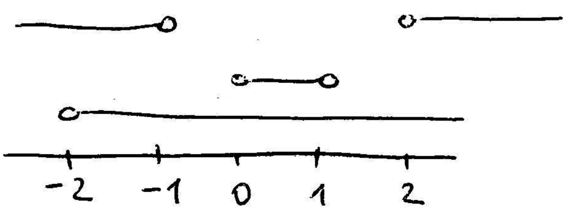
$\log_x(x+2) > \log_x x^2$

$x+2 < x^2$

$x^2-x-2>0$

$x_1=-1,\; x_2=2$

$x\in(-\infty,-1)\cup(2,\infty)$



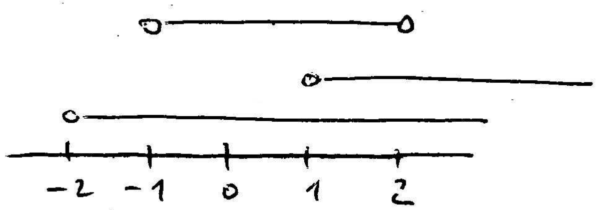
$K_1=\emptyset$

$\boxed{b}$ $x\in(1,\infty) \Rightarrow$ rostoucí fce $\Rightarrow$ znak nerovnosti při odlogaritmování zůstává

$\log_x(x+2) > \log_x x^2$

$x+2 > x^2$

$x\in(-1,2)$



$K_2=(1,2)$

$K=K_1\cup K_2$

$K=(1,2)$

4) $\log_x(x+1) > \log_{\frac{1}{x}}(2-x)$

pozn.: $\log_{\frac{1}{x}}(2-x) = \dfrac{\log_x(2-x)}{\underbrace{\log_x \frac{1}{x}}_{-1}} = -\log_x(2-x) = \log_x\frac{1}{2-x}$

$D_f:$ $\to x\in(0,1)\cup(1,\infty)$

$\to x+1>0 \Rightarrow x\in(-1,\infty)$

$\to \frac{1}{x}\in(0,1)\cup(1,\infty)$ čili

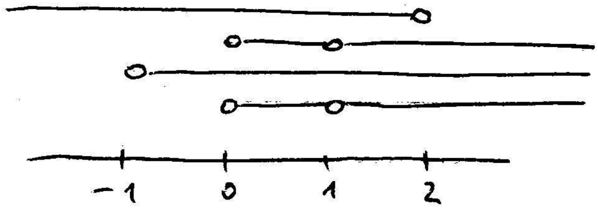
$\frac{1}{x}\neq 1 \wedge \frac{1}{x}>0$

$x\neq 1 \wedge x>0$

$x\in(0,1)\cup(1,\infty)$

$\to x\neq 0$

$\to 2-x>0 \Rightarrow x\in(-\infty,2)$



$\Rightarrow D_f=(0,1)\cup(1,2)$

a) $x \in (0,1)$

$$\log_x (x+1) > \log_x \frac{1}{2-x}$$

$$x+1 < \frac{1}{2-x}$$

$$x+1-\frac{1}{2-x} < 0$$

$$\frac{(x+1)(2-x)-1}{2-x} < 0$$

$$\frac{-x^2+x+2-1}{2-x} < 0$$

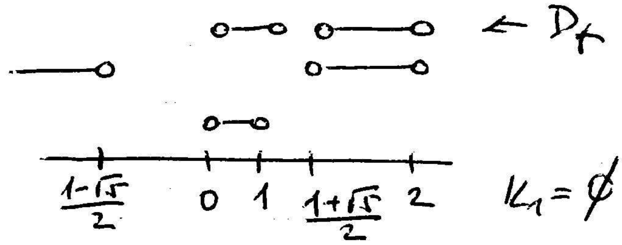
$$\frac{x^2-x-1}{x-2} < 0$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

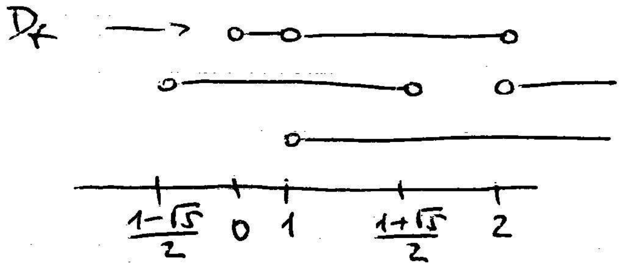
$x-\frac{1+\sqrt{5}}{2}$	$-$	$-$	$+$	$+$
$x-\frac{1-\sqrt{5}}{2}$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
	$-$	$+$	$-$	$+$

$\frac{1-\sqrt{5}}{2}$, $\frac{1+\sqrt{5}}{2}$, 2

$$\Rightarrow x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{1+\sqrt{5}}{2}, 2\right)$$



$K_1 = \emptyset$



b) $x \in (1, \infty)$

$$\log_x (x+1) > \log_x \frac{1}{2-x}$$

$$x+1 > \frac{1}{2-x}$$

$$\vdots$$

$$\frac{x^2-x-1}{x-2} > 0 \Rightarrow x \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right) \cup (2, \infty)$$

$$K_2 = \left(1, \frac{1+\sqrt{5}}{2}\right)$$

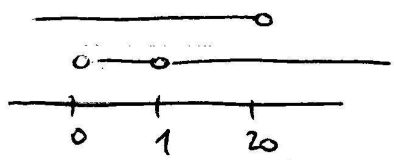
$$K = K_1 \cup K_2$$

$$K = \left(1, \frac{1+\sqrt{5}}{2}\right)$$

5) $\log_x \sqrt{20-x} > 1$

$D_f: \rightarrow x \in (0,1) \cup (1, \infty)$

$\rightarrow 20-x > 0 \Rightarrow x \in (-\infty, 20)$



$D_f = (0,1) \cup (1, 20)$

a) $x \in (0,1)$

$$\log_x \sqrt{20-x} > \log_x x$$

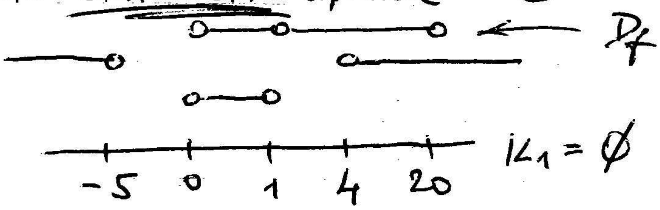
$\sqrt{20-x} < x \quad /^2$ ... obě strany nerovnice jsou kladné $\Rightarrow$ umocnění je zde proto ekvivalentní úpravou ! :)

$$20-x < x^2$$

$$x^2+x-20 > 0$$

$$x_1 = -5, \; x_2 = 4$$

$$x \in (-\infty, -5) \cup (4, \infty)$$



$K_1 = \emptyset$

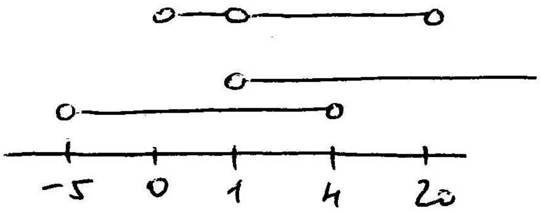
$\boxed{b}$ $x \in (1, \infty)$

$\log_x \sqrt{20-x} > \log_x x$

$\sqrt{20-x} > x$

$x^2 + x - 20 < 0$

$x \in (-5, 4)$



$K_2 = (1, 4)$

$K = K_1 \cup K_2$

$K = (1, 4)$

6) $x^{\log_2 x} > 2 \quad / \log_2$

$D_f = (0, \infty)$

$\log_2 x^{\log_2 x} > \log_2 2$

$\log_2 x \cdot \log_2 x > 1$

subst.: $a = \log_2 x$

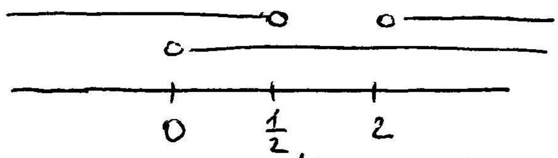
$a^2 > 1$

$a_1 = 1, \quad a_2 = -1$

$a \in (-\infty, -1) \cup (1, \infty)$ čili $a < -1$ ... $\log_2 x < -1 \rightarrow \log_2 x < \log_2 2^{-1} \rightarrow x < \frac{1}{2}$

nebo $a > 1$ ... $\log_2 x > 1 \rightarrow \log_2 x > \log_2 2 \rightarrow x > 2$

$\Rightarrow \quad x \in (-\infty, \frac{1}{2}) \cup (2, \infty)$



$K = (0, \frac{1}{2}) \cup (2, \infty)$

7) $\log_2 x \leqq \dfrac{2}{\log_2 x - 1}$

$$D_f: \begin{array}{l} \to x > 0 \Rightarrow x \in (0, \infty) \\ \to \log_2 x - 1 \neq 0 \\ \qquad \log_2 x \neq 1 \\ \qquad x \neq 2 \end{array} \Bigg\} \Rightarrow D_f = (0,2) \cup (2, \infty)$$

subst.: $a = \log_2 x$

$$a \leqq \frac{2}{a-1}$$

$$a - \frac{2}{a-1} \leqq 0$$

$$\frac{a(a-1) - 2}{a-1} \leqq 0$$

$$\frac{a^2 - a - 2}{a-1} \leqq$$

$$\frac{(a+1)(a-2)}{a-1} \leqq 0$$

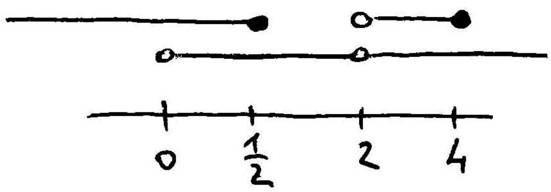
$a+1$	$-$		$+$		$+$		$+$
$a-2$	$-$		$-$		$-$		$+$
$a-1$	$-$		$-$		$+$		$+$
	$\ominus$	-1	$+$	1	$\ominus$	2	$+$

$$a \leqq -1 \quad \vee \quad (a > 1 \wedge a \leqq 2)$$

$$\log_2 x \leqq -1 \quad \vee \quad (\log_2 x > 1 \quad \wedge \quad \log_2 x \leqq 2)$$

$$x \leqq 2^{-1} \quad \vee \quad (x > 2 \quad \wedge \quad x \leqq 2^2)$$

$$\underbrace{\underbrace{x \leqq \tfrac{1}{2}}_{x \in (-\infty, \frac{1}{2}\rangle} \quad \vee \quad \underbrace{(x > 2 \quad \wedge \quad x \leqq 4)}_{x \in (2, 4\rangle}}_{x \in (-\infty, \frac{1}{2}\rangle \cup (2, 4\rangle}$$



$$K = (0, \tfrac{1}{2}\rangle \cup (2, 4\rangle$$