SIGNÁLY A LINEÁRNÍ SYSTÉMY

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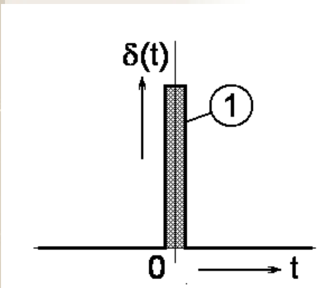
IV.
KONVOLUCE
&
VZORKOVACÍ TEORÉM

MU IBA

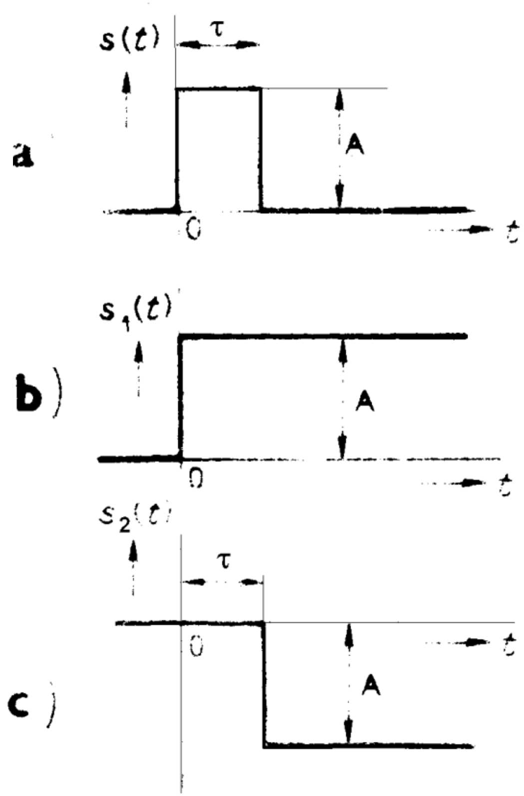
SPEKTRUM DIRACOVA IMPULZU

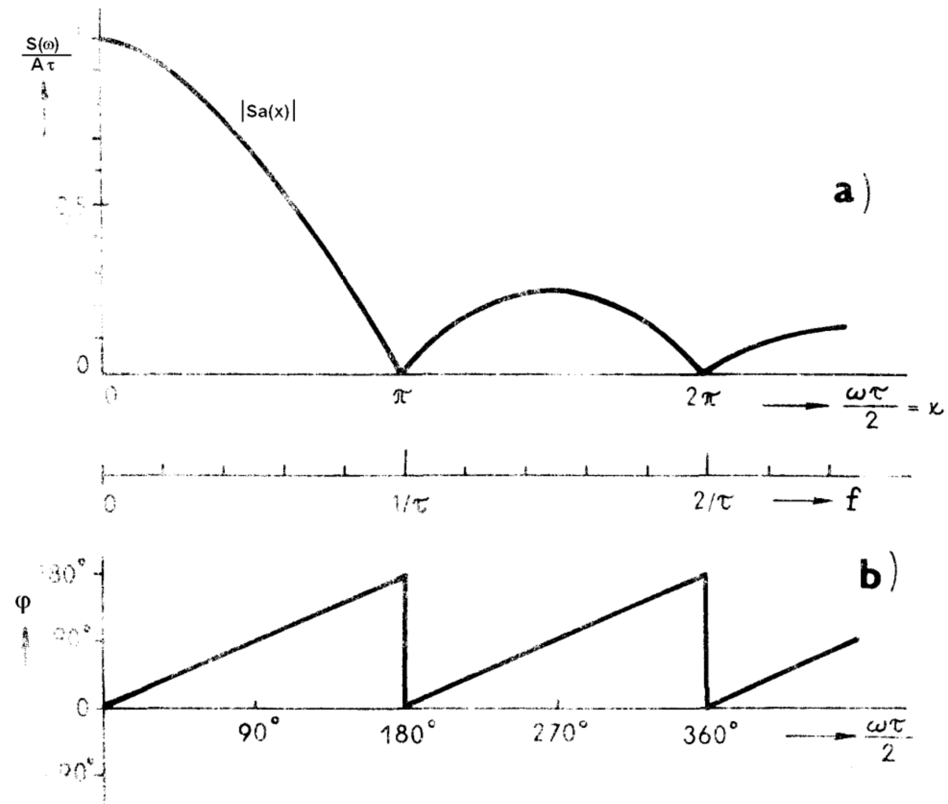


$\tau \rightarrow 0$

$A \rightarrow \infty$

$A.\tau = 1$





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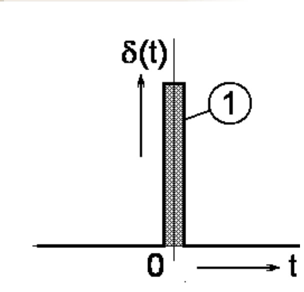

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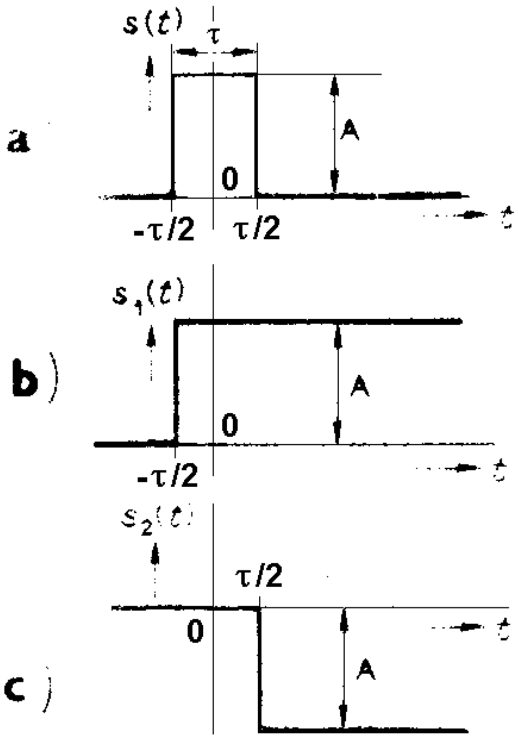

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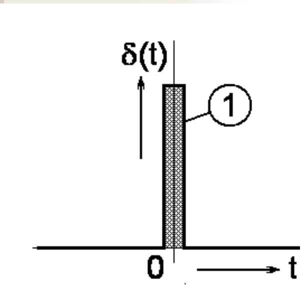

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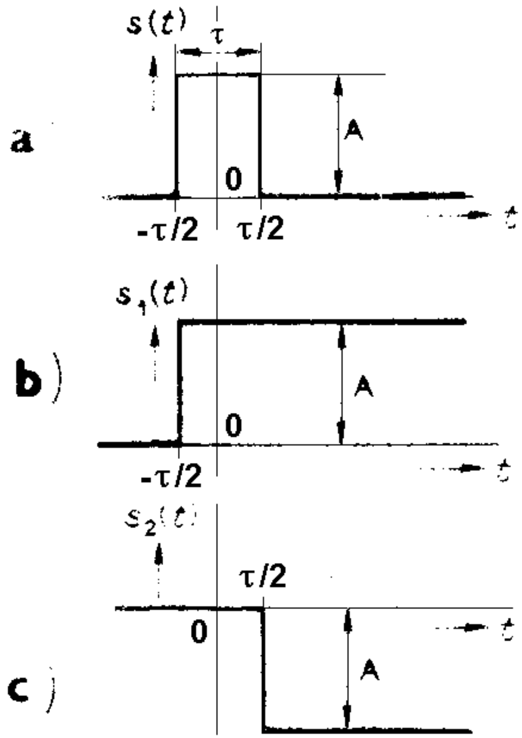
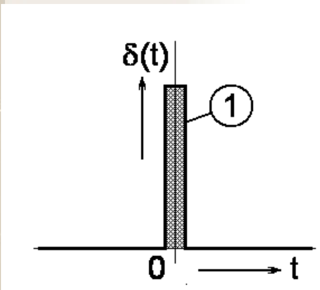


$$S(\omega) = A.\left(\frac{1}{j\omega}e^{j\omega\tau/2} - \frac{1}{j\omega}e^{-j\omega\tau/2}\right) =$$

$$= A.\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega} = \; . =$$

$$= \frac{2A}{\omega}\sin\frac{\omega\tau}{2} = \; . = A.\tau\,\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}.$$

SPEKTRUM DIRACOVA IMPULZU



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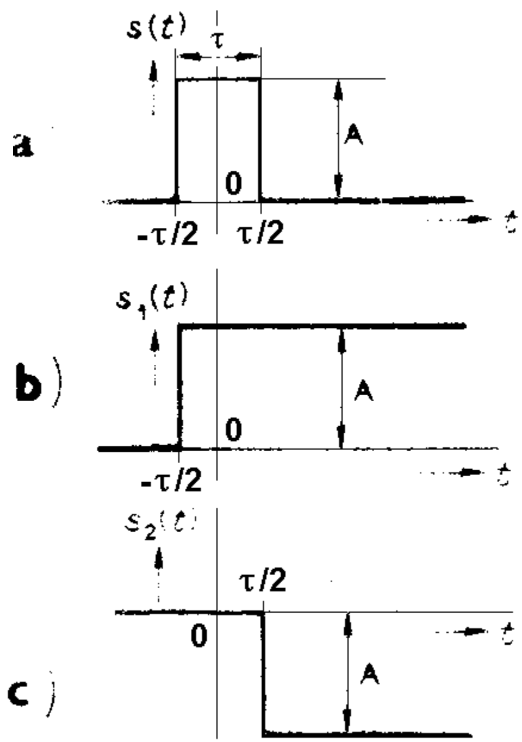

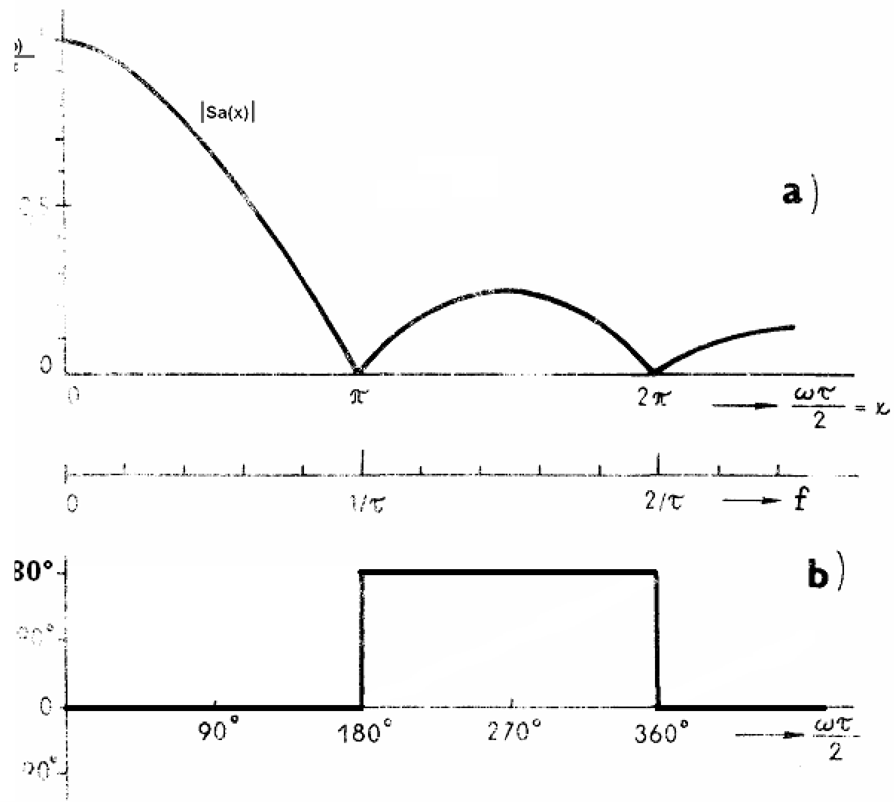

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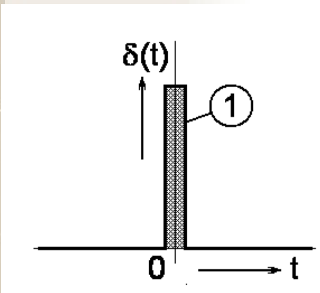

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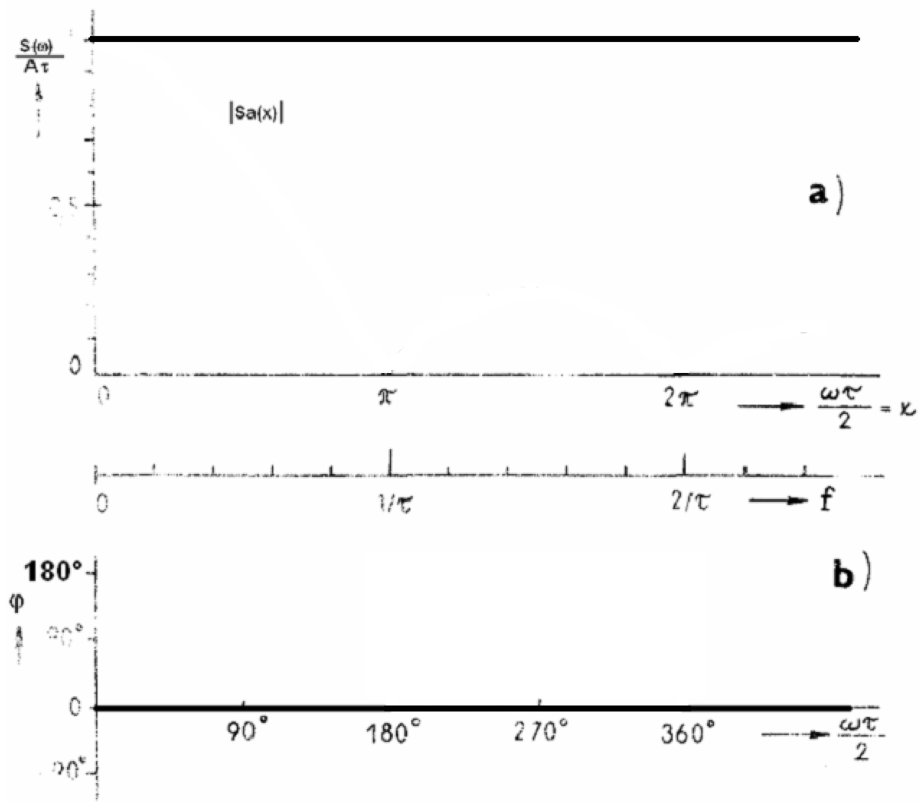
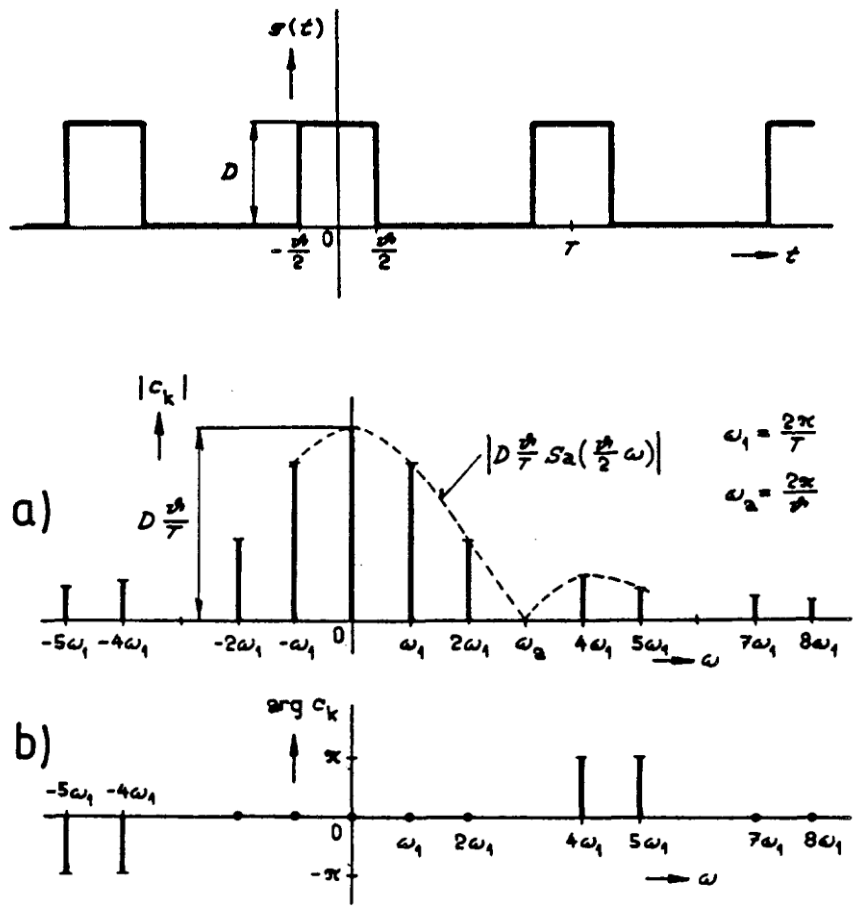


SPEKTRUM PULZU DIRACOVÝCH IMPULZŮ



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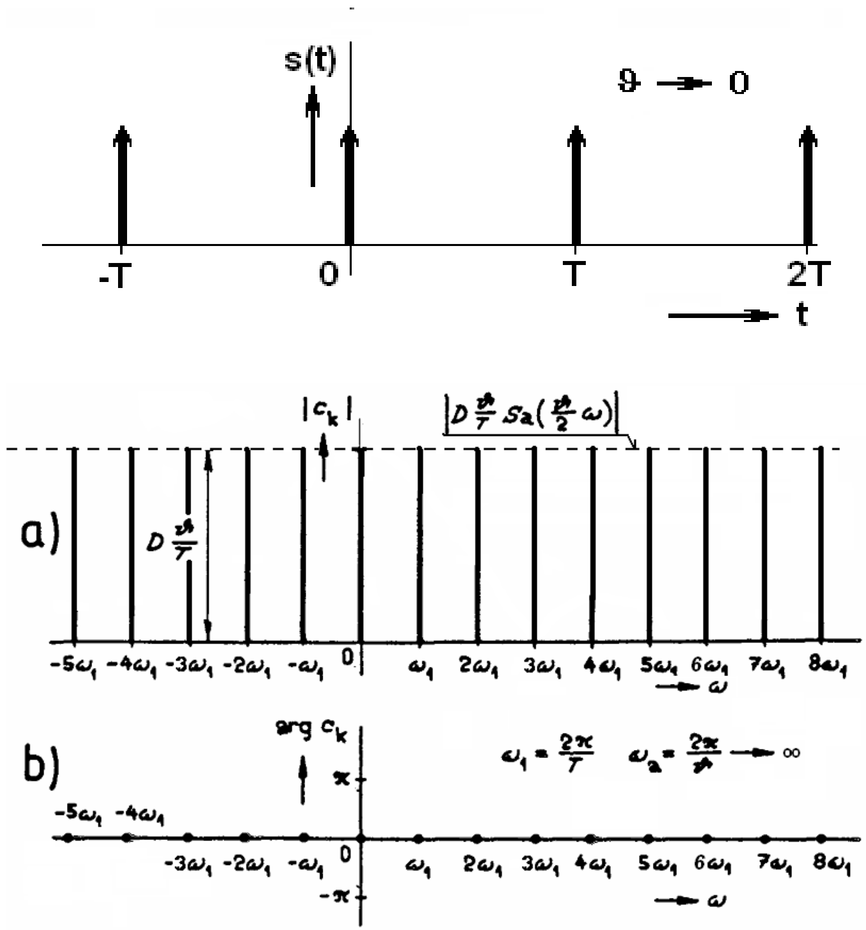


KONVOLUCE

$$s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\tau).s_2(t-\tau).d\tau =$$

$$= \int_{-\infty}^{\infty} s_1(t-\tau).s_2(\tau).d\tau \approx S_1(\omega).S_2(\omega)$$

Důkaz:

$$s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\tau).s_2(t-\tau).d\tau = \begin{vmatrix} x = t-\tau \\ \tau = t-x \\ d\tau = -dx \end{vmatrix} =$$

$$= -\int_{\infty}^{-\infty} s_2(x).s_1(t-x).dx = s_2(t) * s_1(t)$$

KONVOLUCE

Distributivní zákon:

$$f_1(t) * \left[f_2(t) * f_3(t) \right] = \left[f_1(t) * f_2(t) \right] * f_3(t)$$

Asociativní zákon:

$$f_1(t) * \left[f_2(t) + f_3(t) \right] = f_1(t) * f_2(t) + f_1(t) * f_3(t)$$

KONVOLUCE

Zákon o posunu v čase

Je –

$$f_1(t) * f_2(t) = g(t),$$

pak

$$f_1(t) * f_2(t-\tau) = g(t-\tau),$$

$$f_1(t-\tau) * f_2(t) = g(t-\tau)$$

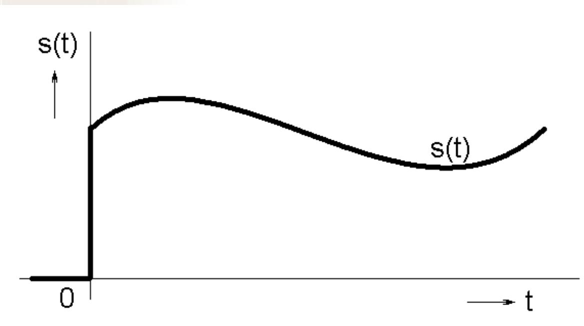
a

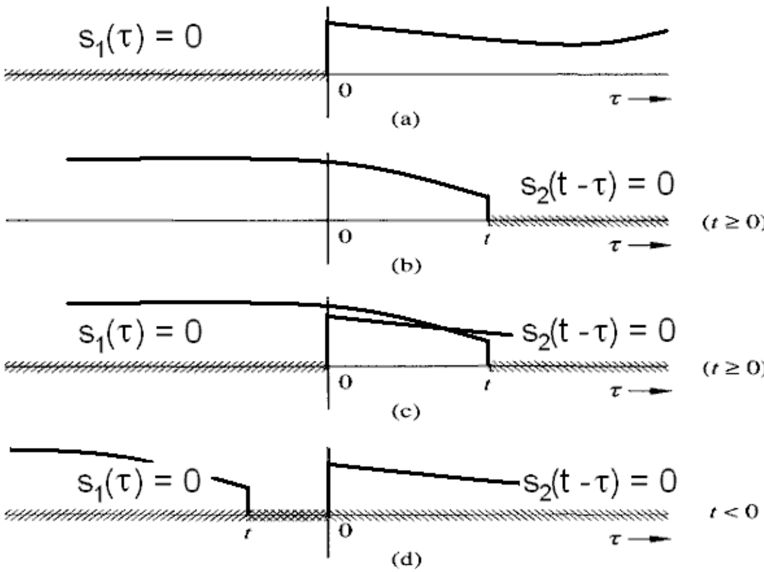
$$f_1(t-\tau_1) * f_2(t-\tau_2) = g(t-\tau_1-\tau_2)$$

KONVOLUCE

Konvoluce kauzálních signálů:

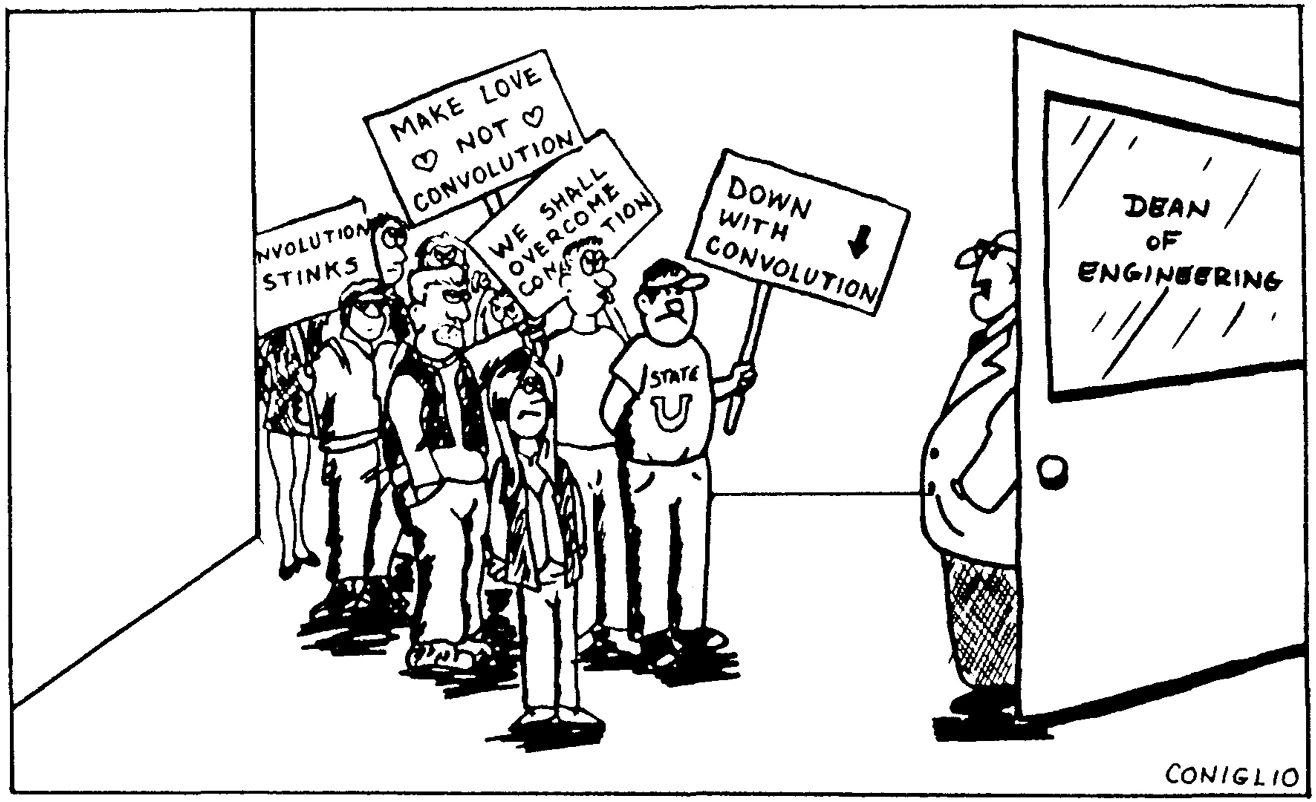
Pro kauzální signály platí s(t) = 0 pro t < 0





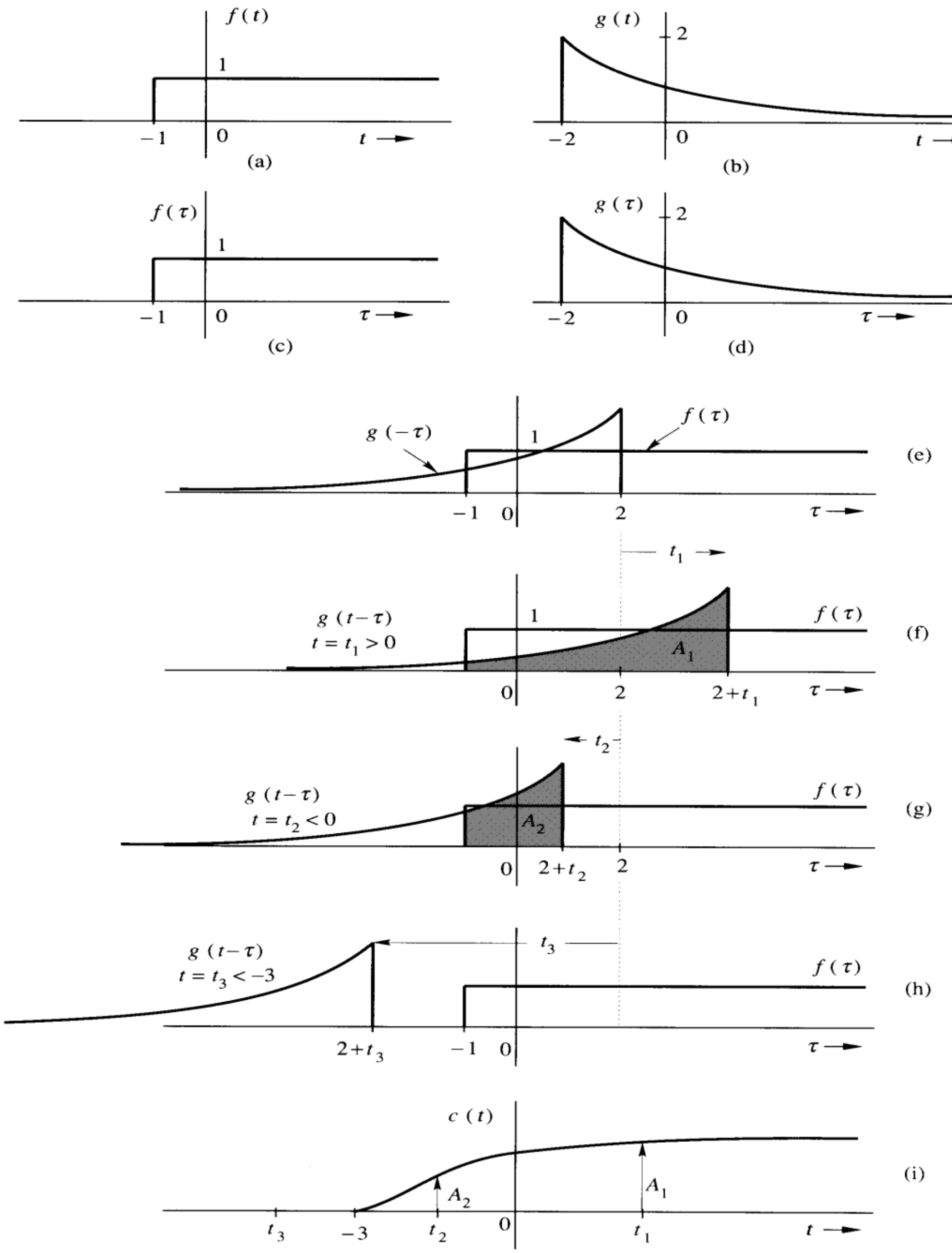
$$s_1(t) * s_2(t) = \int_0^t s_1(\tau).s_2(t - \quad).d\tau$$

KONVOLUCE

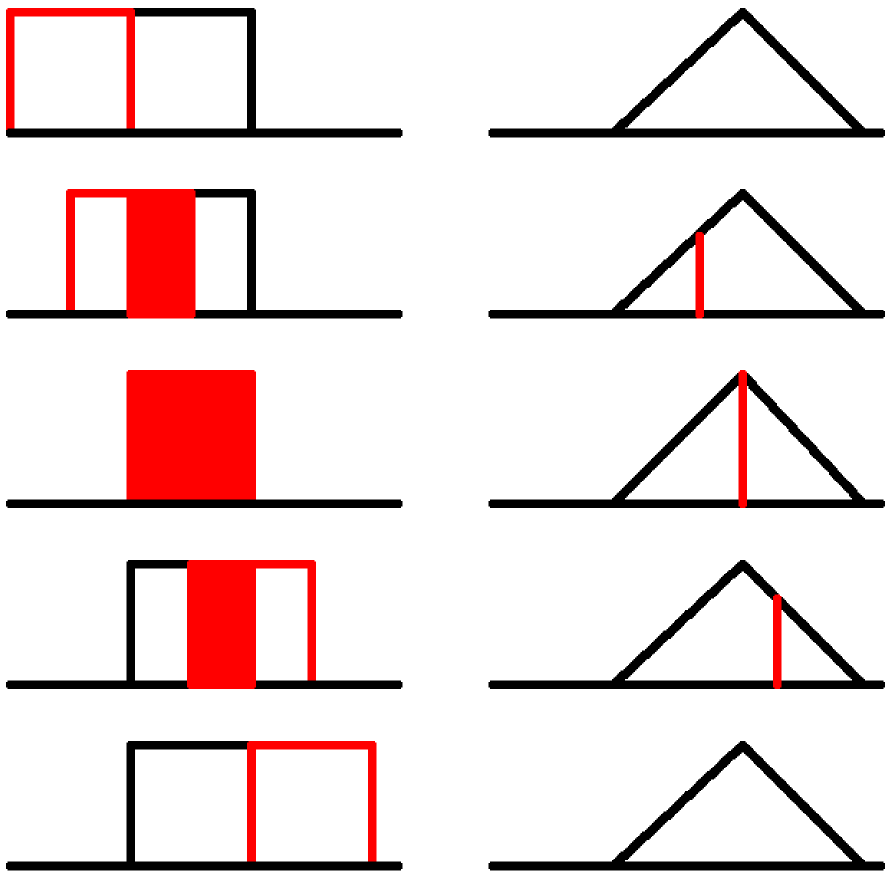


Convolution: its bark is worse than its bite!

KONVOLUCE


f(t)
1
−1
0
t
(a)
g(t)
2
−2
0
t
(b)
f(τ)
1
−1
0
τ
(c)
g(τ)
2
−2
0
τ
(d)
g(−τ)
f(τ)
1
−1
0
2
τ
(e)
t1
g(t−τ)
t = t1 > 0
1
f(τ)
A1
0
2
2+t1
τ
(f)
t2
g(t−τ)
t = t2 < 0
f(τ)
A2
0
2+t2
2
τ
(g)
g(t−τ)
t = t3 < −3
t3
f(τ)
2+t3
−1
0
τ
(h)
c(t)
A2
A1
t3
−3
t2
0
t1
t
(i)

KONVOLUCE



KONVOLUCE

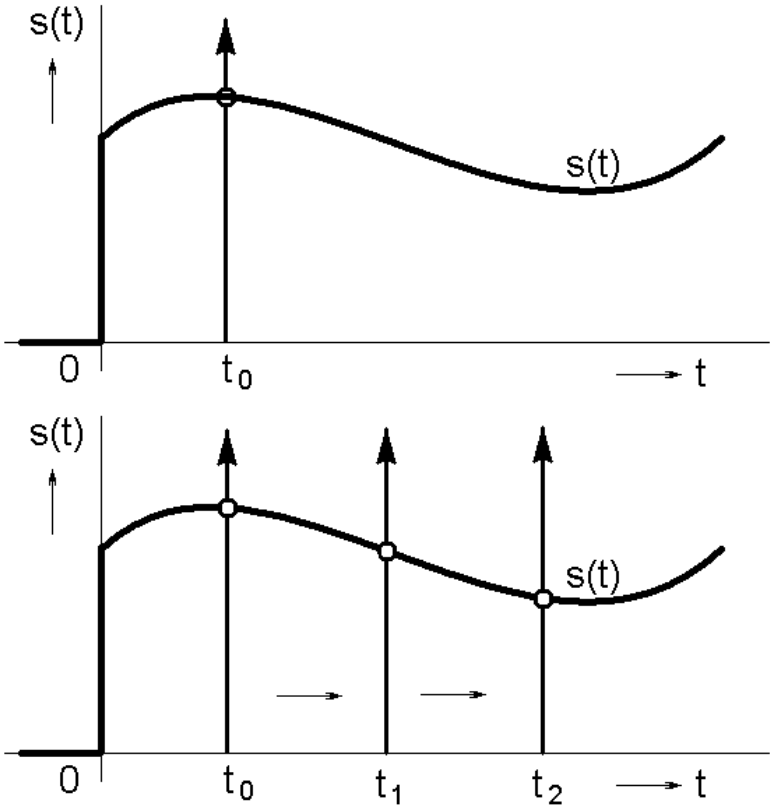
signálu s jednotkovým impulsem

definice:

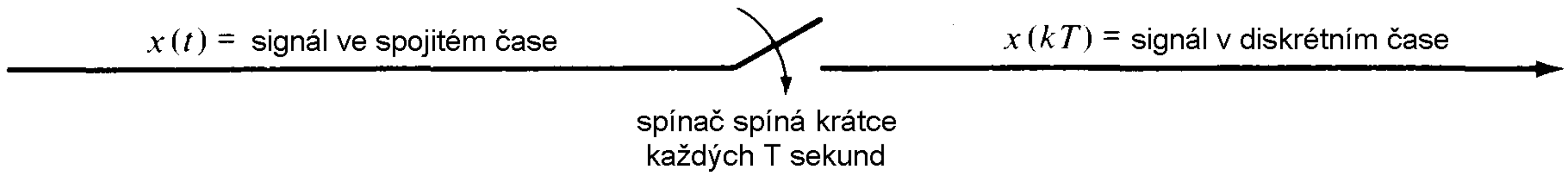
$$\int_{-\infty}^{\infty} s(t).\delta(t-t_0)\,dt = s(t_0)$$

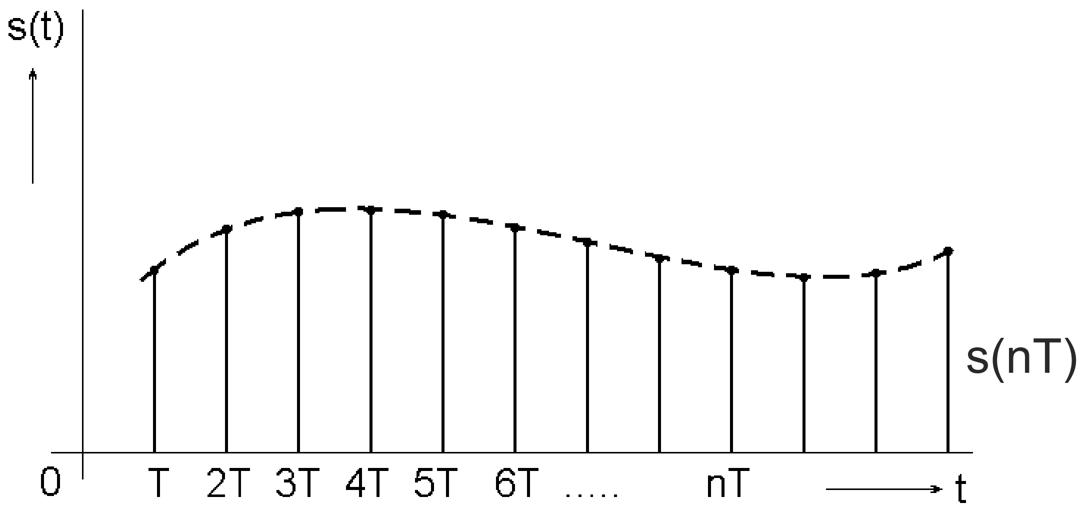
konvoluce:

$$s(t) * \delta(t) = \int_{-\infty}^{\infty} s(t).\delta(t-\tau)\,d\tau = s(t)$$



DISKRÉTNÍ SIGNÁL - VZORKOVÁNÍ

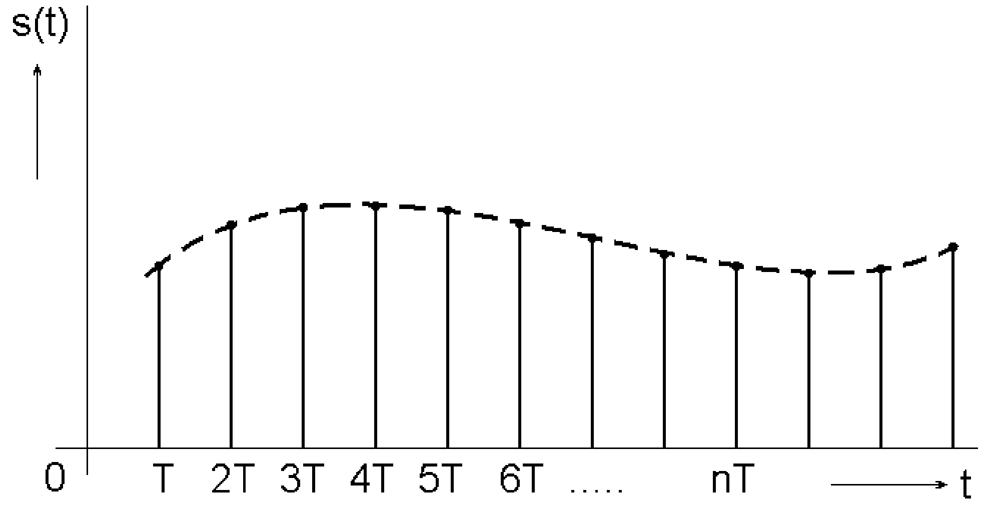




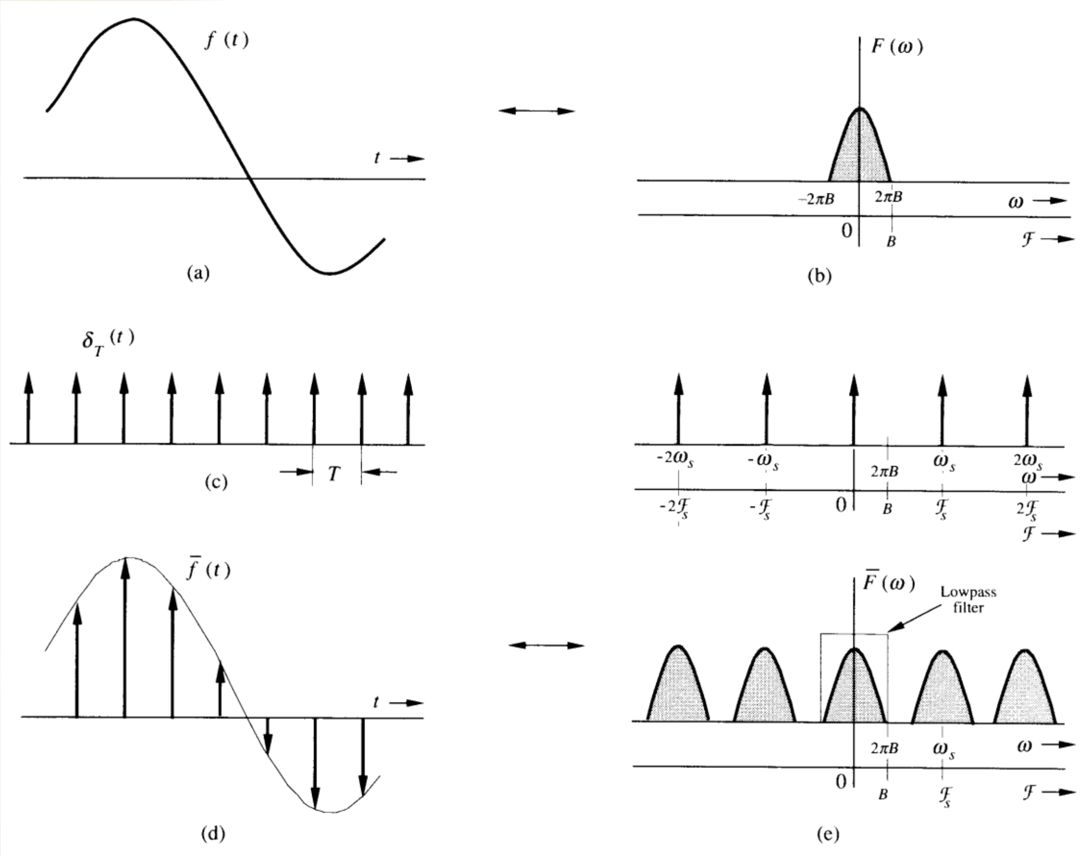
VZORKOVACÍ TEORÉM

$$s(t) \rightarrow s(T_1), s(T_2), s(T_3), \dots, s(T_n), \dots$$

$$s(t) \rightarrow s(T), s(2T), s(3T), \dots, s(nT), \dots$$



VZORKOVACÍ TEORÉM



Vzorkovací frekvence:

$f_s \geq 2B = f_N$,

kde B je maximální kmitočet ve vzorkovaném signálu

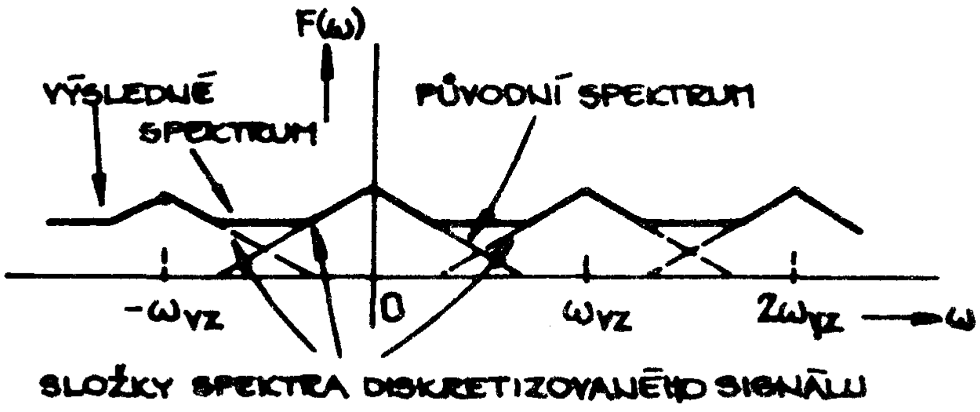
f_N –

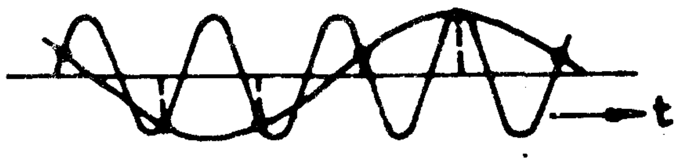
Nyquistův, (Shannonův, Kotelnikovův) kmitočet

$T_N = 1/f_N = 1/2B$

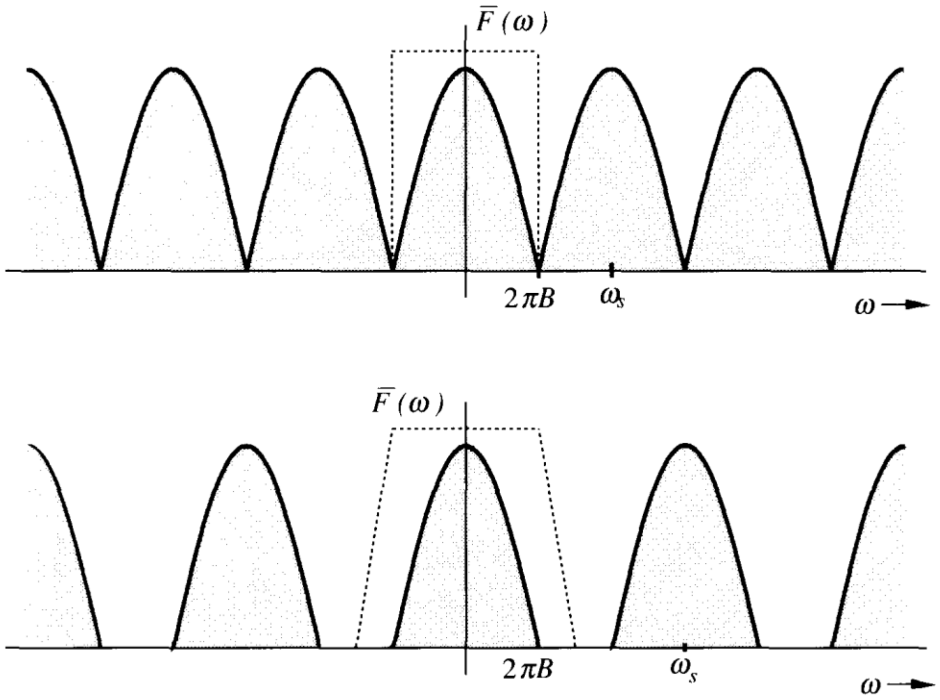
Nyquistův interval (perioda), vzorkovací interval (perioda)

VZORKOVACÍ TEORÉM





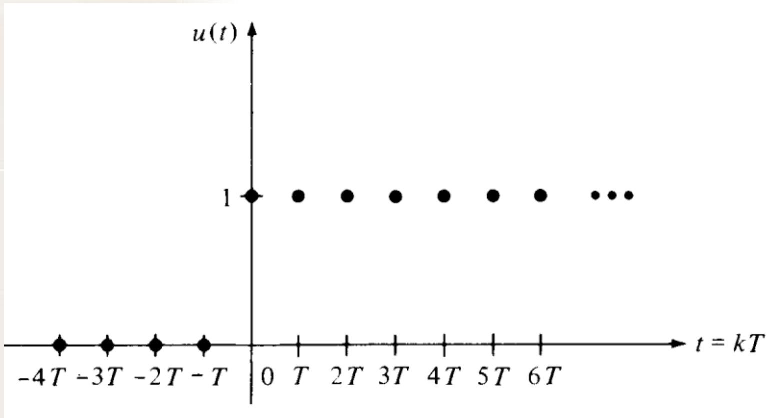
VZORKOVACÍ TEORÉM



$f_{sr} = (4 \div 5).f_N$

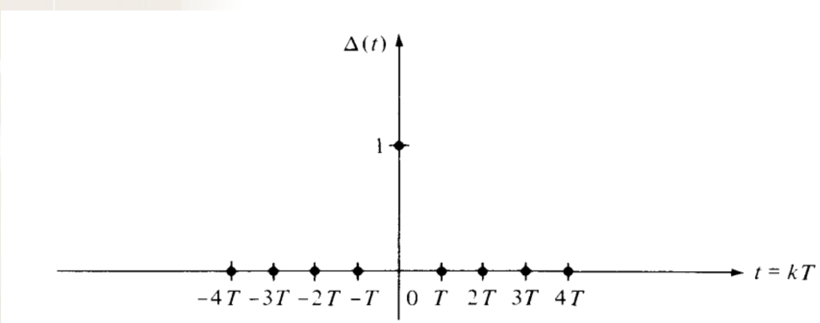
V. DISKRÉTNÍ SIGNÁL
POPIS V ČASOVÉ OBLASTI

JEDNORÁZOVÉ DISKRÉTNÍ SIGNÁLY



- ☑ jednotkový skok

$$\Sigma(t) = \begin{cases} 0, & t = kT, k = \ldots, -2, -1, \\ 1, & t = kT, k = 0, 1, 2, \ldots \end{cases}$$



- ☑ jednotkový impuls

$$\Delta(t) = \begin{cases} \;, & t = \\ \;, & t = \; T, k \neq \end{cases}$$

PERIODICKÉ DISKRÉTNÍ SIGNÁLY

- ☑ diskrétní signál x(kT) je periodický s periodou NT, když platí

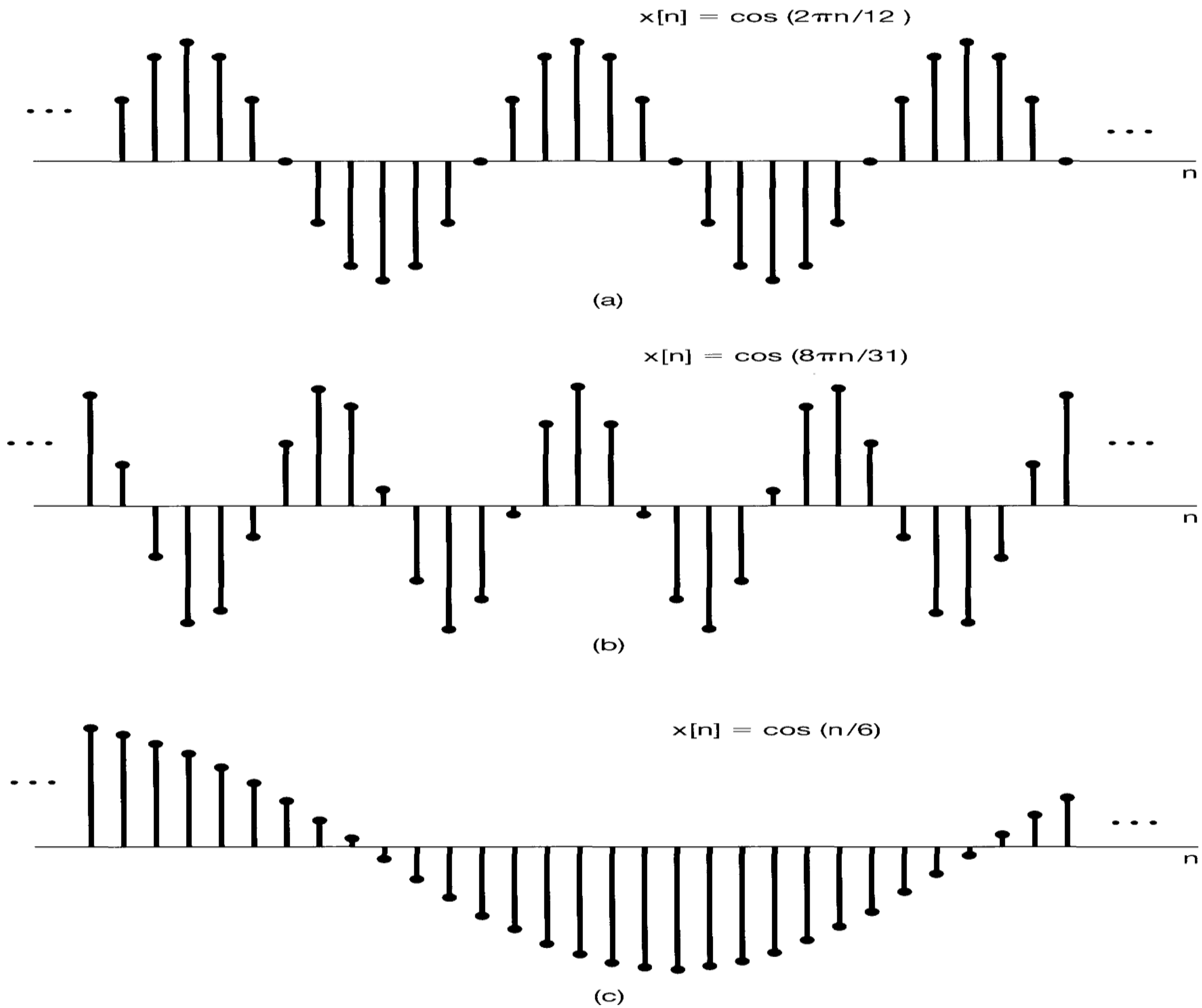
  $x[(k+N)T] = x(kT), \ \text{pro } k = 0, \ 1, \ 2, \ldots$

- ☑ příklady
  - ➔ $x(kT) = A.\cos(2\pi k/N)$
  - ➔ $x(kT) = A.\sin(2\pi k/N)$
  - ➔ $x(kT) = A.\exp(j2\pi k/N)$

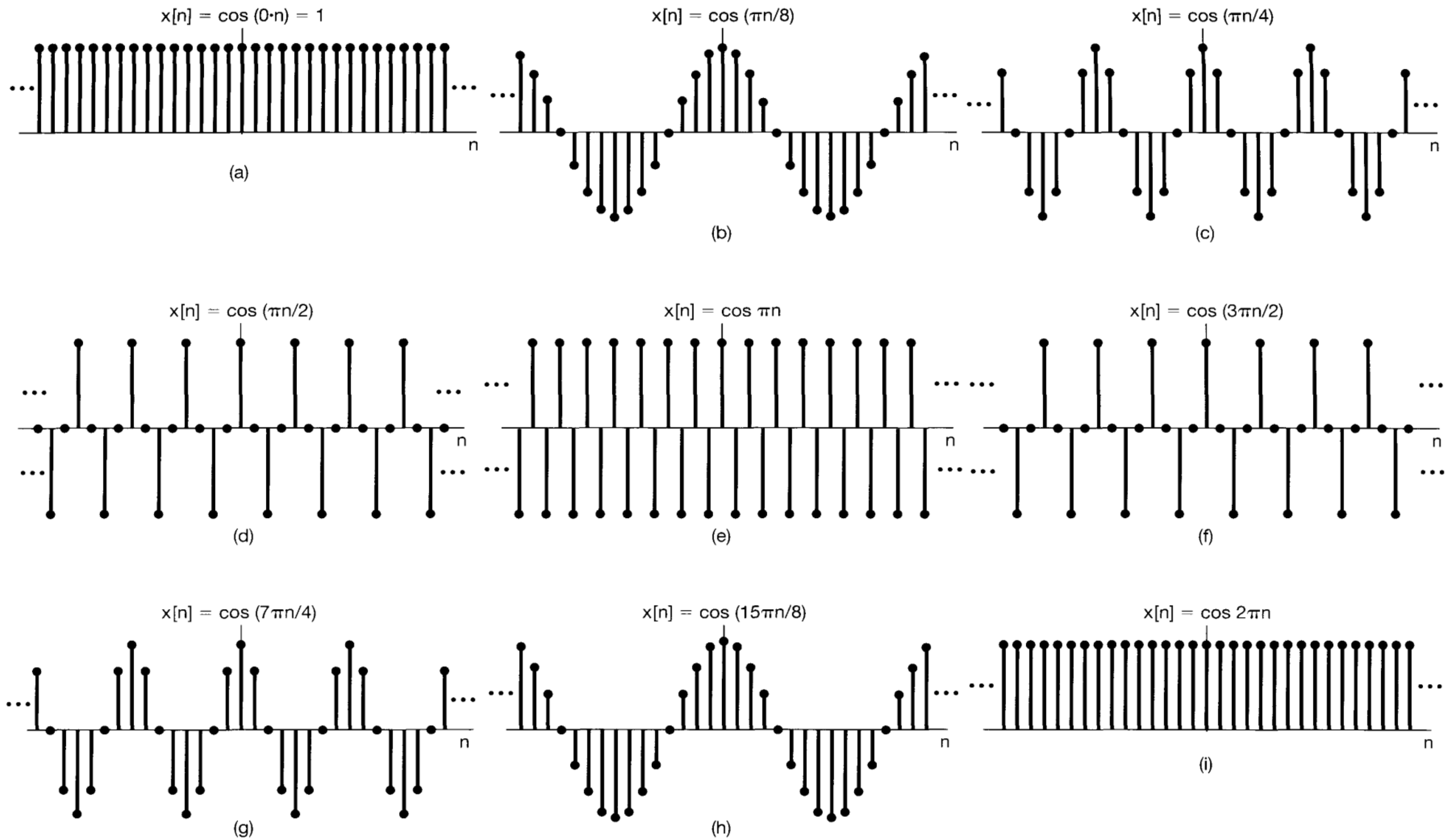
$$x[(k+N)T] = \exp\frac{j2\pi(k+N)}{N} = \exp\frac{j2\pi k}{N}.\exp(j2\pi)$$

$$\exp(j2\pi) = \cos 2\pi + j\sin 2\pi$$

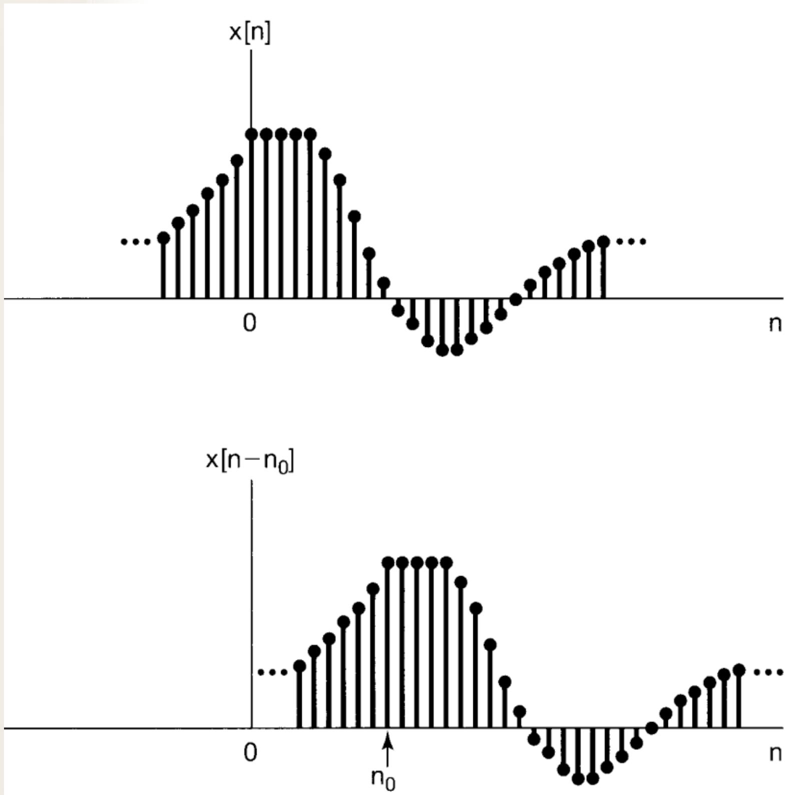
HARMONICKÝ DISKRÉTNÍ SIGNÁL

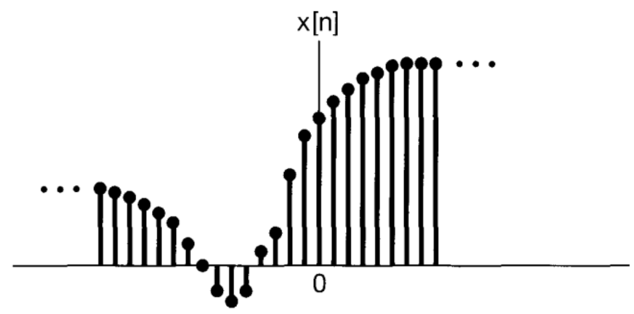


HARMONICKÝ DISKRÉTNÍ SIGNÁL

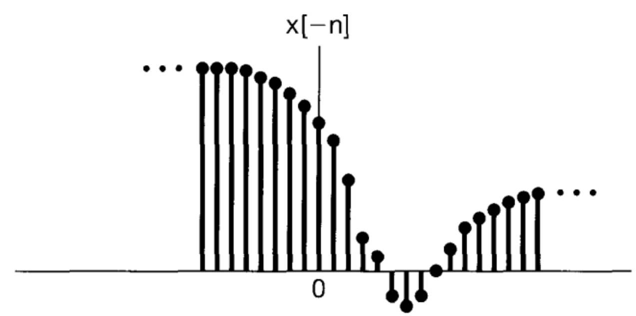


DISKRÉTNÍ SIGNÁLY





(a)



(b)

KONVOLUCE

- spojité signály

$$s_1(t) * s_2(t) = \int_{-\infty}^{t} s_1(\tau).s_2(t-\tau).d\tau \approx S_1(\omega).S_2(\omega)$$

- diskrétní signály

$$s_1(nT) * s_2(nT) = \sum_{i=0}^{n} s_1(iT).s_2(nT - iT) \approx S_1(z).S_2(z)$$

DISKRÉTNÍ KONVOLUCE