

Global analysis. Exercises 9

1) Find the metric on the sphere $S^2 \subset \mathbb{R}^3$ using the following coordinates:

- the coordinates x, y obtained using the stereographic projection;
- the spherical coordinates θ, φ (for $A \in S^2$, θ is the angle between OA and Oz and φ is the angle between OA' and Ox , where A' is the projection of A to the plane Oxy).

2) Find the Christoffel symbols for the metric on the sphere $S^2 \subset \mathbb{R}^3$ in the coordinates of Exercise 1.

3) Let g be the metric on \mathbb{R}^2 obtained from the metric on the sphere using the stereographic projection. With respect to the metric g find:

- the middle of the segment AB with $A = (0.5; 0.5)$ and $B = (0.9; 0.3)$;
- the length of the circle $(x + 1)^2 + (y - 5)^2 = 1$.