Enemy-Victim Models

"Populační ekologie živočichů"

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Predator-prey model

- continuous model of Lotka & Volterra (1925-1928)

H .. density of prey
r .. intrinsic rate of prey population
a .. predation rate

P .. density of predators
m .. predator mortality rate
b .. reproduction rate of predators

- in the absence of predator, prey grows exponentially $\rightarrow$ $$\frac{dH}{dt} = rH$$
- in the absence of prey, predator dies exponentially $\rightarrow$ $$\frac{dP}{dt} = -mP$$

- predation rate is linear function of the number of prey .. aHP
- each prey contributes identically to the growth of predator .. bHP

$$\frac{dH}{dt} = rH - aHP$$
$$\frac{dP}{dt} = bHP - mP$$

Analysis of the model

Zero isoclines:

- for prey population:

$$\frac{dH}{dt} = 0 \qquad 0 = rH - aHP \qquad \boxed{P = \frac{r}{a}}$$

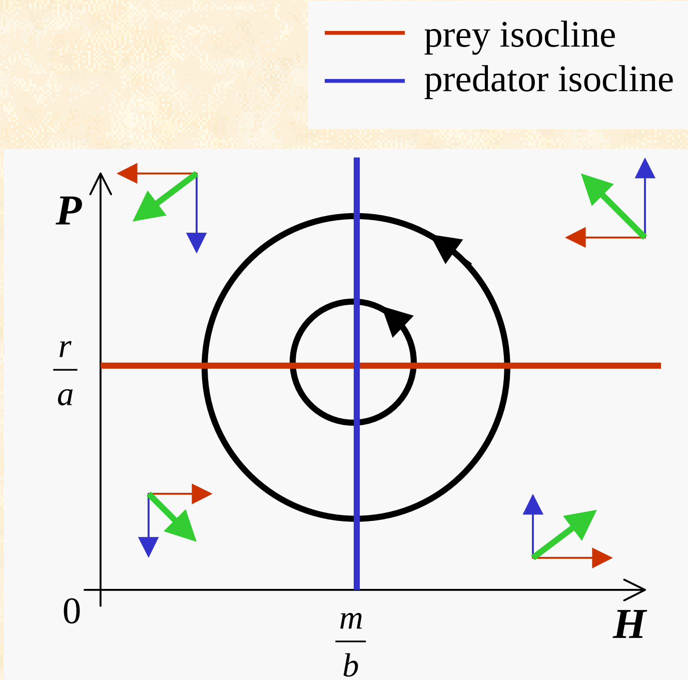
- for predator population:

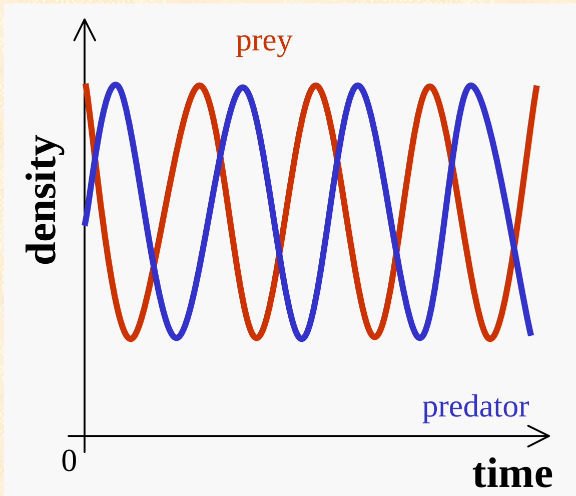
$$\frac{dP}{dt} = 0 \qquad 0 = bHP - mP \qquad \boxed{H = \frac{m}{b}}$$

- prey population would grow to infinity

$\rightarrow$ **neutral stability**

- do not converge, has no asymptotic stability (trajectories are closed lines)
- unstable system, amplitude of the cycles is determined by initial numbers
- POOR model





Incorporation of density-dependence

- in the absence of the predator prey population reaches carrying capacity *K*

$$\frac{dH}{dt} = rH\left(1 - \frac{H}{K}\right) - aHP$$

$$\frac{dP}{dt} = bHP - mP$$

- for given parameter values: $r = 3$, $m = 2$, $a = 0.1$, $b = 0.3$, $K = 10$

$$\frac{dH}{dt} = 3H\left(1 - \frac{H}{10}\right) - 0.1HP \qquad \frac{dP}{dt} = 0.3HP - 2P$$

Zero isoclines:

- for prey population: $\dfrac{dH}{dt} = 0$ $\quad 0 = 3H\left(1 - \dfrac{H}{10}\right) - 0.1HP$

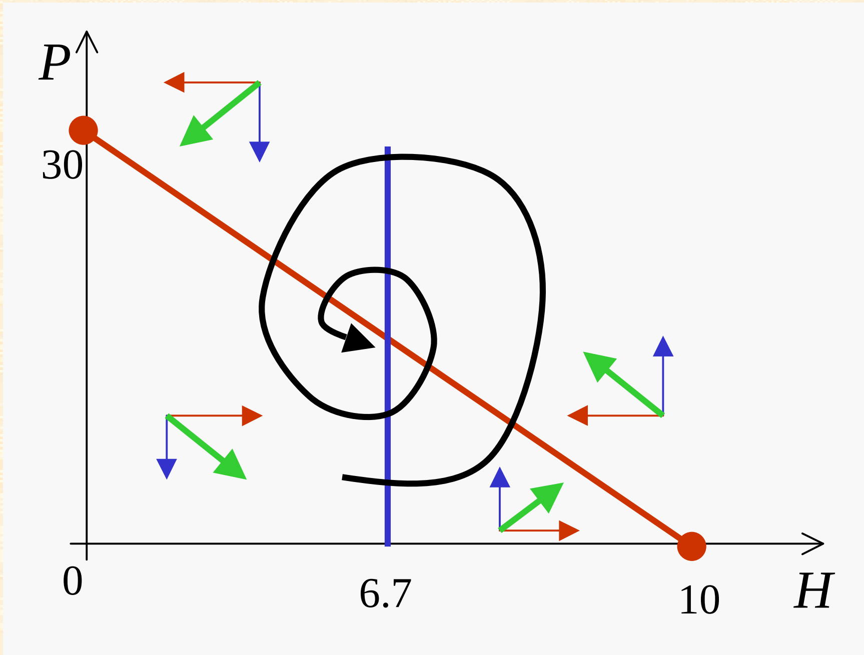
if H = 0 (trivial solution) or if $\quad 0 = 3\left(1 - \dfrac{H}{10}\right) - 0.1P$ $\quad \boxed{P = 30 - 3H}$

- for predator population: $\dfrac{dP}{dt} = 0$ $\quad 0.3HP - 2P = 0$

if P = 0 (trivial solution)
or if $0.3H - 2 = 0$

$\boxed{H = 6.667}$

- if gradient of prey isocline is negative .. approached stable equilibrium



Incorporation of functional response

- functional response Type II: $$H_a = \frac{aHT}{1+aHT_h}$$

- rate of consumption by all predators: $$\frac{H_a P}{T} = \frac{aHP}{1+aHT_h}$$

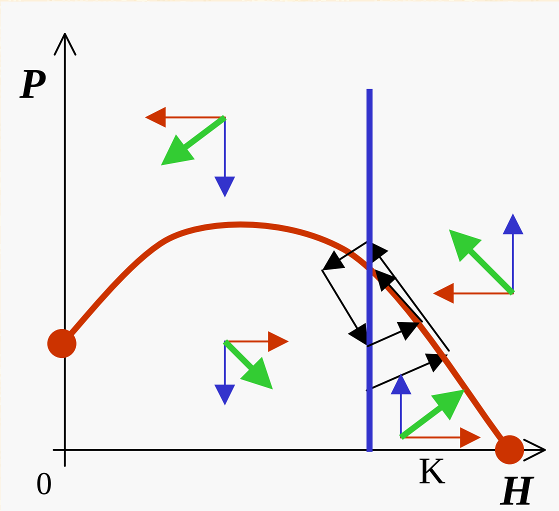
$$\frac{dH}{dt} = r_H H\left(1-\frac{H}{K}\right) - \frac{aHP}{1+aHT_h} \qquad \frac{dP}{dt} = bHP - mP$$

- for parameters: $r_H = 3$, $a = 0.1$, $T_h = 2$, $K = 10$

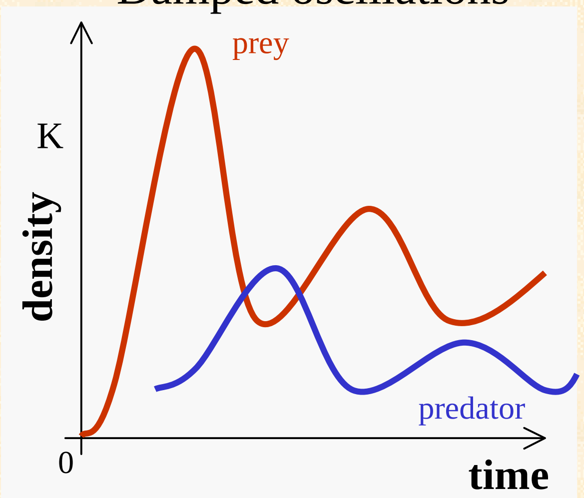
$$\frac{dH}{dt} = 0 \qquad 0 = 3H\left(1-\frac{H}{10}\right) - \frac{0.1HP}{1+0.1H2}$$

prey isocline: $$P = 30 + 6H - 0.6H^2$$

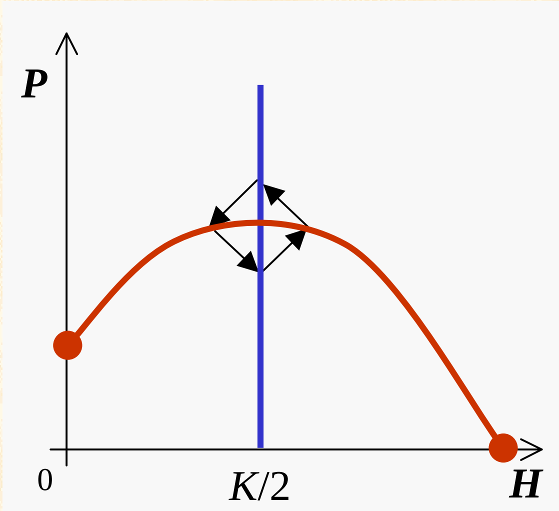
- predator exploits prey close to *K*
- isocline: *H* = 9



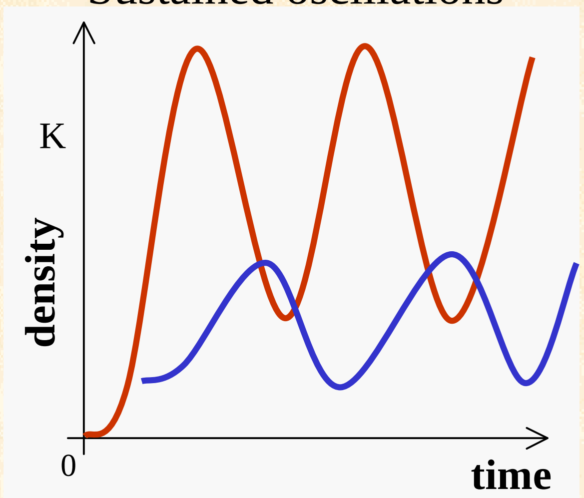
Damped oscillations



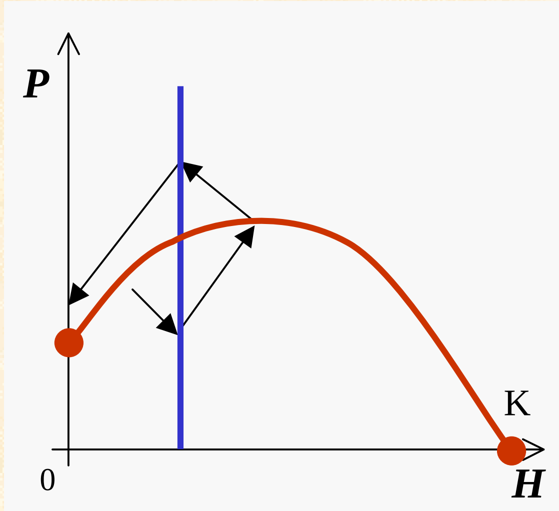
- predator exploits prey close to *K*/2
- isocline: *H* = 5



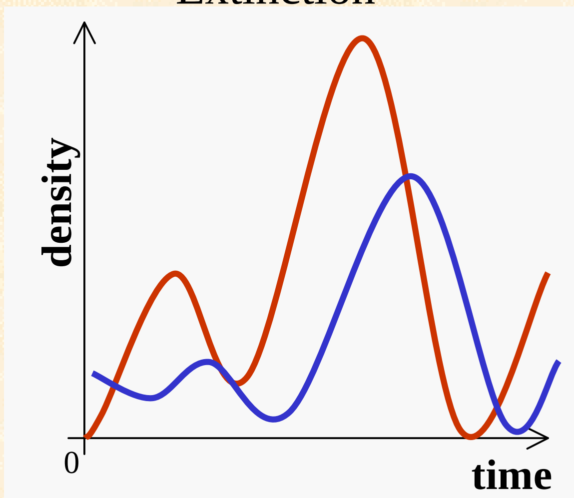
Sustained oscillations



- predator exploits prey at low density
- isocline: *H* = 2



Extinction



Rosenzweig & MacArthur (1963)

Incorporation of predator's carrying capacity

- logistic model with carrying capacity proportional to *H*
- *k* .. carrying capacity of the predator
- $r_P = bH - m$

$$\frac{dP}{dt} = bHP - mP$$

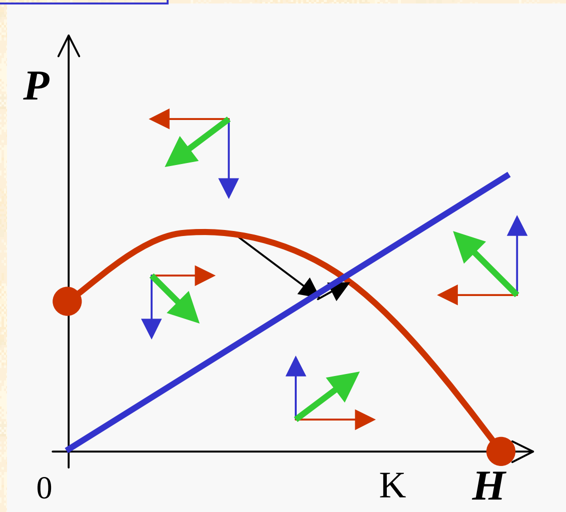
$$\frac{dP}{dt} = r_P P\left(1 - \frac{P}{kH}\right) \qquad \frac{dH}{dt} = r_H H\left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h}$$

- for parameters: $r_P = 2, k = 0.2$

$$\frac{dP}{dt} = 0 \qquad 0 = 2P\left(1 - \frac{P}{0.2H}\right)$$

predator isocline: $H = 5P$

prey isocline: $P = 30 + 6H - 0.6H^2$



Host-parasitoid model

- discrete model of Nicholson & Bailey (1935)

H_t = number of hosts in time t
H_a = number of attacked hosts
λ = finite rate of increase of the host

P_t = number of parasitoids
c = conversion rate, no. of parasitoids for 1 host (=1)

$$H_{t+1} = \lambda(H_t - H_a)$$

$$P_{t+1} = cH_a = H_a$$

Incorporation of random search

- parasitoid searches randomly, has unlimited ability to lay eggs
- encounters (x) are random (Poisson distribution)

$$p_x = \frac{\mu^x e^{-\mu}}{x!} \qquad x = 0, 1, 2, ... \qquad p_0 = e^{-\mu}$$

p_0 = proportion of not encountered, μ .. mean number of encounters

E_t = total number of encounters
a = searching efficiency (proportion of hosts encountered)

$$E_t = a\, H_t\, P_t \qquad \mu = \frac{E_t}{H_t} = aP_t \qquad p_0 = e^{-aP_t}$$

- proportion of encounters (1 or more times): $p = (1 - p_0)$

$$p = (1 - e^{-aP_t})$$

$$H_a = H_t\left(1 - e^{-aP_t}\right)$$

$$H_{t+1} = \lambda(H_t - H_a)$$

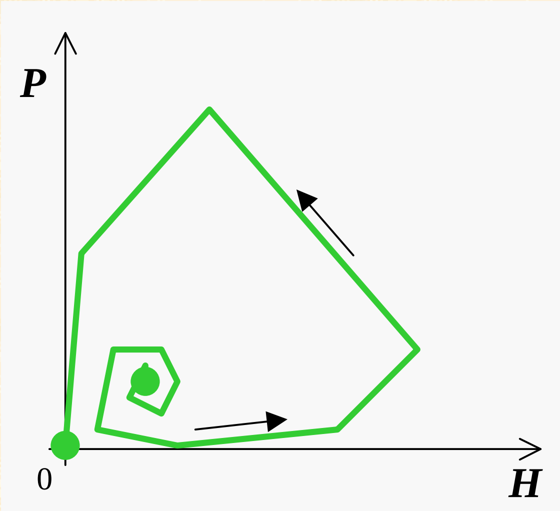
$$P_{t+1} = H_a$$

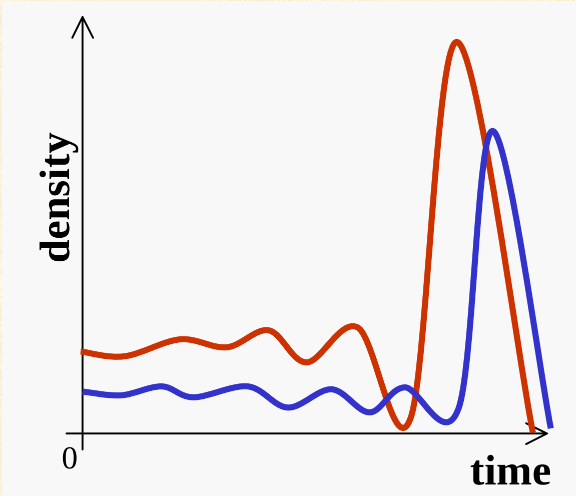
$$H_{t+1} = \lambda H_t e^{-aP_t}$$

$$P_{t+1} = H_t\left(1 - e^{-aP_t}\right)$$

- highly unstable model for all parameter values:

- equilibrium is possible but the slightest disturbance leads to divergent oscillations (extinction of parasitoid)





Incorporation of density-dependence

- exponential growth of hosts is replaced by logistic equation

$$H_{t+1} = \lambda H_t e^{\left(1-\frac{H_t}{K}\right)-aP_t}$$

$$P_{t+1} = H_t\left(1-e^{-aP_t}\right)$$

$$q = \frac{H^*}{K}$$

*H**.. new host carrying capacity

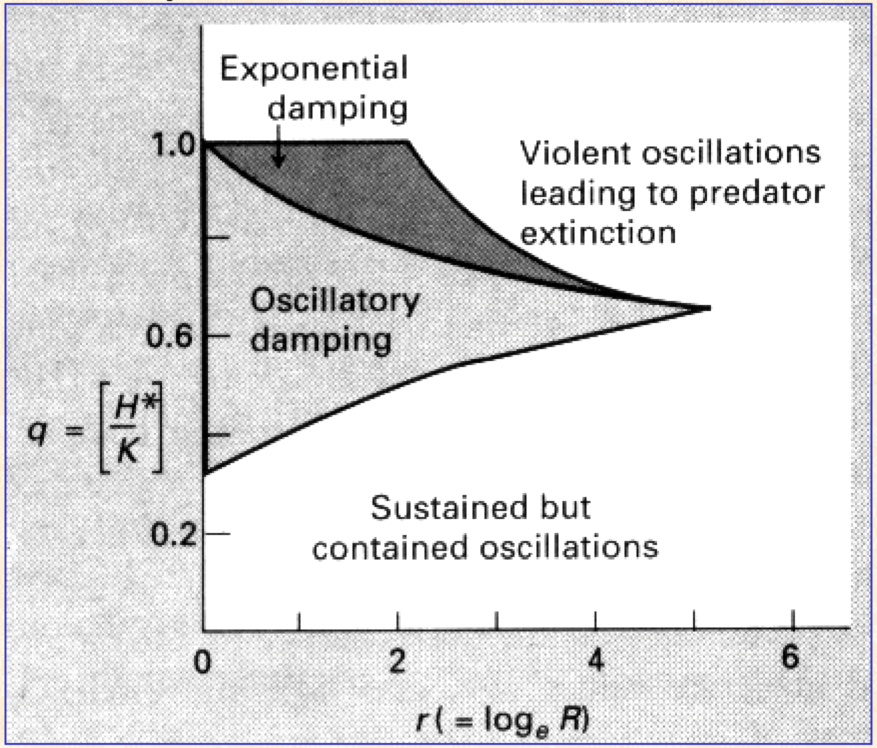
- depends on parasitoids' efficiency

- when a is low then $q \rightarrow 1$

- when a is high then $q \rightarrow 0$

- density-dependence have stabilising effect for moderate r and q

Stability boundaries



Beddington et al. (1975)

Incorporation of the refuge

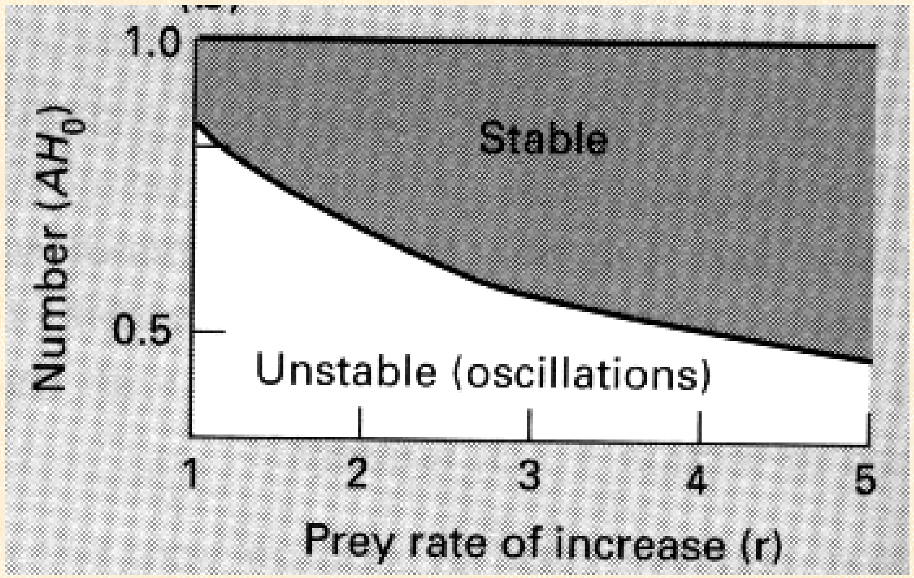
- if hosts are distributed non-randomly in the space

<u>Fixed number refuge</u>: H_0 hosts are always protected

$$H_{t+1} = \lambda H_0 + \lambda (H_t - H_0) e^{-aP_t}$$

$$P_{t+1} = (H_t - H_0)\left(1 - e^{-aP_t}\right)$$

- have strong stabilising effect even for large r



Hassell & May (1973)

Incorporation of aggregated distribution

- distribution of encounters is not random but aggregated (negative binomial distribution)

- proportion of hosts not encountered (p_0):

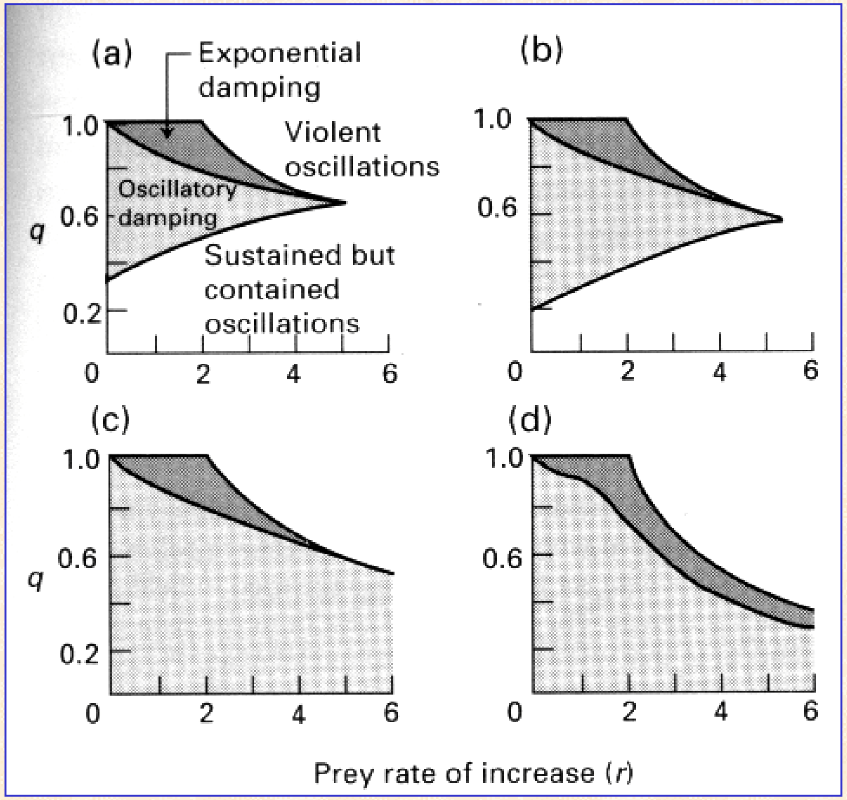
$$p_0 = \left(1 + \frac{aP_t}{k}\right)^{-k}$$

where k = degree of aggregation

$$H_{t+1} = \lambda H_t e^{\left(1-\frac{H_t}{K}\right)\left(1+\frac{aP_t}{k}\right)^{-k}}$$

$$P_{t+1} = H_t\left(1-\left(1+\frac{aP_t}{k}\right)^{-k}\right)$$

- very stable model system if $k \le 1$



Stability boundaries:
a) ***k***=∝, b) ***k***=2, c) ***k***=1, d) ***k***=0

Hassell (1978)

Example 16

You want to control population of mites. Before introduction of predatory mites you want to simulate the predator-prey dynamic using the following model:

$$\frac{dH}{dt} = r_H H\left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h} \qquad \frac{dP}{dt} = \frac{aHP}{1 + aHT_h} - dP$$

To estimate parameters you need to run the following experiments:

1. Keep prey population without predators, record densities over a month and estimate intrinsic rate of increase (r_H) and carrying capacity (K). You found that $r_H = 0.2$ and $K = 500$.
2. Keep predators at constant density of prey, record predator densities over a month and estimate natural predators' mortality (d). You found that $d = 0.1$.
3. Offer one predator different prey densities and estimate the functional response. You find that $a = 0.001$ and $T_h = 0.5$.
4. How long it takes for the predatory mite to control mite pests if pests has a density of 200 individuals and the predators is only 1?

```
predprey<-function(t,y,pa){
H<-y[1]
P<-y[2]
with(as.list(pa),{
dH.dt<-rH*H*(1-H/K)-a*H*P/(1+a*H*Th)
dP.dt<- a*H*P/(1+a*H*Th)-d*P
return(list(c(dH.dt,dP.dt)))}))}

H<-200; P<-1
time<-seq(0,500,0.1)
pa<-c(rH=0.2,K=500,a=0.001,Th=0.5,d=0.1)
library(deSolve)
out<-data.frame(ode(c(H,P),time,predprey,pa))
matplot(time,out[,-1],type="l",lty=1:2,col=1)
legend("right",c("H","P"),lty=1:2)
```

Example 17

Aphids has increased their population density to 50 individuals/plant. You have observed that their $\lambda = 3$, $K = 800$, $T_h = 0.3$ You need to control aphids using a parasitoid. You can choose from three parasitoid species (A, B, C). The three species differ in the number of hosts they infect (*c*) and in their search efficiency (*a*):

	A	B	C
c	1	2	5
a	0.3	0.07	0.001

Use the discrete Nicholson-Bailey host-parasitoid model with functional response of the type II within POPULUS. Introduce a single parasitoid and find which of the three species will achieve the quickest control.