Global analysis. Exercises 3

1) Find the Lie brackets of the following vector fields:

- $X = \sin u \frac{\partial}{\partial v} + \cos v \frac{\partial}{\partial u}$, $Y = u \frac{\partial}{\partial u} + v \frac{\partial}{\partial v}$;
- $X = z^2 \frac{\partial}{\partial x} + xy \frac{\partial}{\partial y}$, $Y = xyz \frac{\partial}{\partial x} + y^2 \frac{\partial}{\partial y} + x \frac{\partial}{\partial z}$.

2) Let M be a 1-dimensional manifold, X, Y vector fields on M, $X_x \neq 0$ for all $x \in M$ and $[X, Y] = 0$. Prove that $Y = cX$, where $c \in \mathbb{R}$ is a constant.

3) Find the vector fields defined by the following flows:

- $Fl_t^X(x, y) = (5t + x, 4t + y)$;
- $Fl_\varphi^X(x, y) = (x \cos\varphi - y \sin\varphi, x \sin\varphi + y \cos\varphi)$.

4) Find the integral curves of the vector fields:

- $X = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$;
- $X = (x + y) \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}$;
- $X = x^2 \frac{\partial}{\partial x} + y^2 \frac{\partial}{\partial y}$.