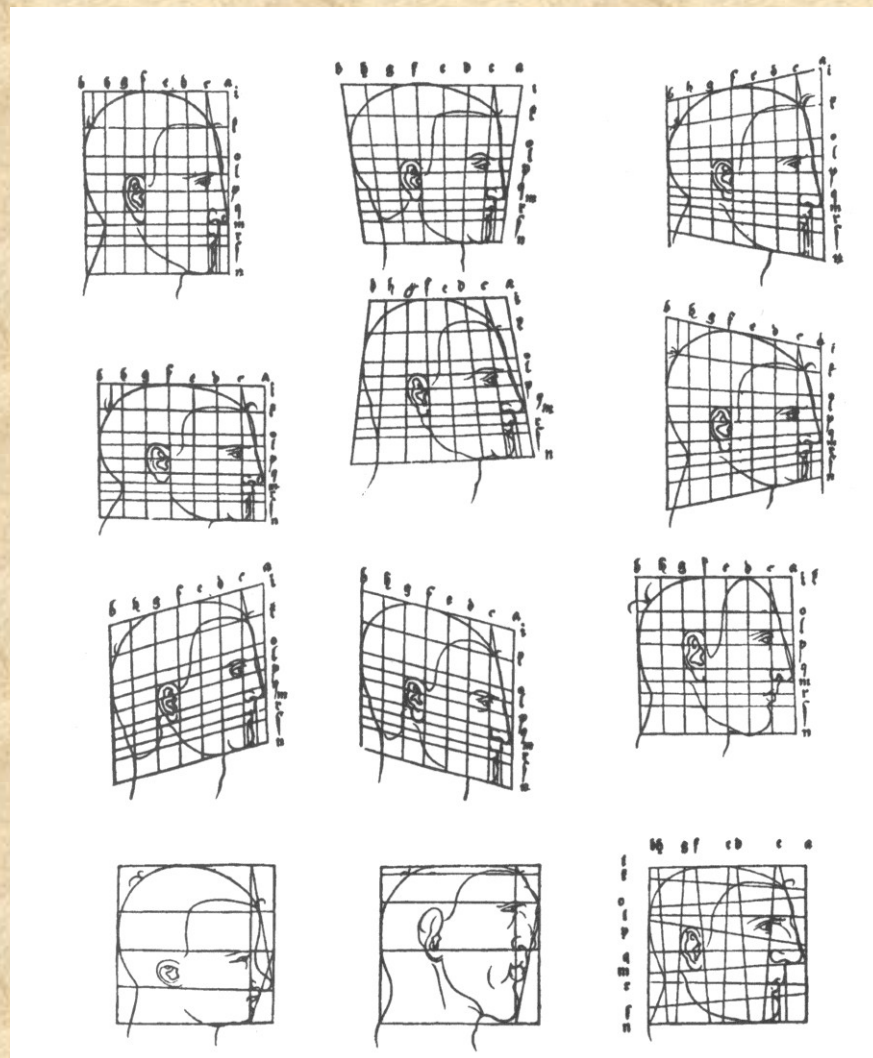


MORFOMETRIE NA PRŮSEČÍKU BIOLOGIE A GEOMETRIE

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Brno
macholan@iach.cz**

Snaha zachytit tvarové změny a rozdíly mezi objekty



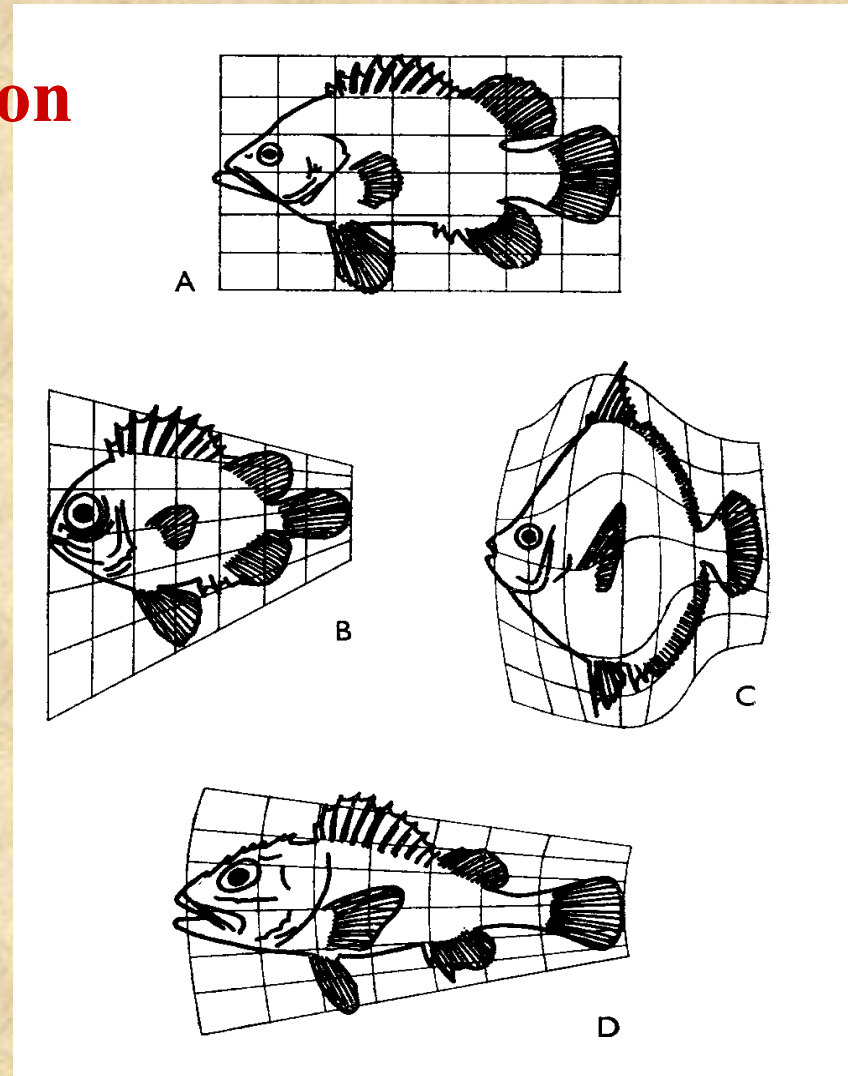
A. Dürer (1524): Vier Bücher von Menlicher Proportion.

V historii zkoumání tvaru biologických objektů 2 odlišné strategie:

1) Sir W. D'Arcy Thompson

Absence kvantifikace
tvarových změn!

W. A. Thompson (1917):
On Growth and Form.



V historii zkoumání tvaru biologických objektů 2 odlišné strategie:

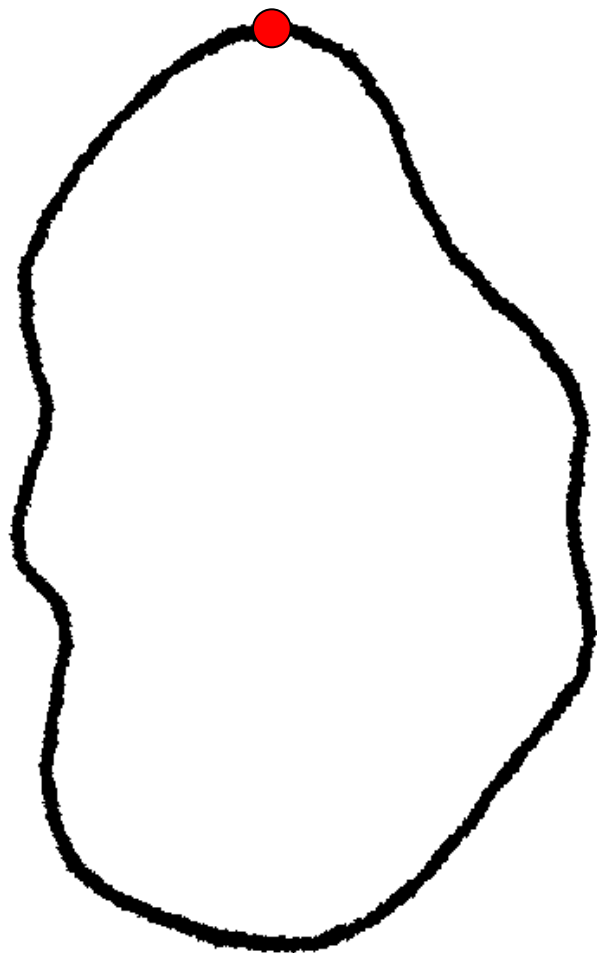
2) Tradiční morfometrie:

- F. Galton, K. Pearson, R. A. Fisher, S. Wright, H. Hotelling, ...**
- rozměry, váhy, úhly, plochy, ...**
- PCA, FA, DFA, CVA, PCOA, shluková a., ...**

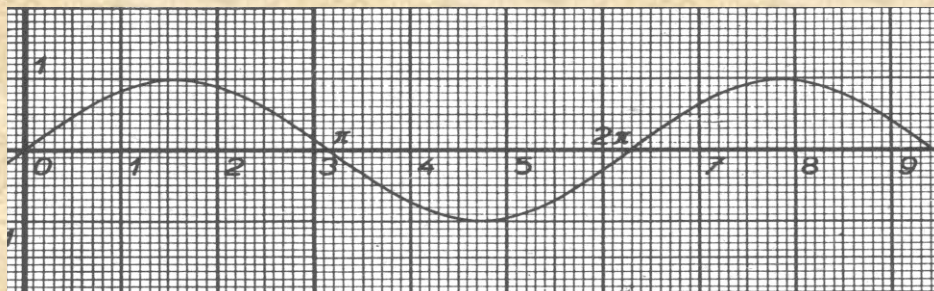
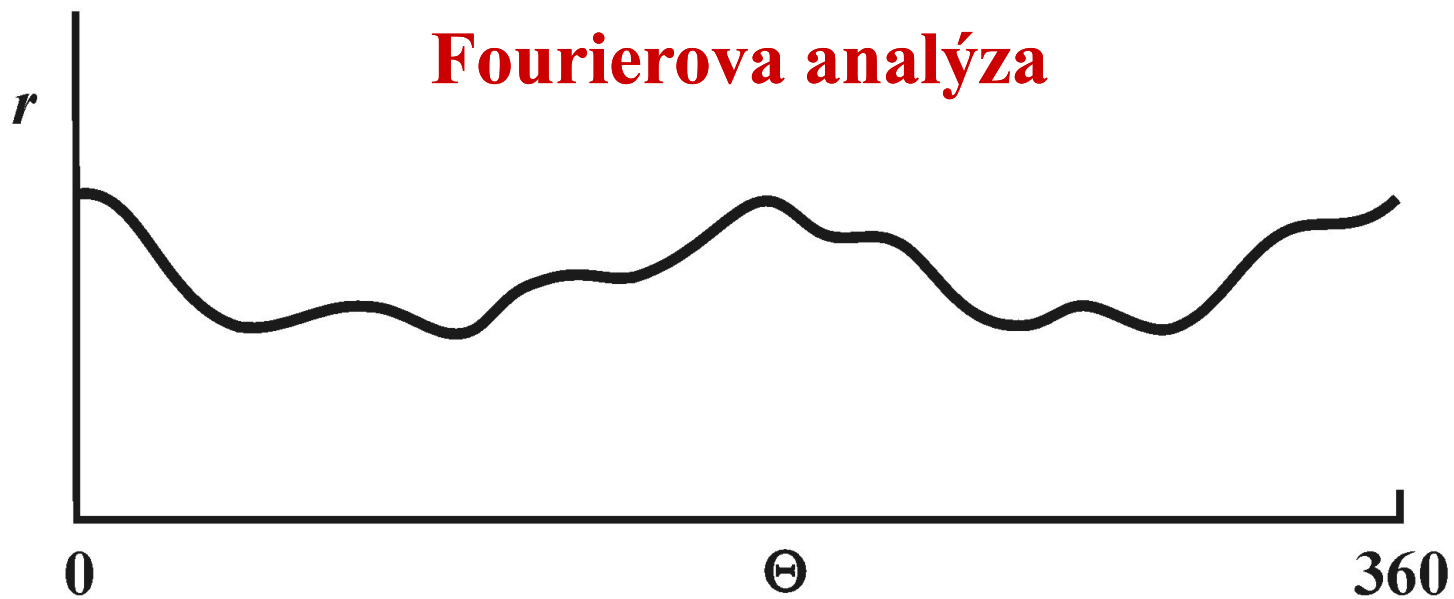
1983-1989

**C. R. Goodall, K. V. Mardia, I. Dryden, D. G. Kendall,
F. L. Bookstein:
„morfometrická syntéza“, „nová morfometrie“**

- **význačné body (landmarks)**
- **kvantifikace tvaru pomocí tvarových koordinát**
- **odlišení různých tvarových složek**
- **informace o tvaru zachována po celou dobu matematické analýzy**
- **velikostní standardizace a možnost samostatné práce s vektorem velikosti**
- **možnost zpracování dat pomocí tradičního morfometrického aparátu**

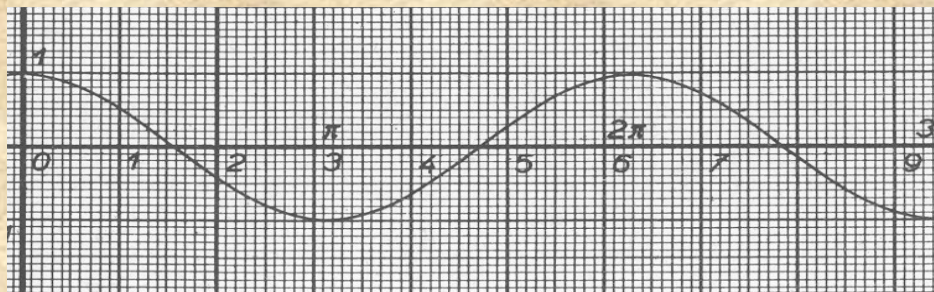


Fourierova analýza

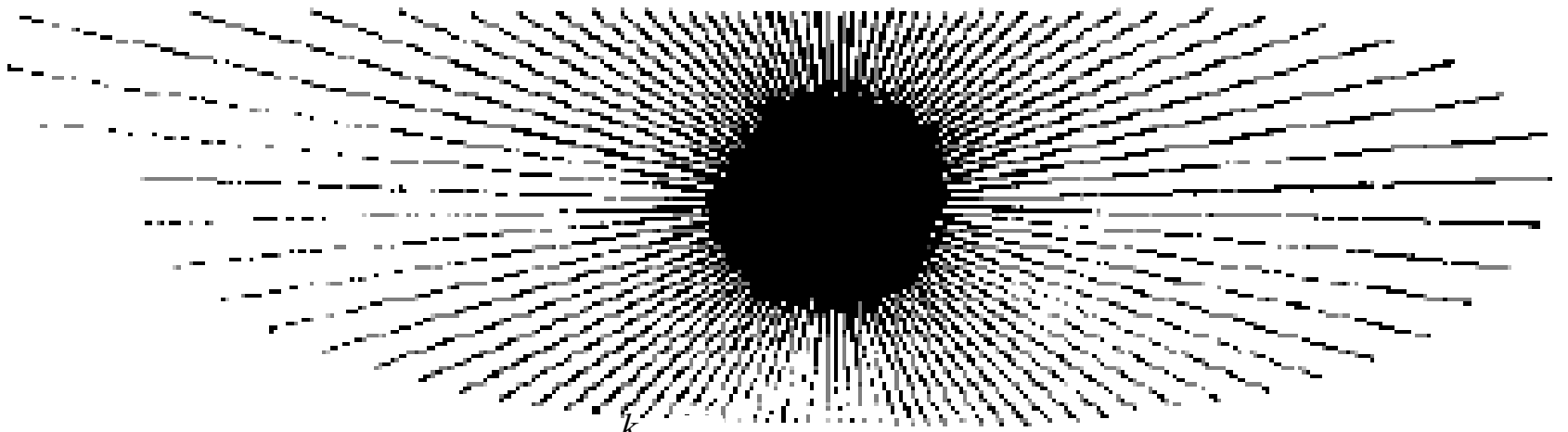
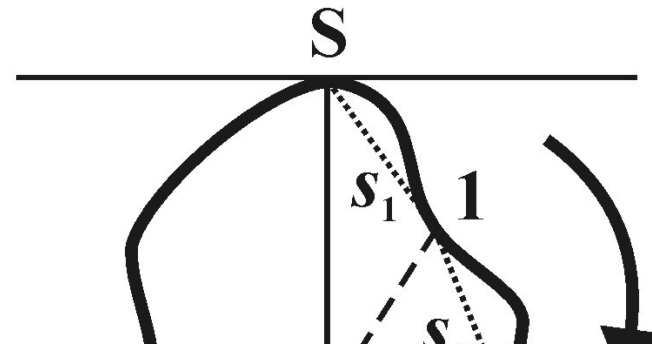
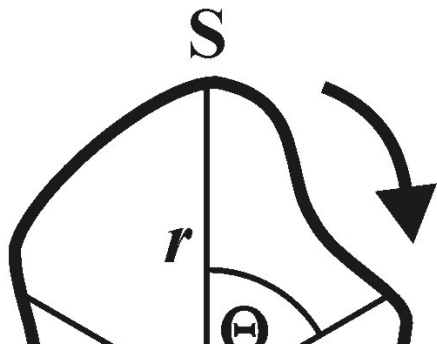


sin

**harmonické složky
(harmonics), koeficienty**



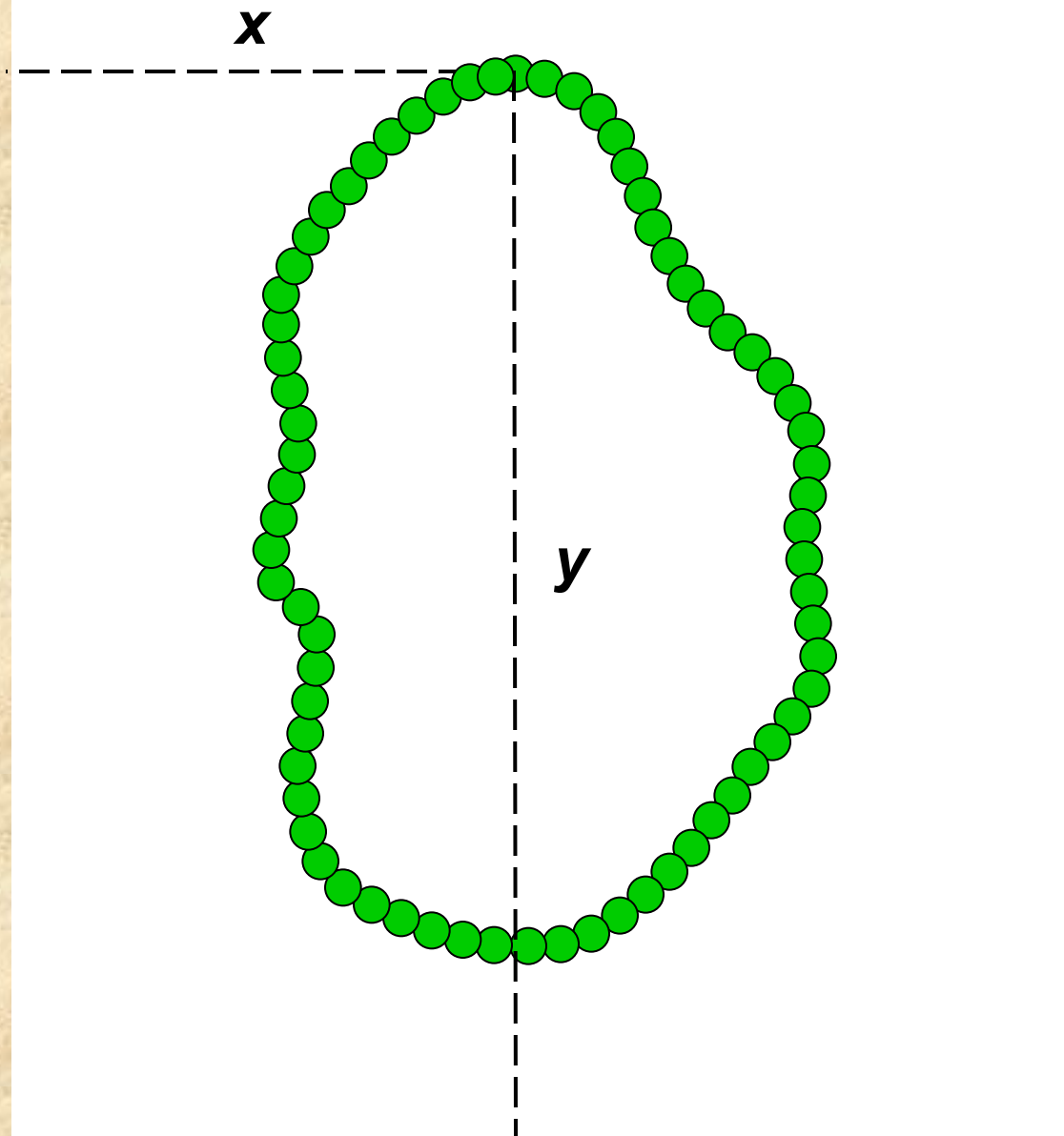
cos

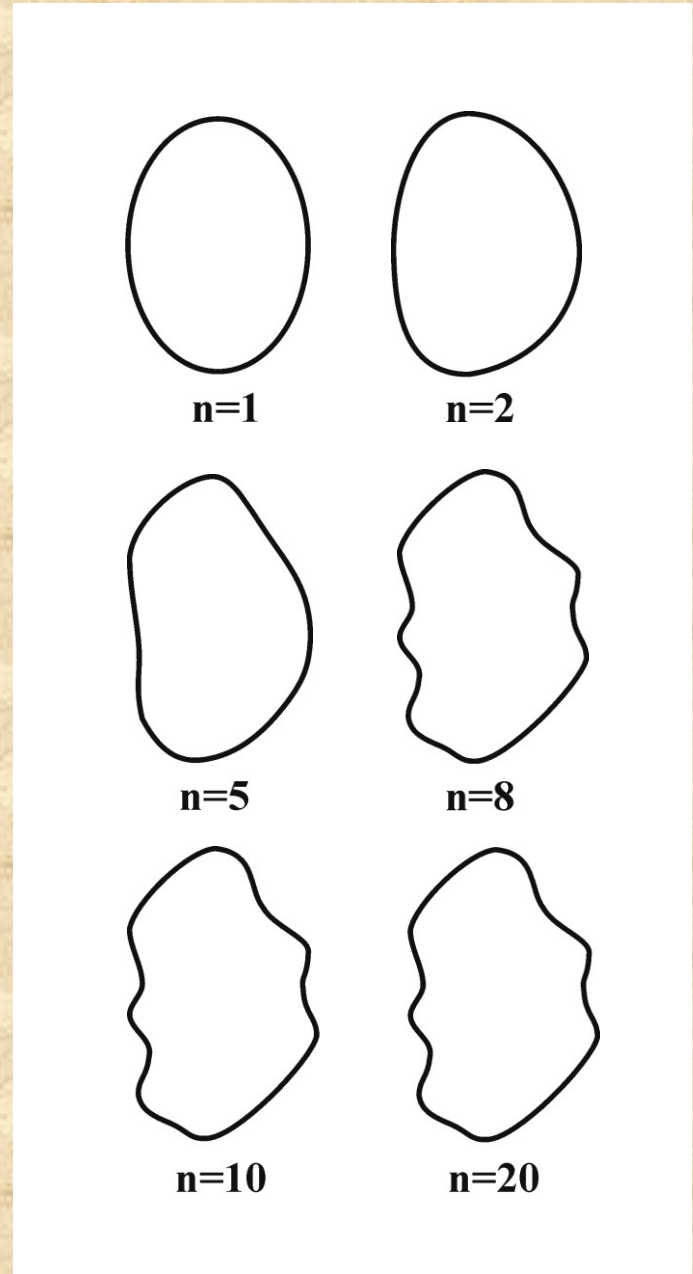
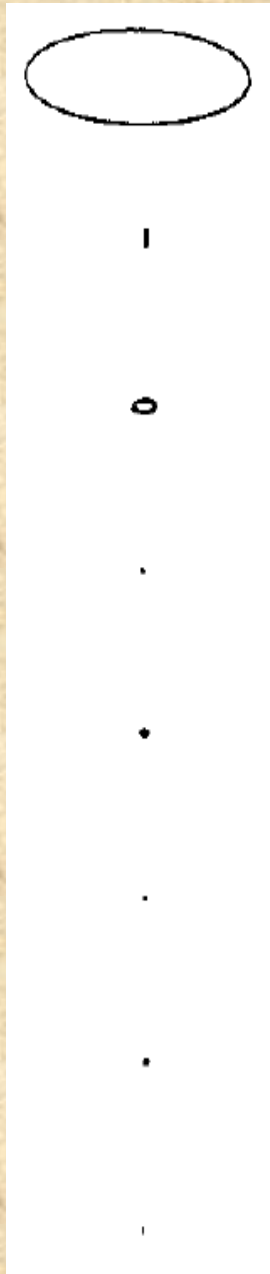
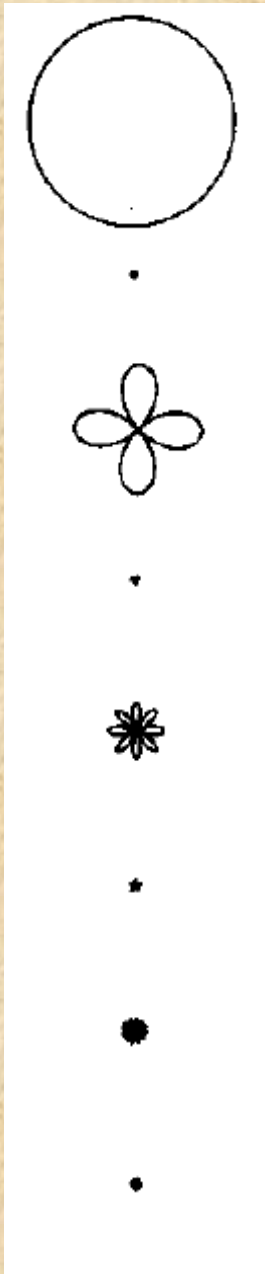


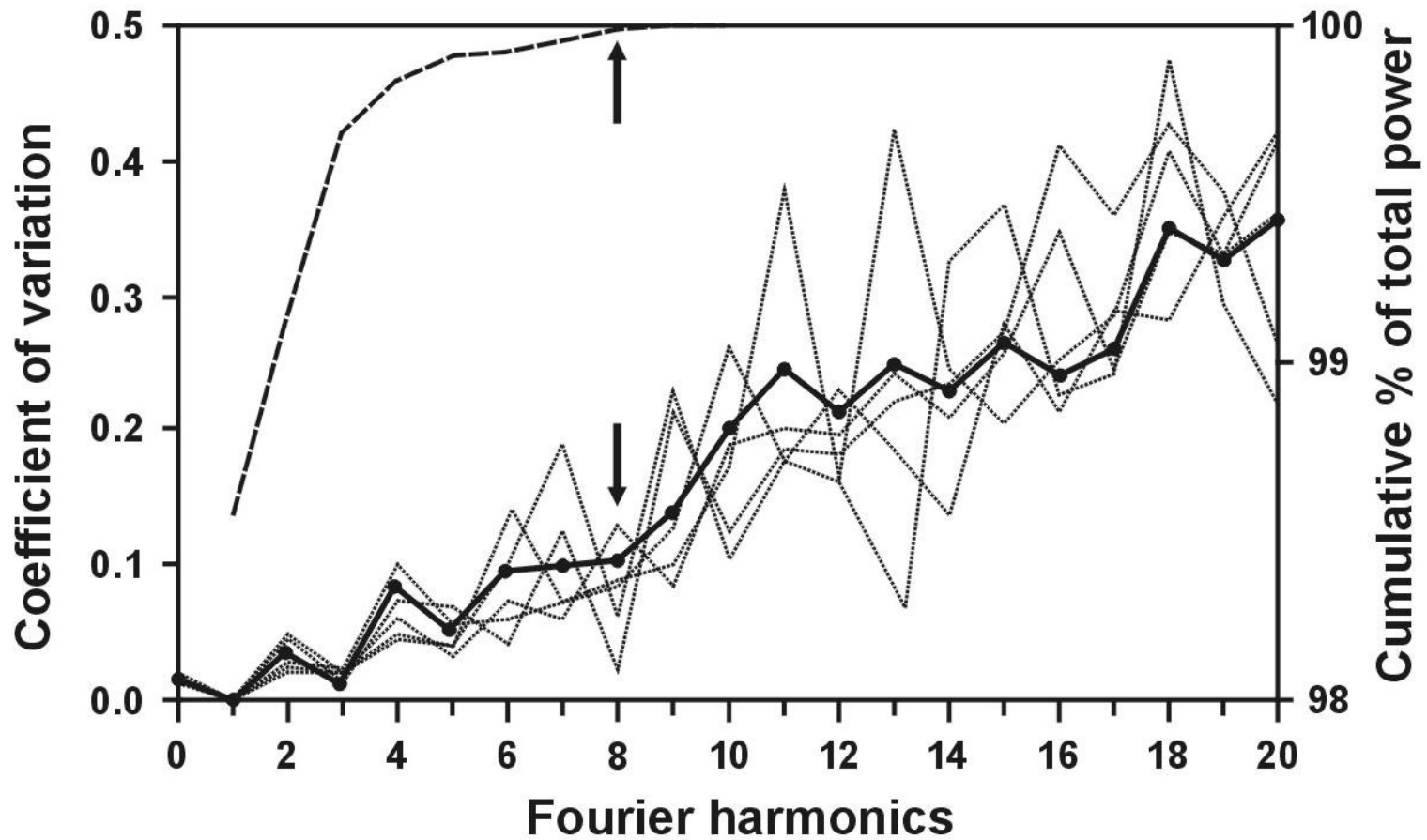
$$r_{\theta} = a_0 + \sum_{i=1}^k a_i \cos_i \theta + b_i \sin_i \theta$$

$$a_0 = \sqrt{\frac{1}{p} \sum_{i=1}^p \theta_i} \quad a_i = \sqrt{\frac{2}{p} \sum_{i=1}^p \theta_i \cos_i \theta_i} \quad b_i = \sqrt{\frac{2}{p} \sum_{i=1}^p \theta_i \sin_i \theta_i}$$

Elíptická Fourierova analýza



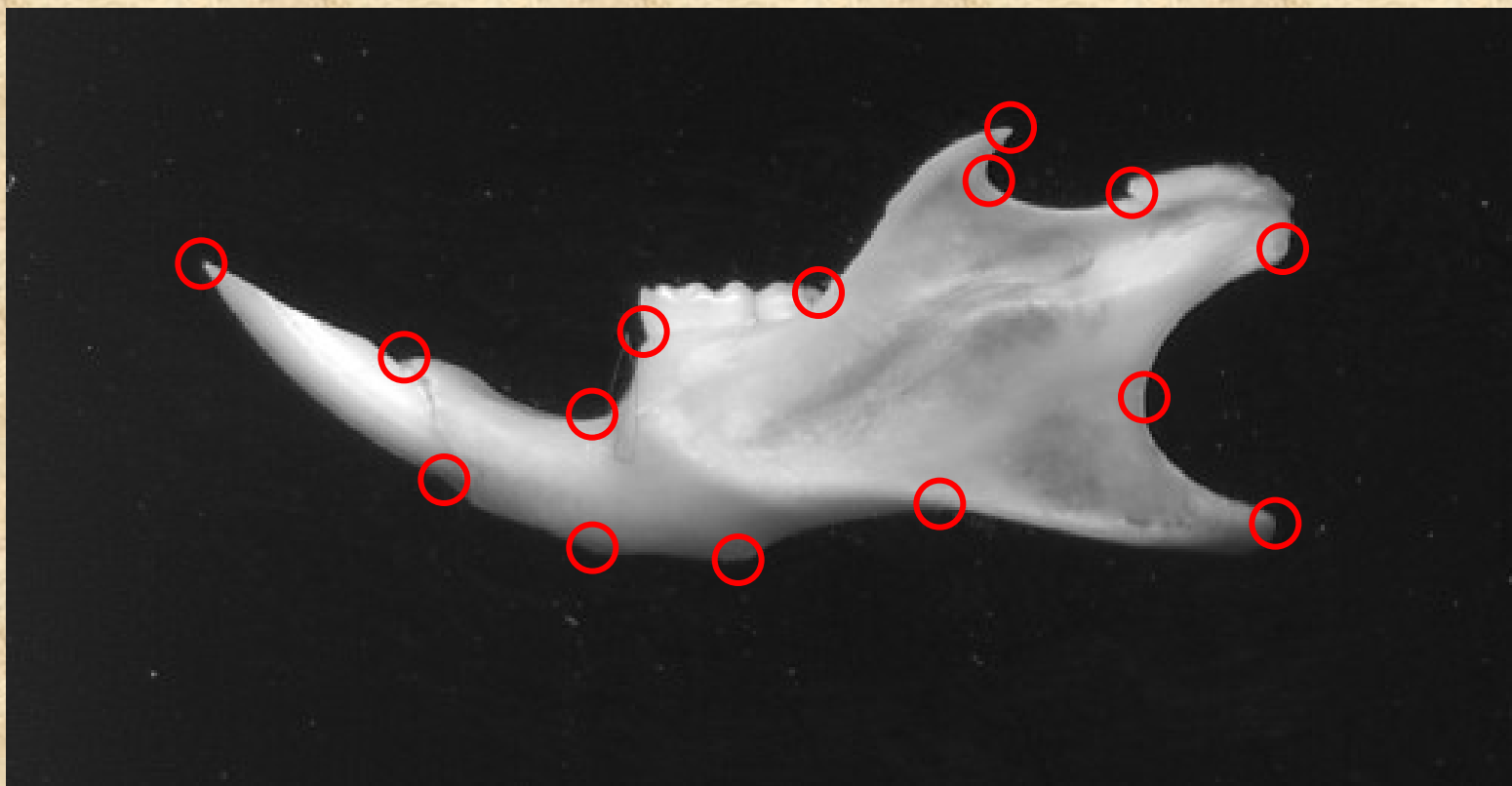




Geometrická morfometrie II.

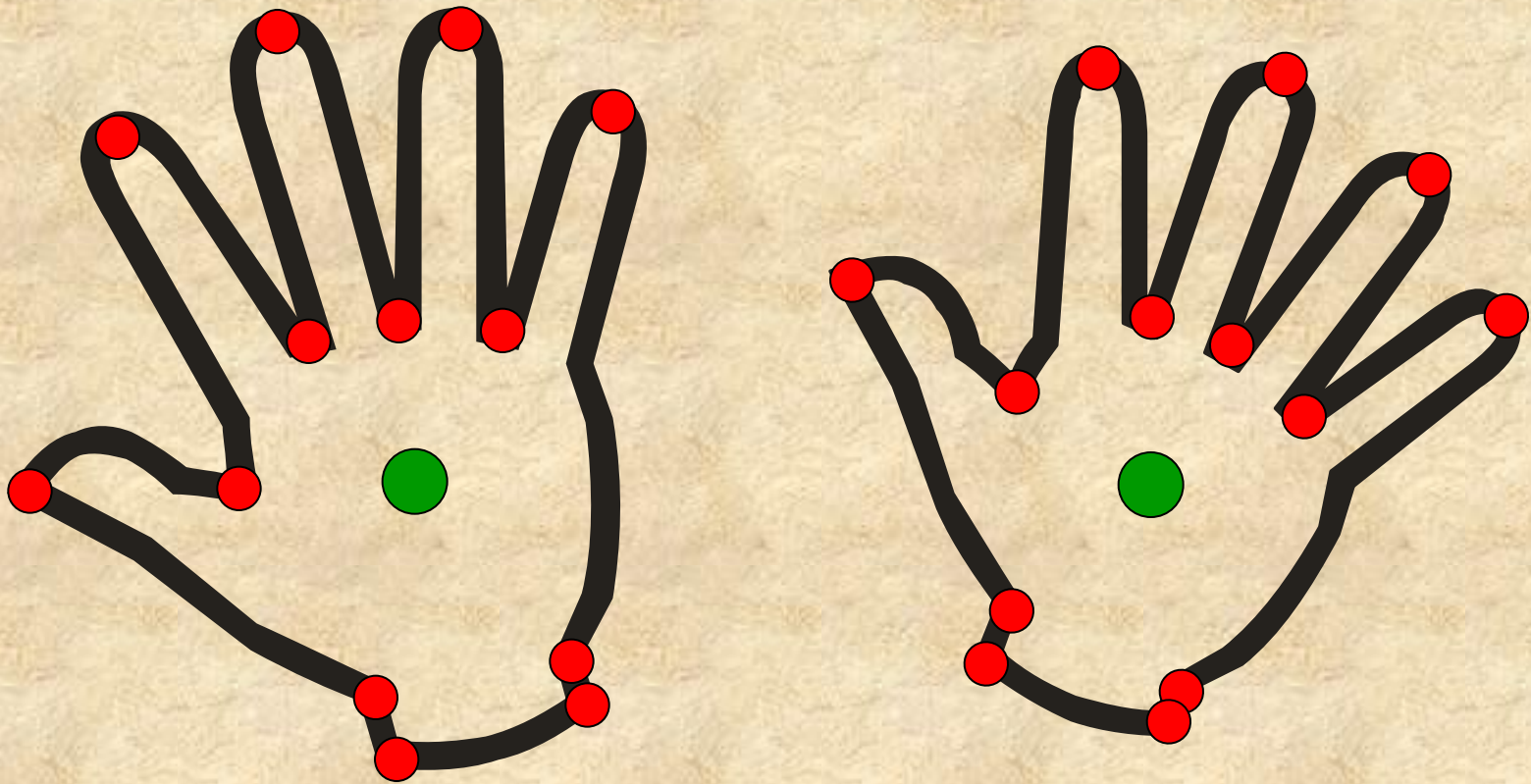
Význačné body, tvarové koordináty

význačné body (landmarks) = body, které lze přesně lokalizovat a které jsou alespoň v geometrickém smyslu mezi objekty homologické



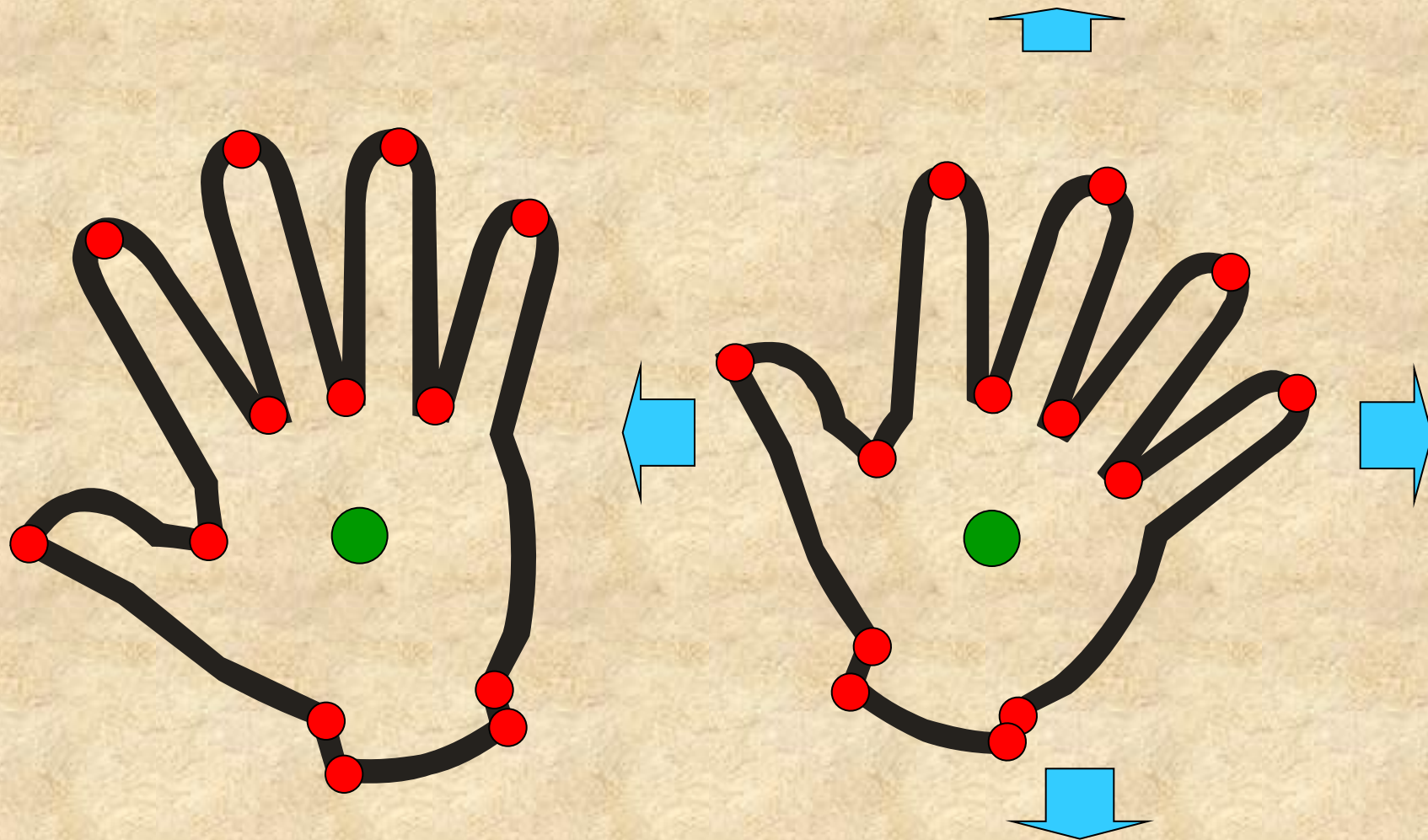
Prokrustovská superpozice = GLS (Generalized Least Squares)

tvar = vše kromě informace o velikosti, pozici a orientaci objektu



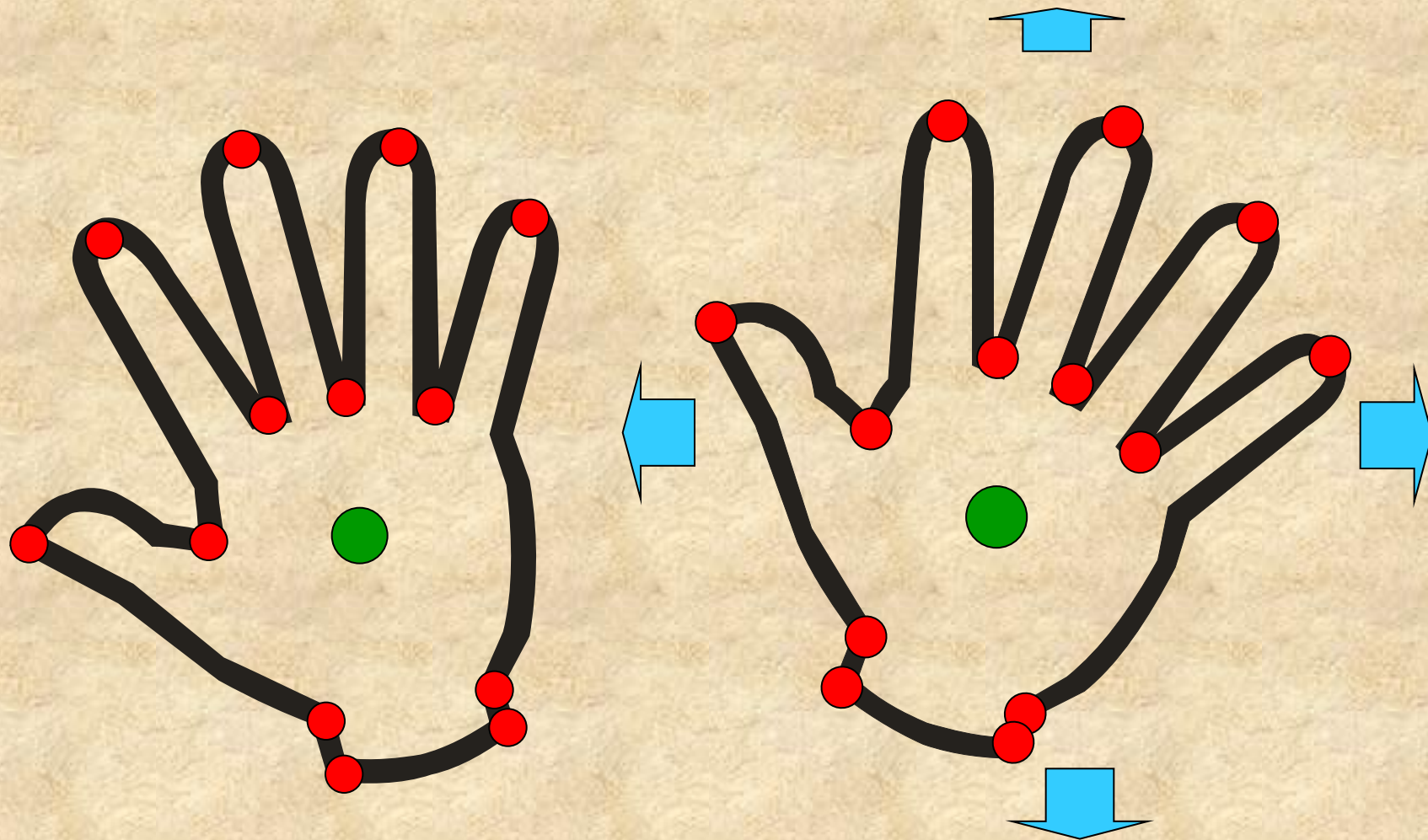
OBRAZOVÝ PROSTOR: p – rozměrů, k – bodů; $n = pk$

Prokrustes – krok 1



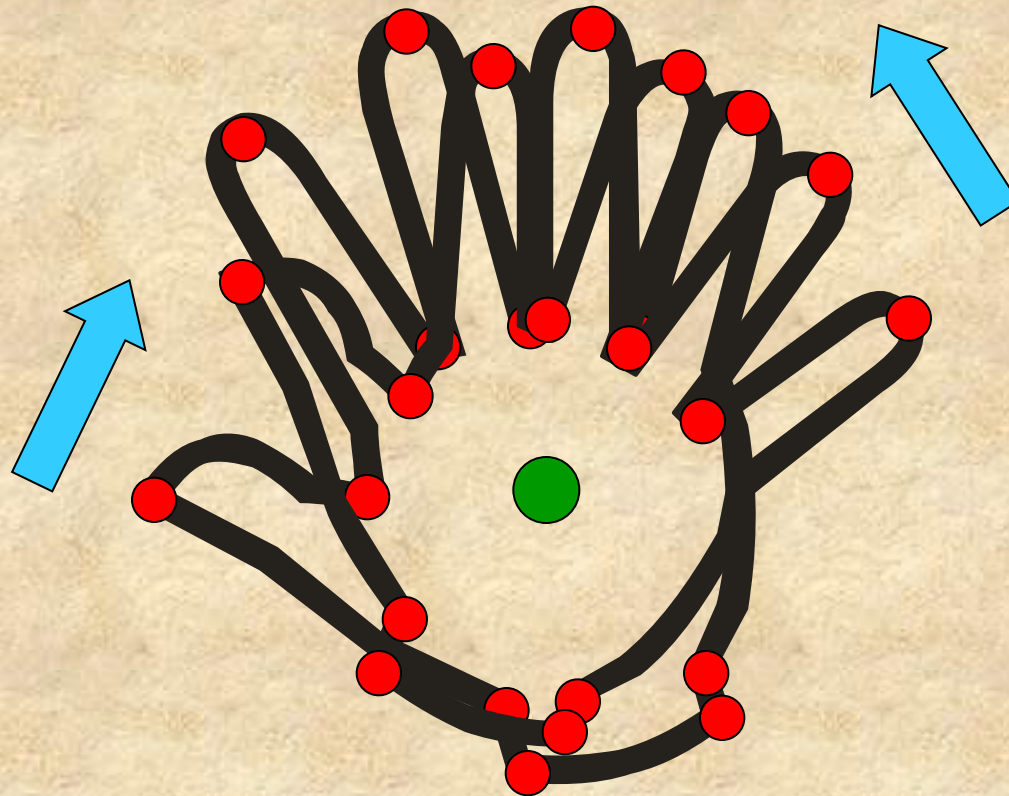
OBRAZOVÝ PROSTOR: p – rozměrů, k – bodů; $n = pk$

Prokrustes – krok 1

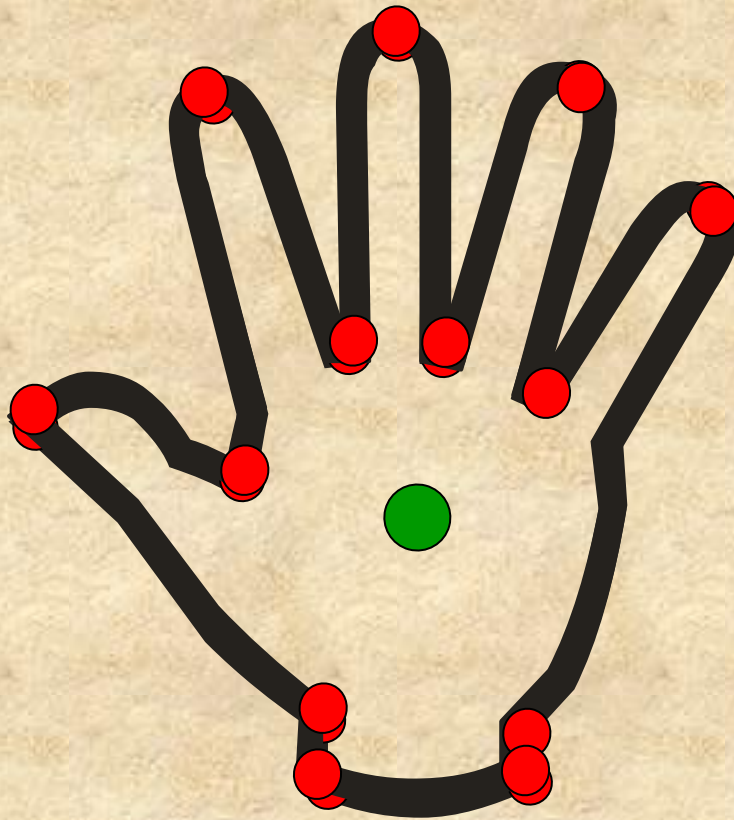


OBRAZOVÝ PROSTOR: p – rozměrů, k – bodů; $n = pk$

Prokrustes – krok 2



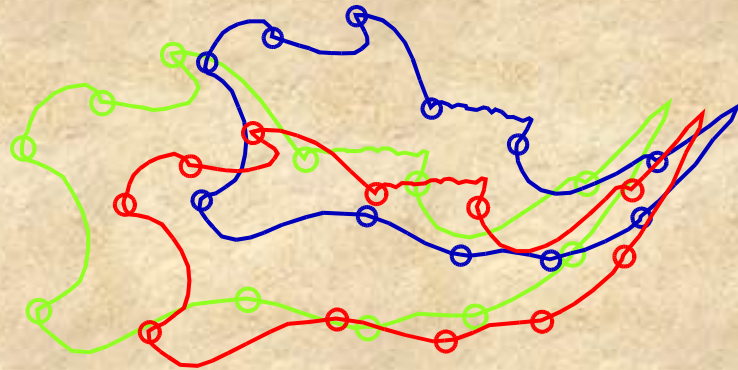
Prokrustes – krok 3



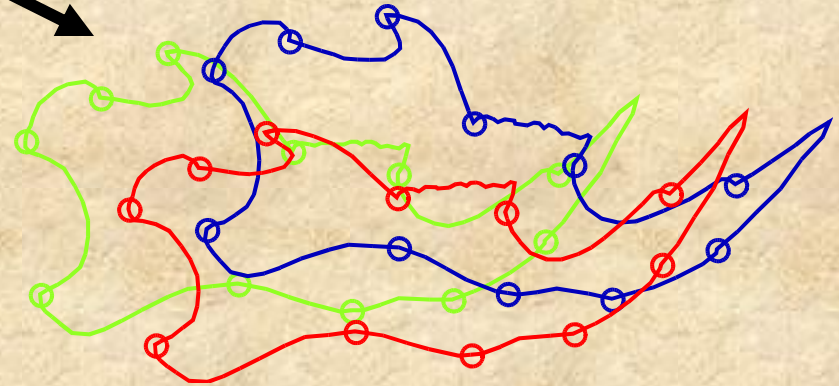
TVAROVÝ PROSTOR: $n = pk - k - k(k-1)/2 - 1$

Extracting shape information: Procrustes superposition

Original landmark configurations



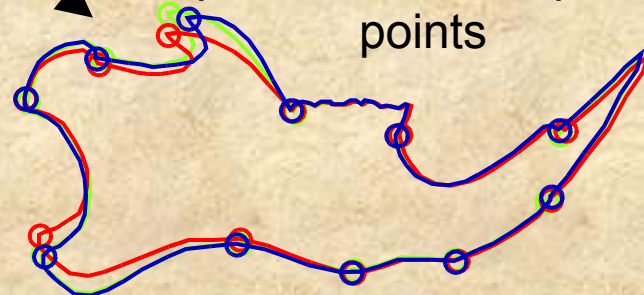
1. Change scale so that all configurations have the same size

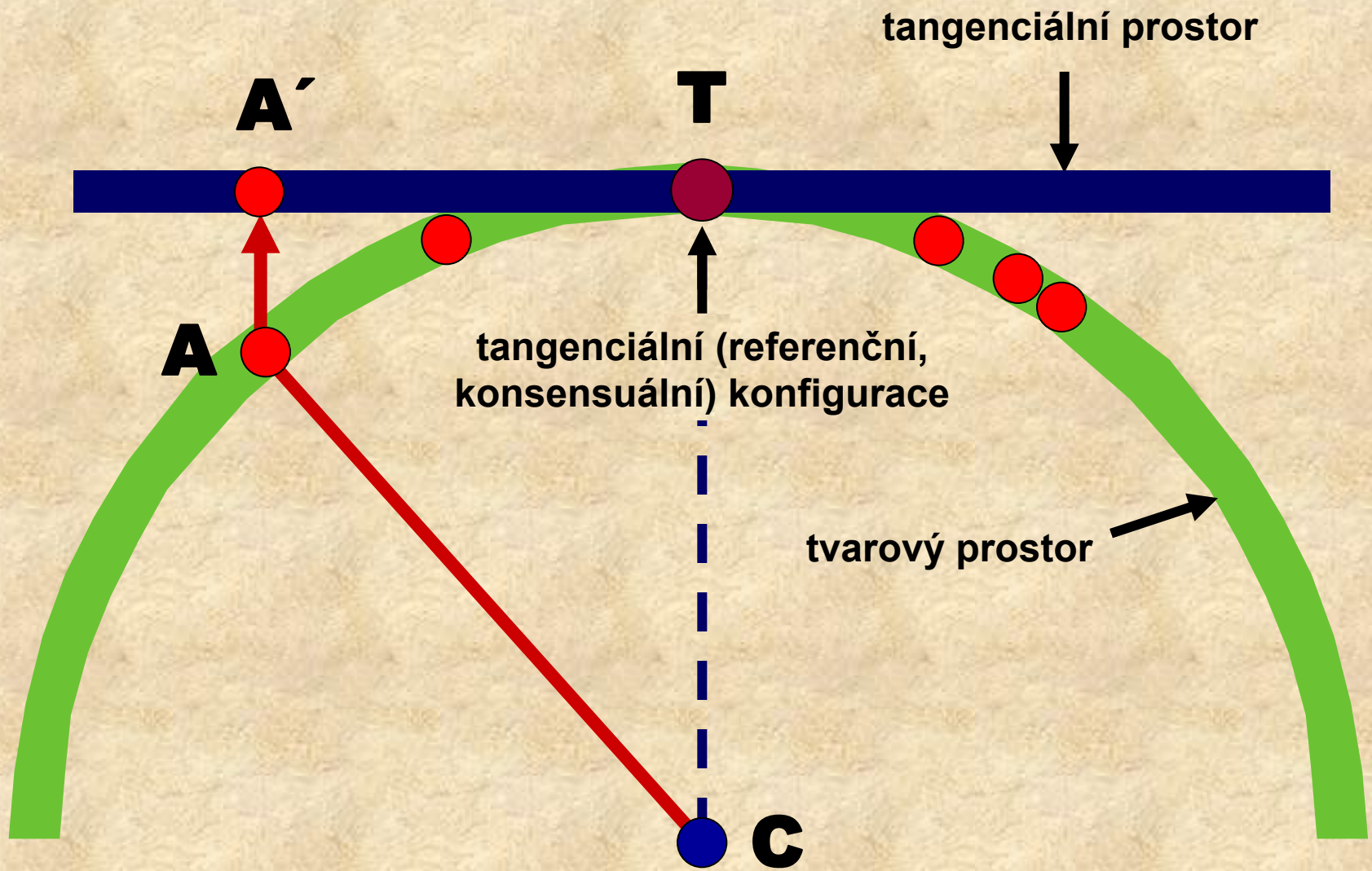


2. Superposition of the centers of gravity on a single point

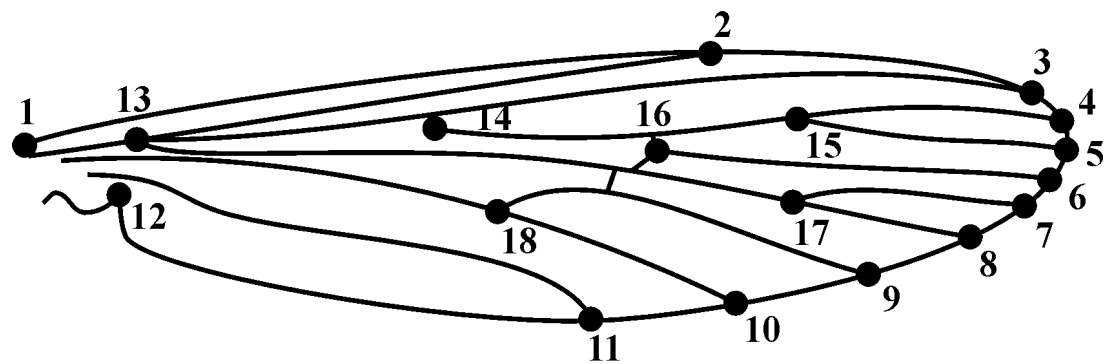


3. Rotation to minimize the dispersion of corresponding points

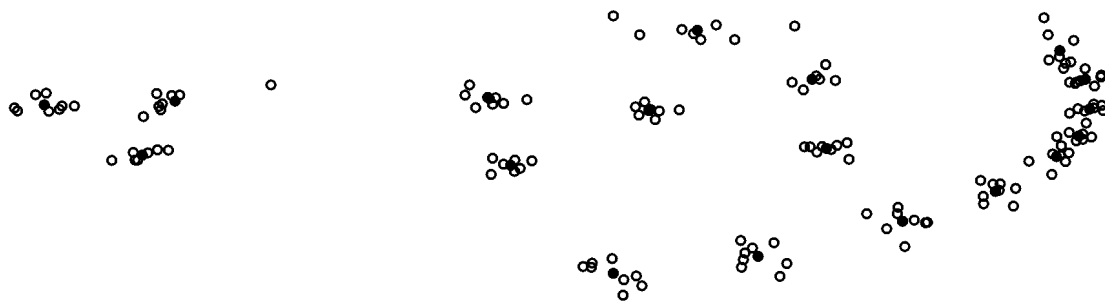




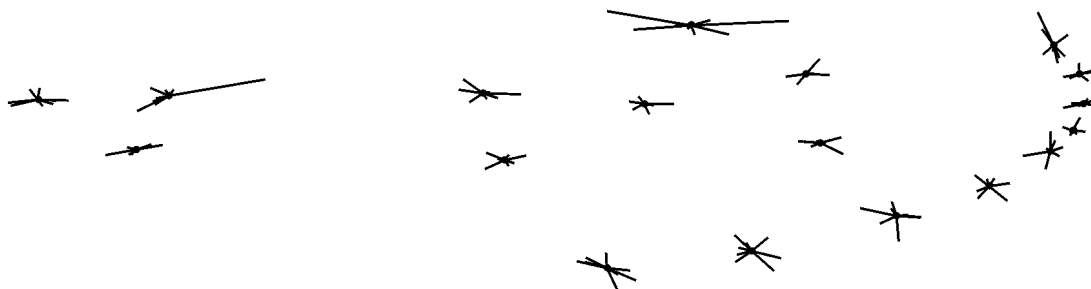
a)



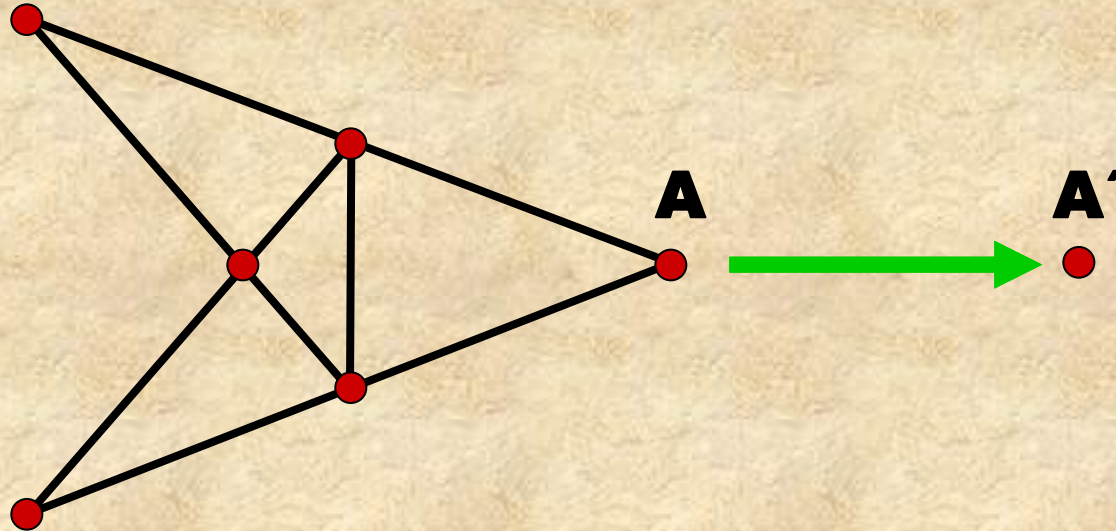
b)



c)



Problém nejmenších čtverců:

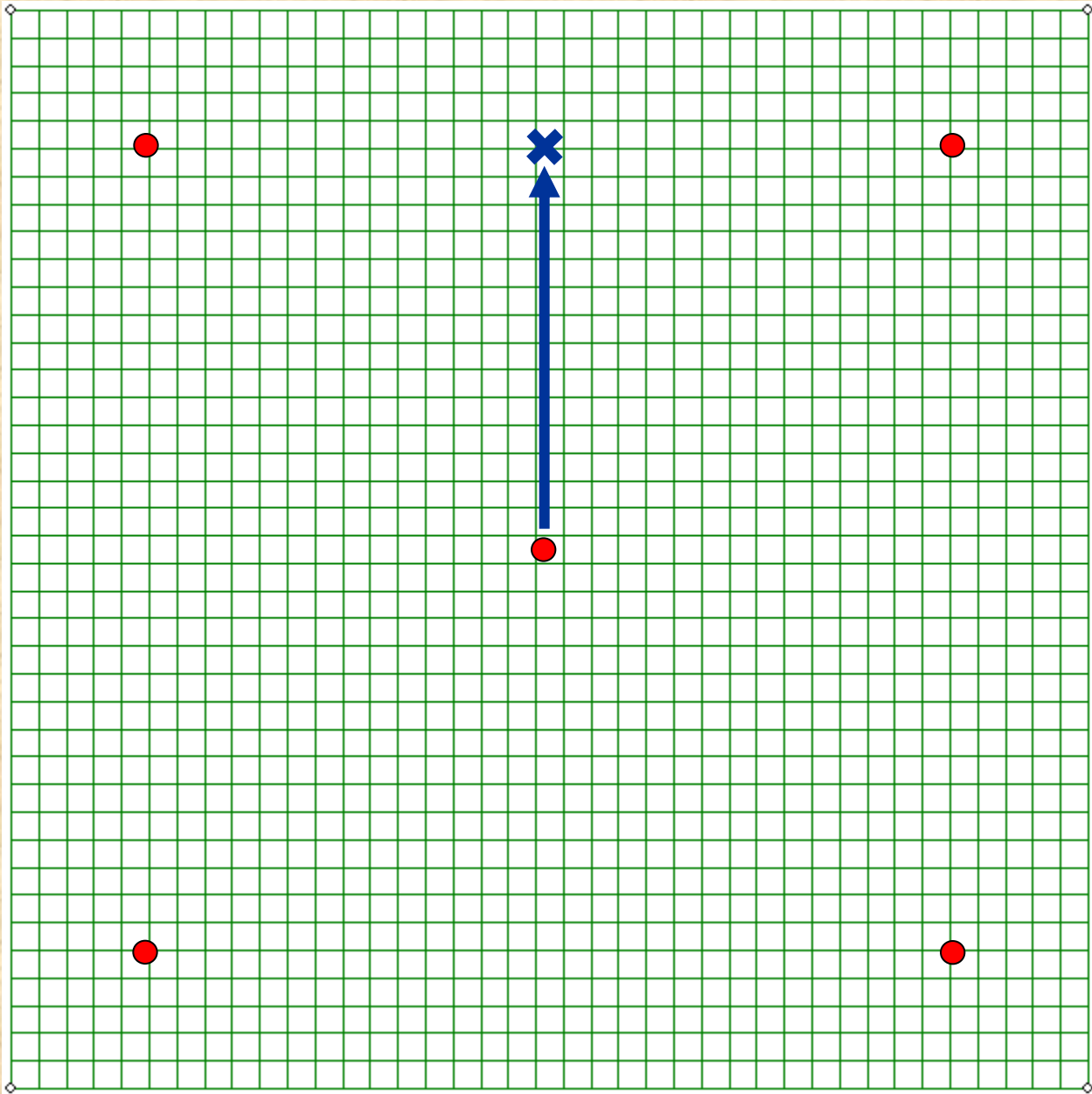


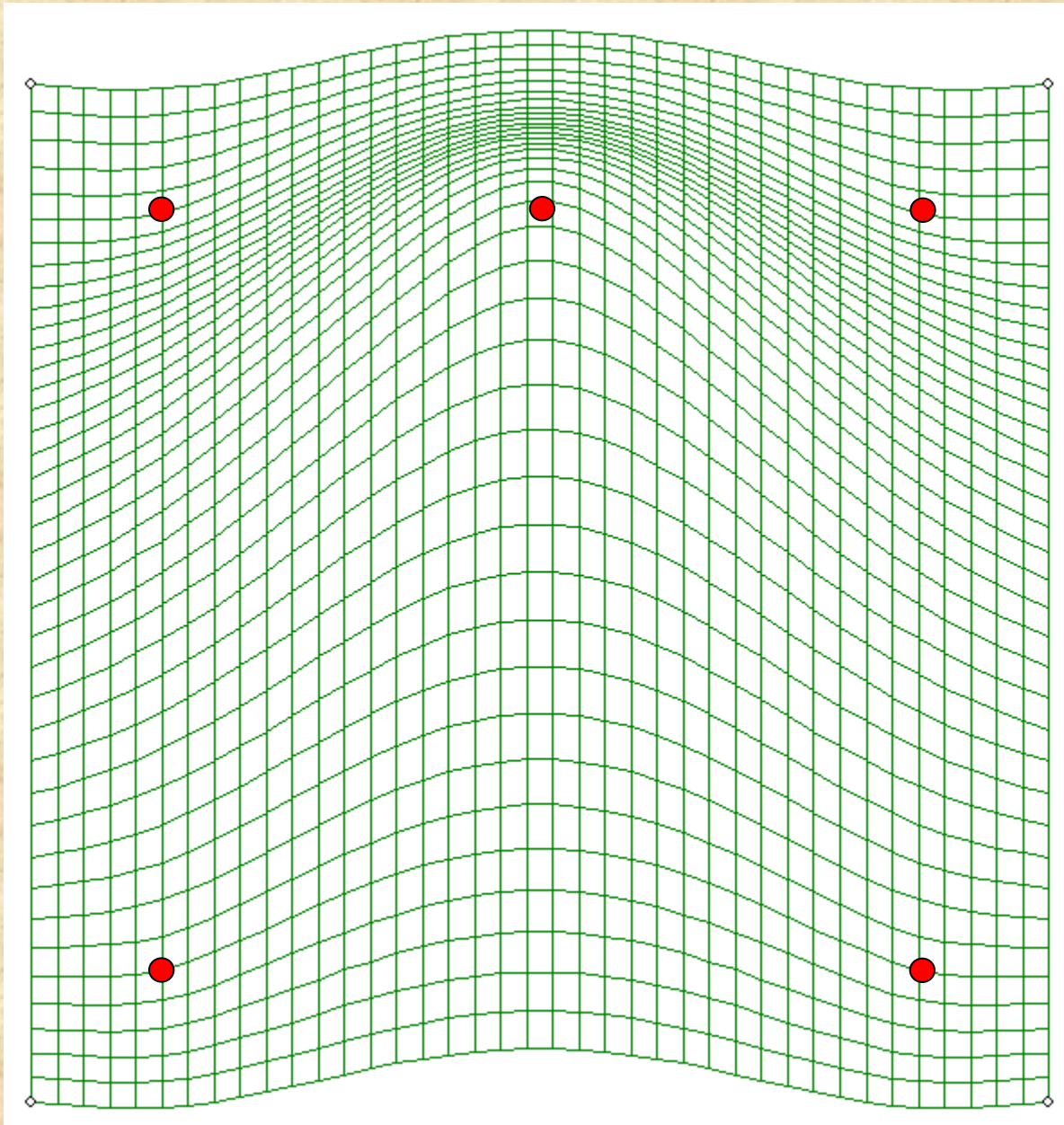
→ Rezistentní přizpůsobení
GRF (Generalized Resistant Fit)

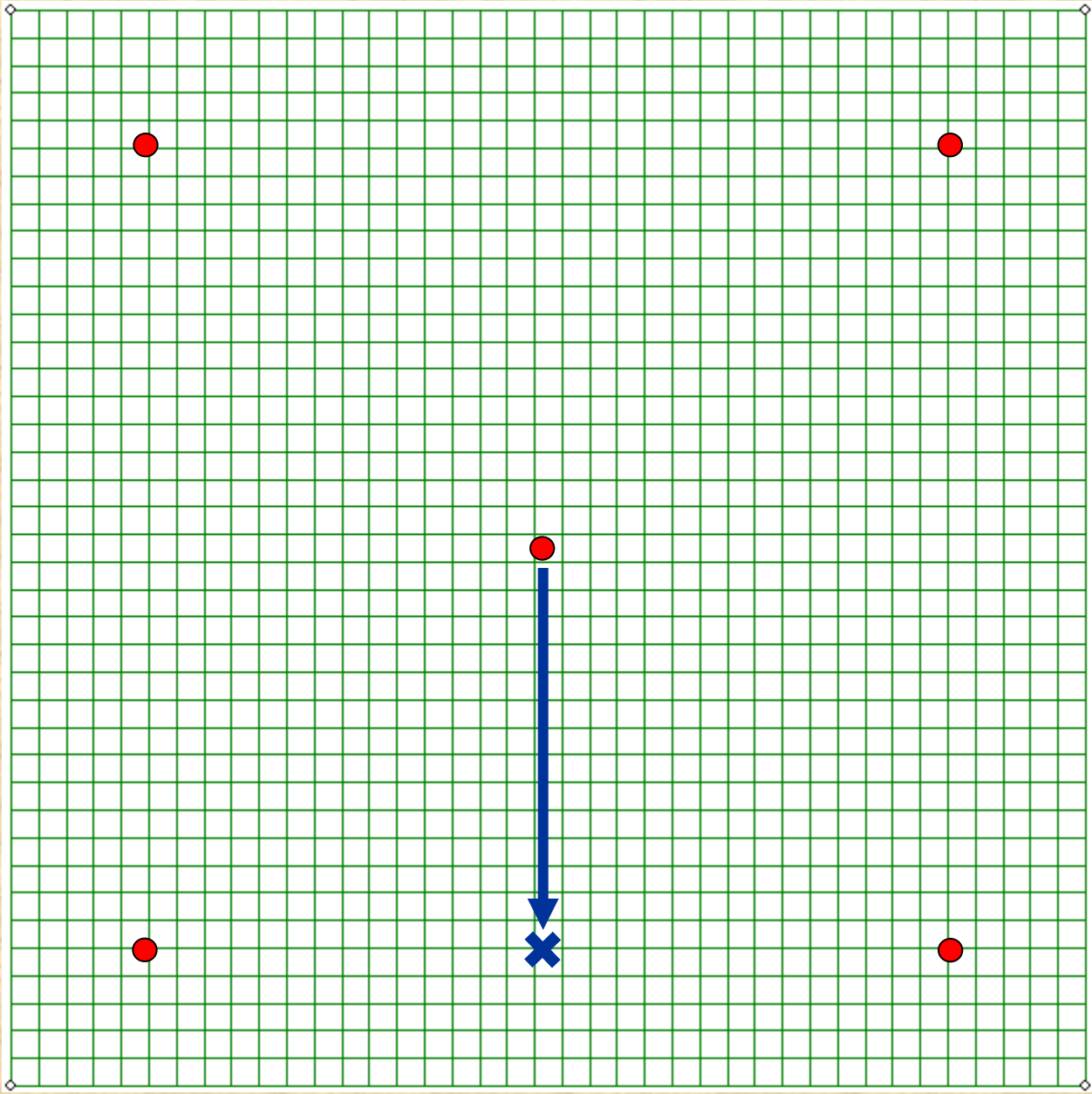
Geometrická morfometrie III. Deformace souřadnicových sítí

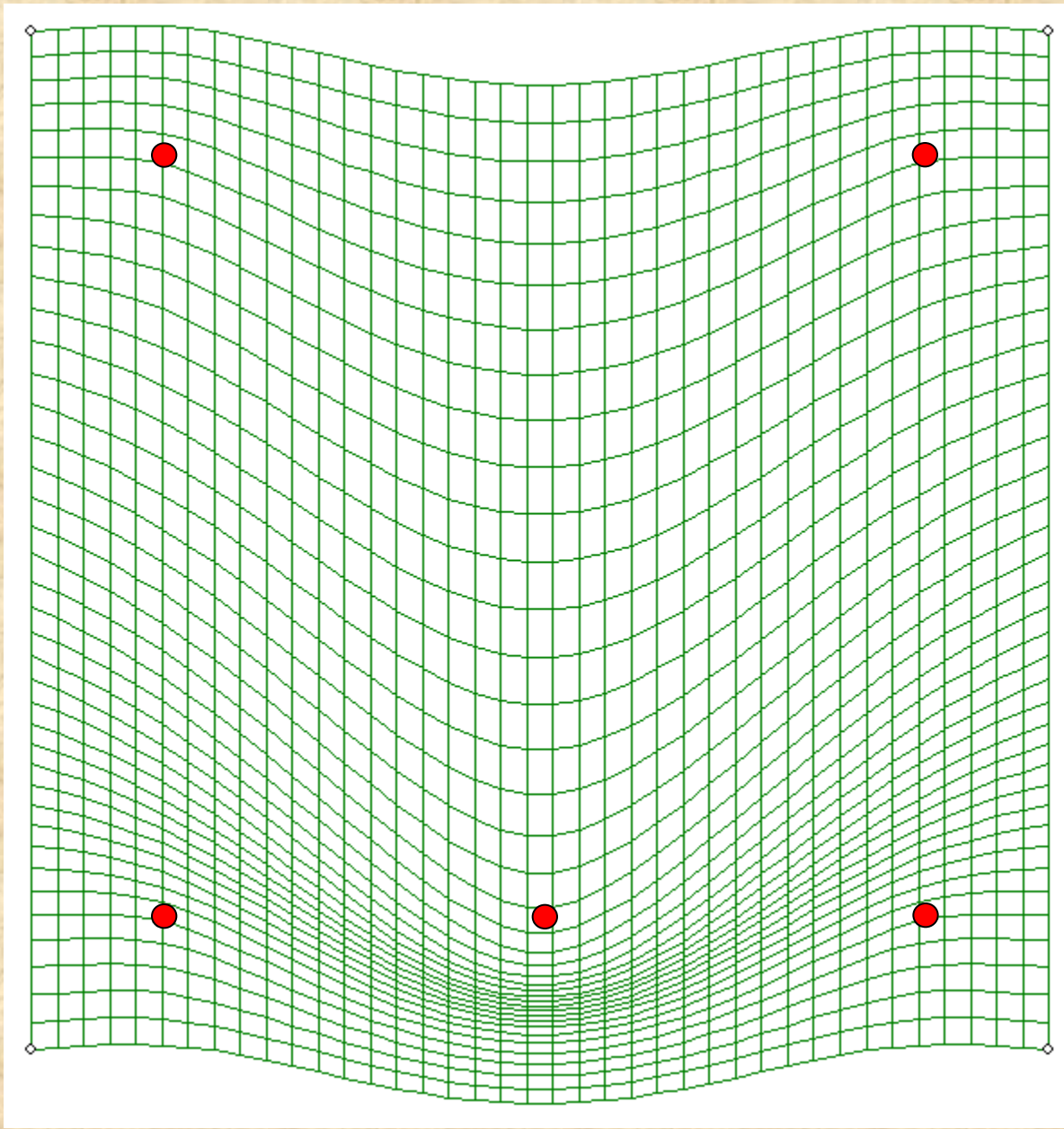
Metoda ohebných pásků, TPS (Thin-Plate Spline)

- **metafora nekonečně velkého, nekonečně tenkého kovového plátu**









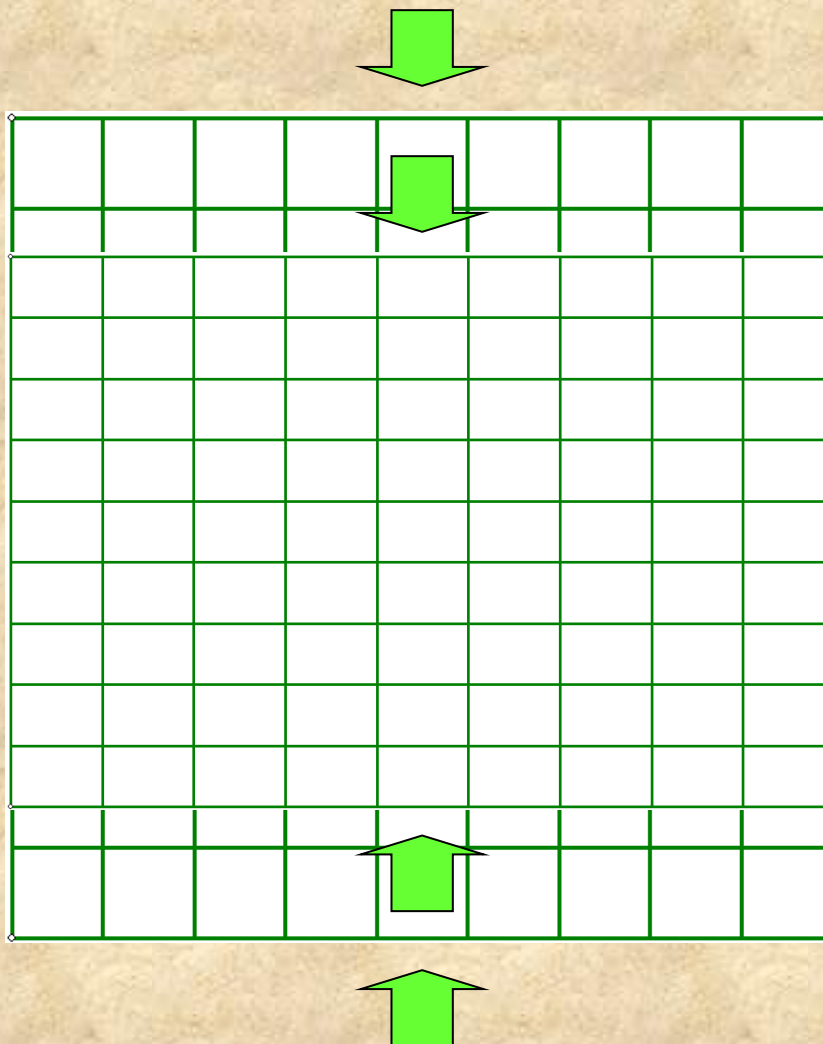
Geometrická morfometrie III.

Deformace souřadnicových sítí

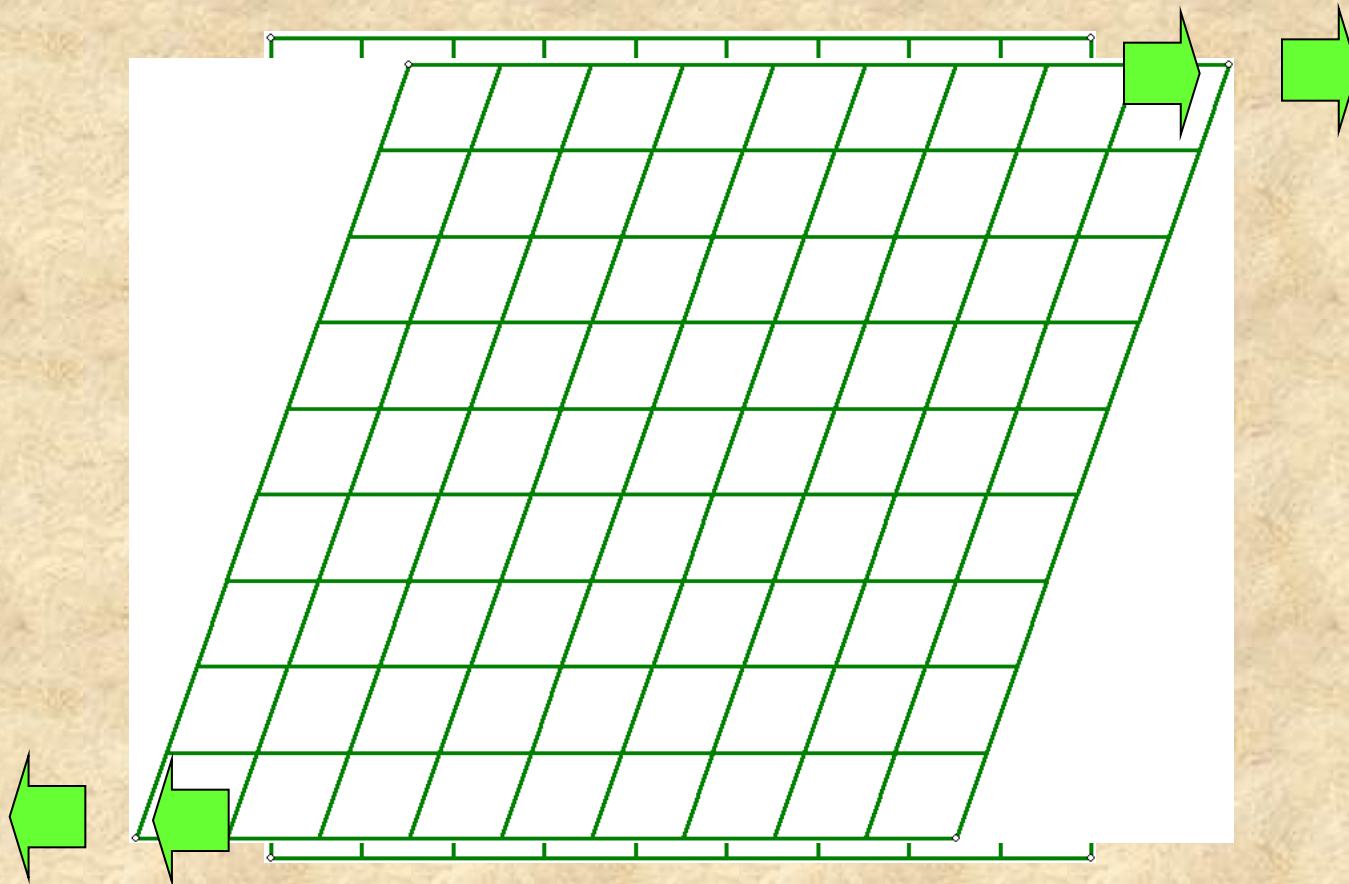
Metoda ohebných pásků, TPS (Thin-Plate Spline)

- metafora nekonečně velkého, nekonečně tenkého kovového plátu
- energie nutná k deformaci plátu = deformační energie (bending energy)
- odlišení afinní a neafinní změny tvaru
- projekce latentních kořenů deformační energie do jednotlivých os = parciální deformace (partial warps)
- parciální deformace 0 ~ uniformní složce

Afinní (uniformní) změna tvaru

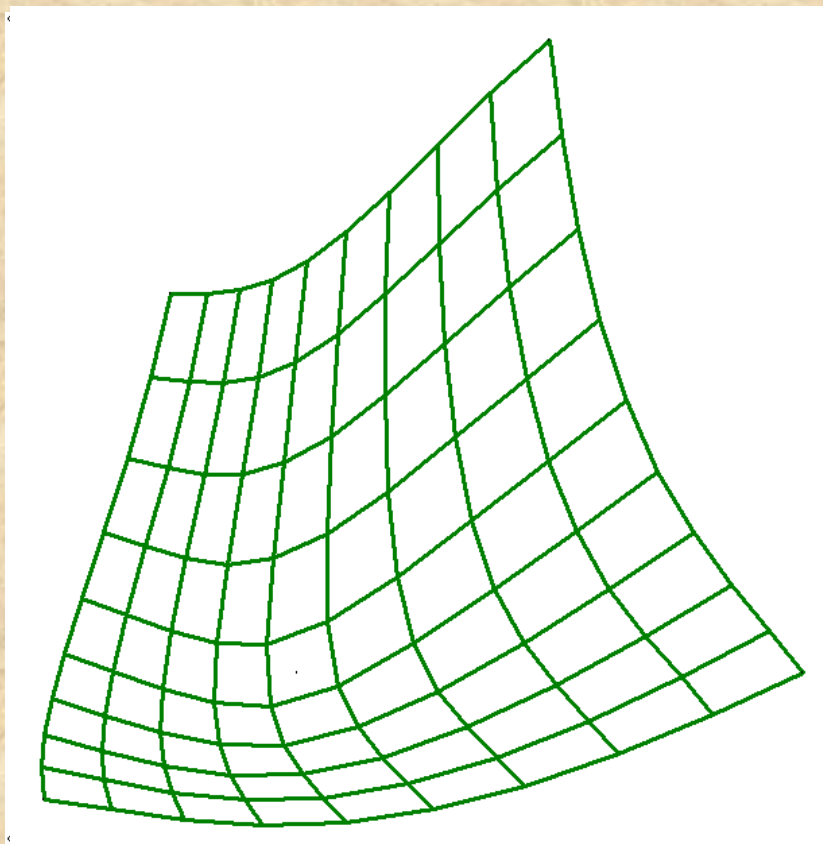


Afinní (uniformní) změna tvaru



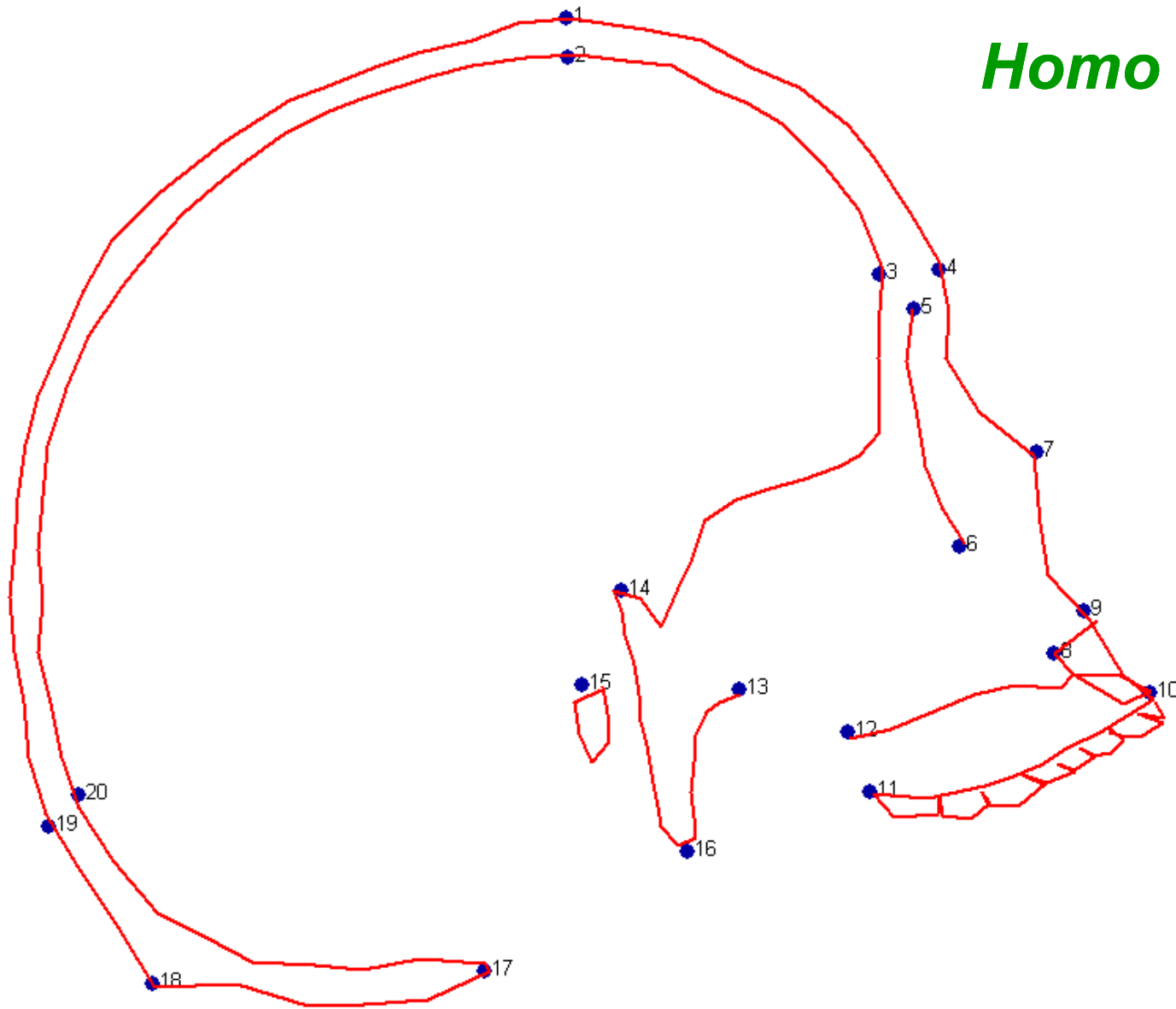
- **rovnoběžky zůstávají rovnoběžné**

Neafinní (neuniformní) změna tvaru

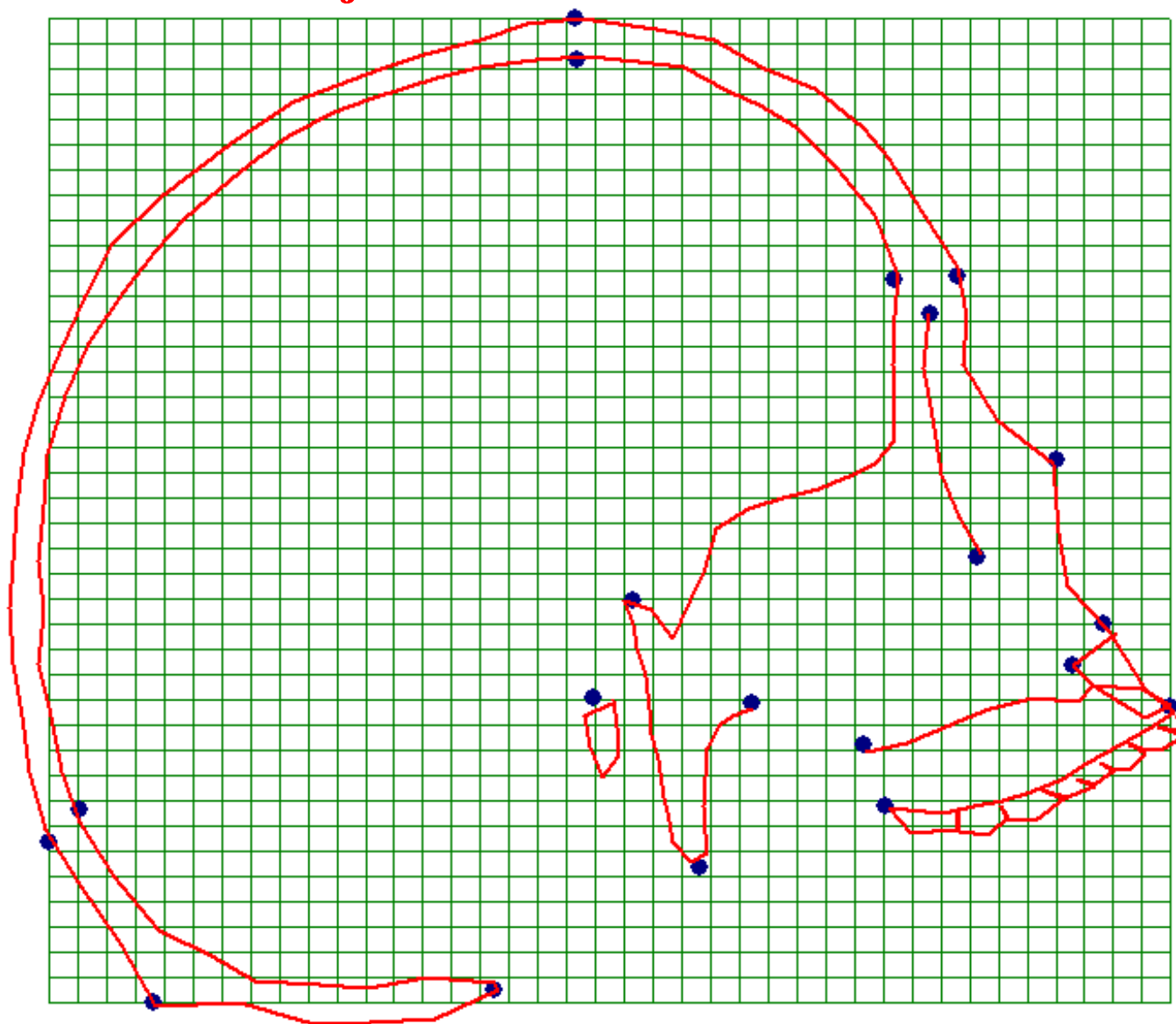


- **změna tvaru sítě lokální**

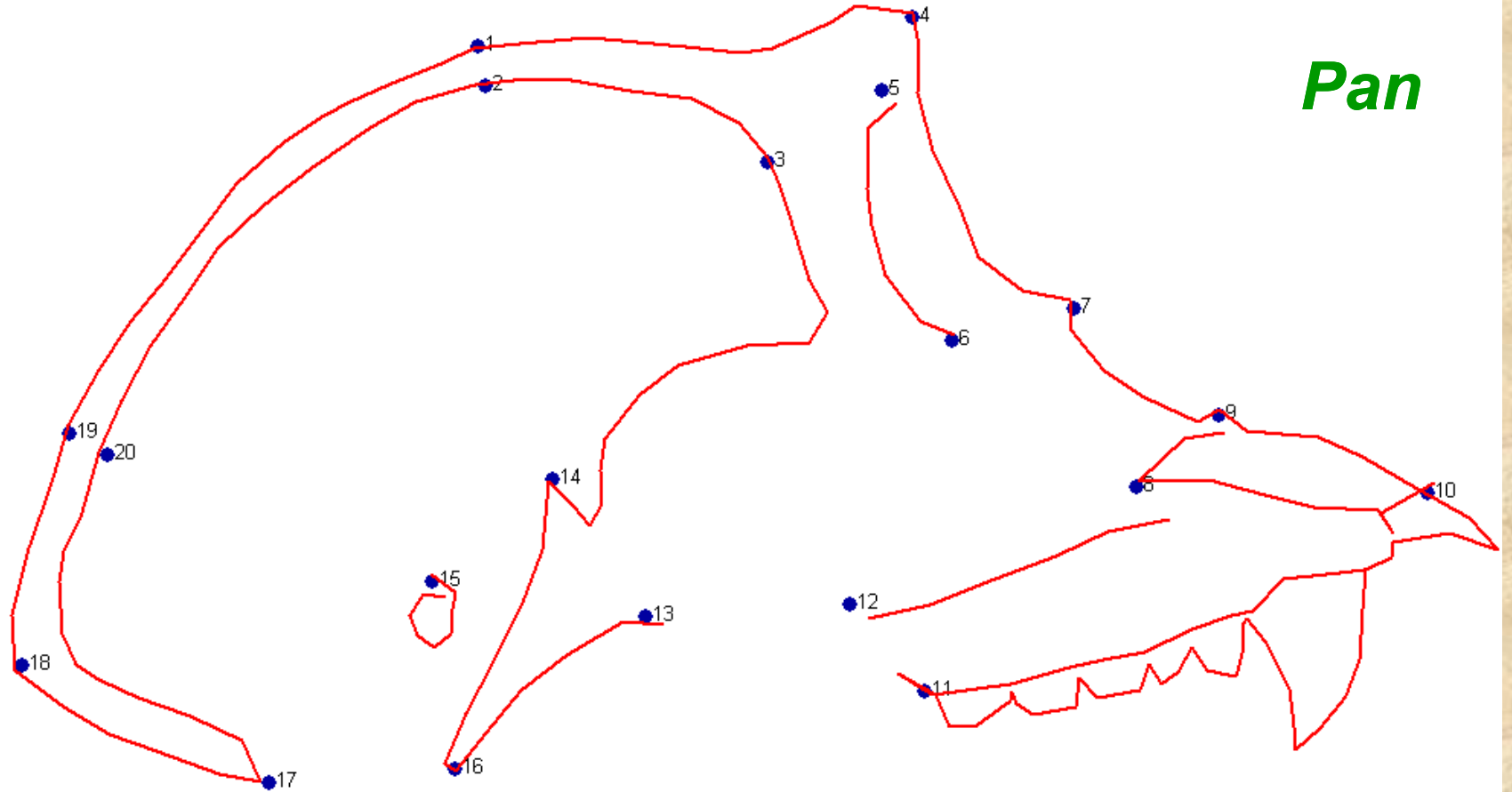
Homo



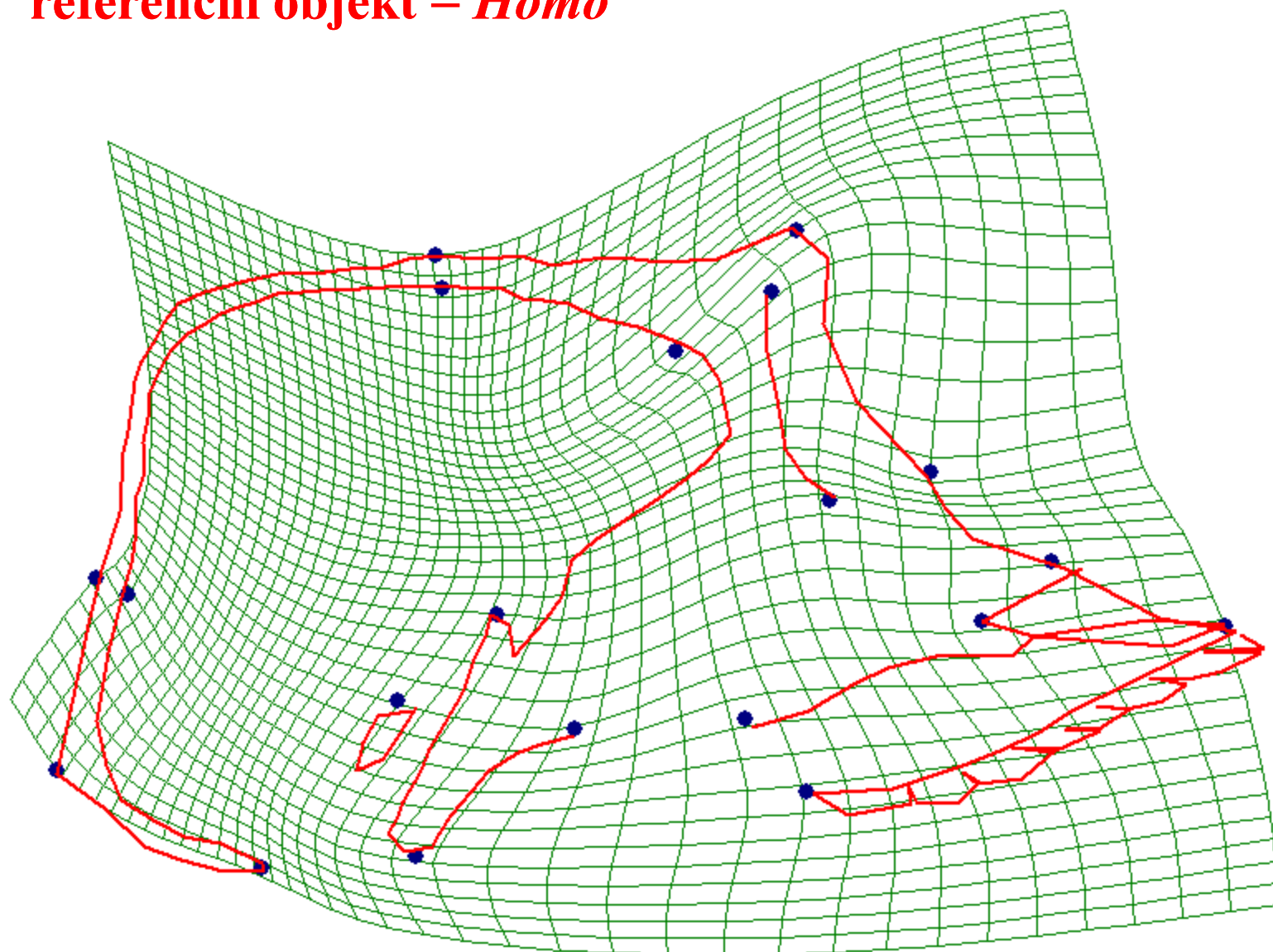
referenční objekt

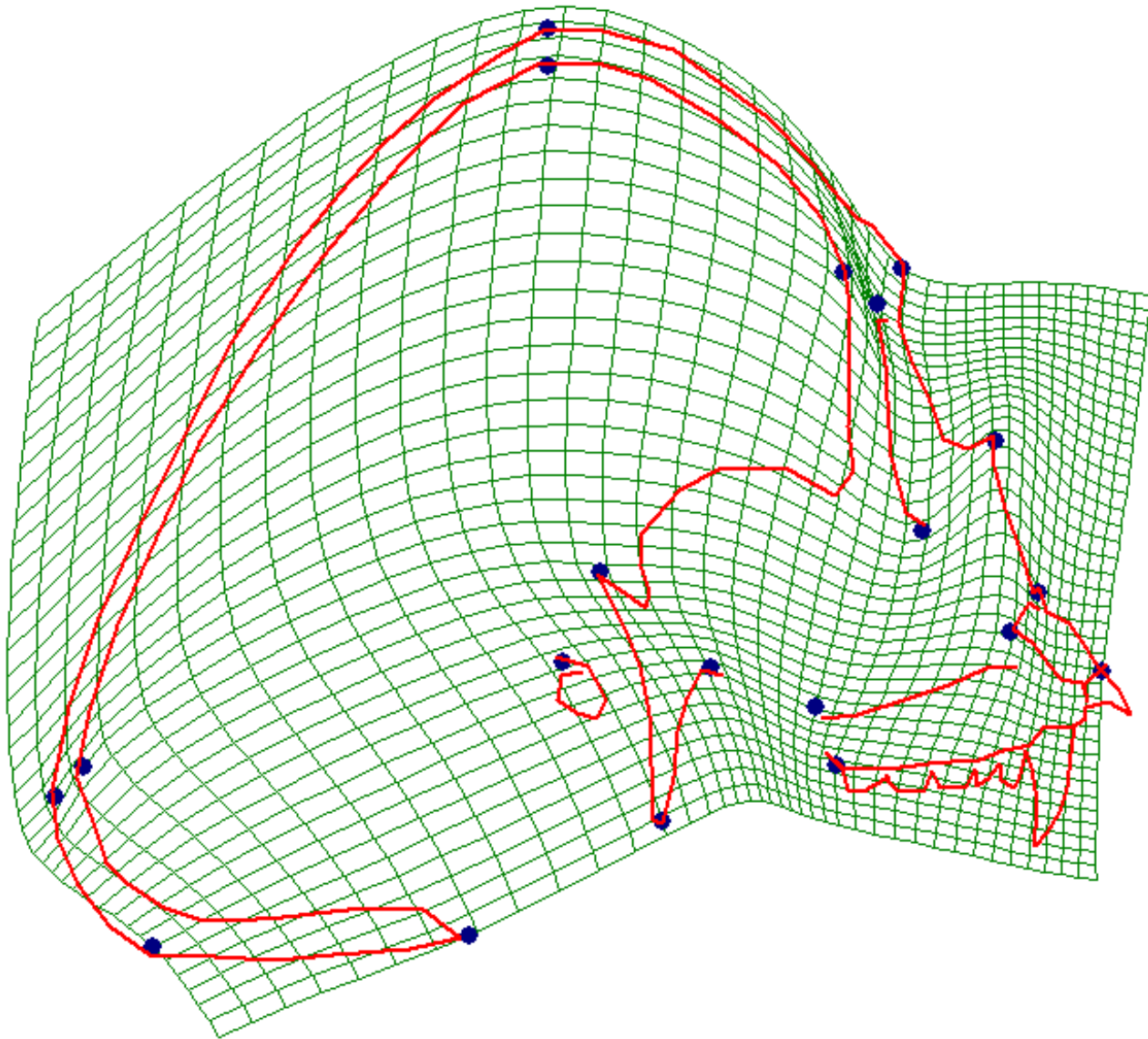


Pan



referenční objekt = *Homo*

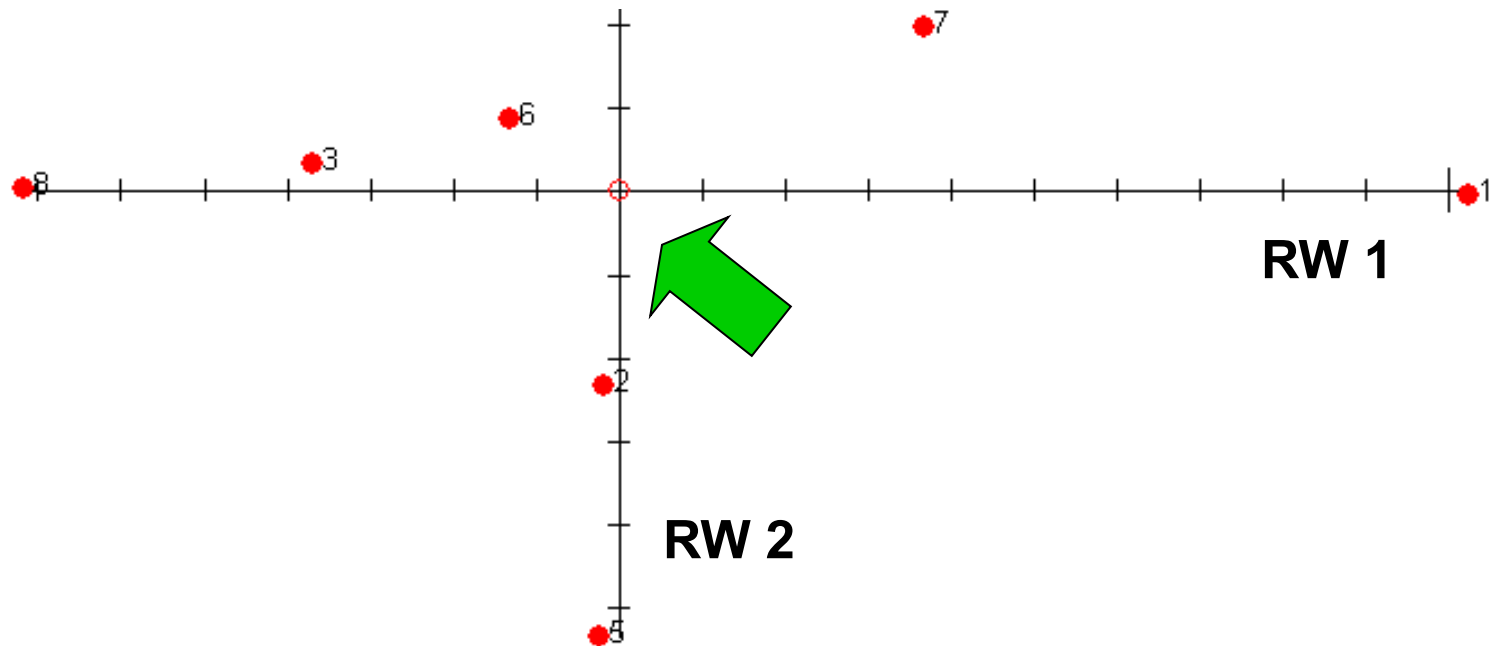


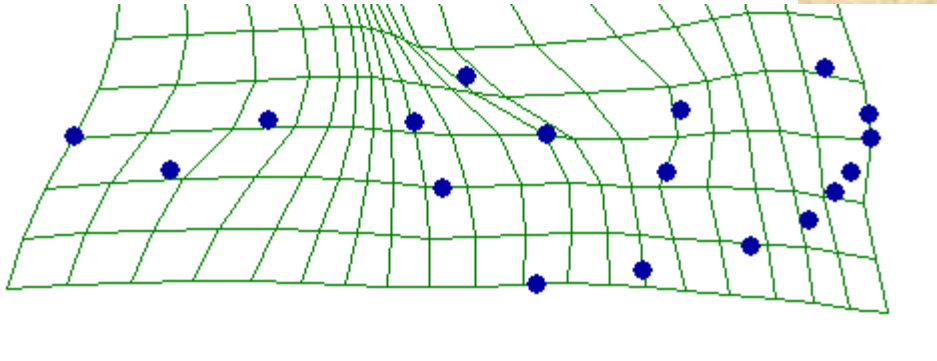
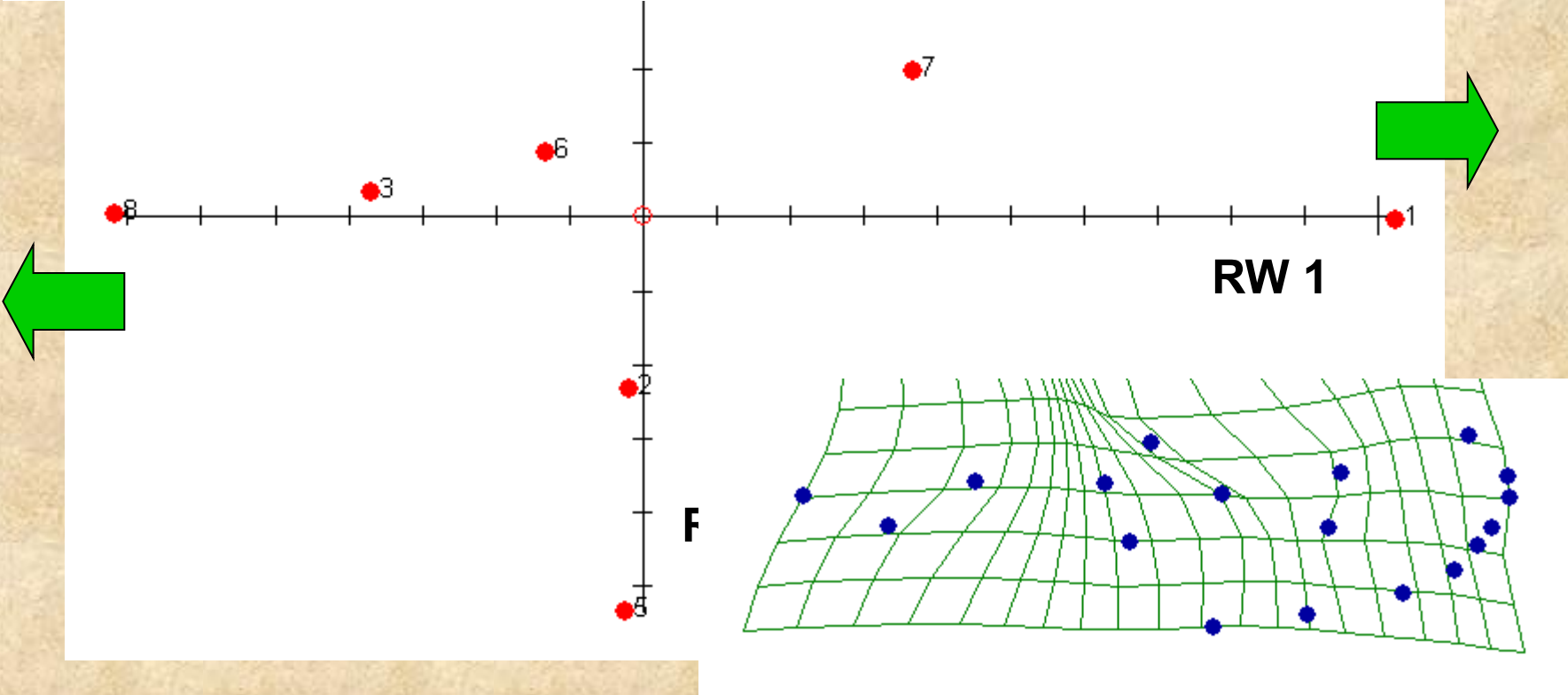
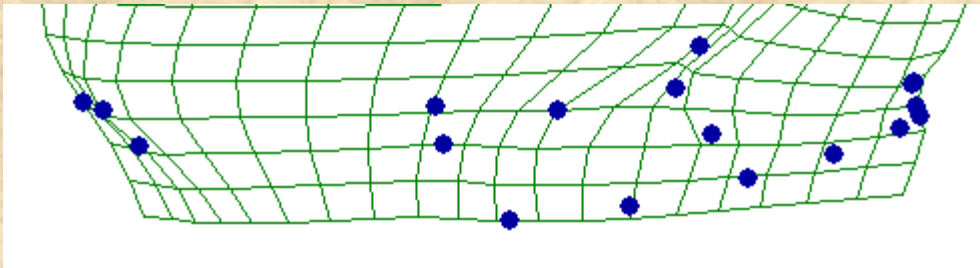


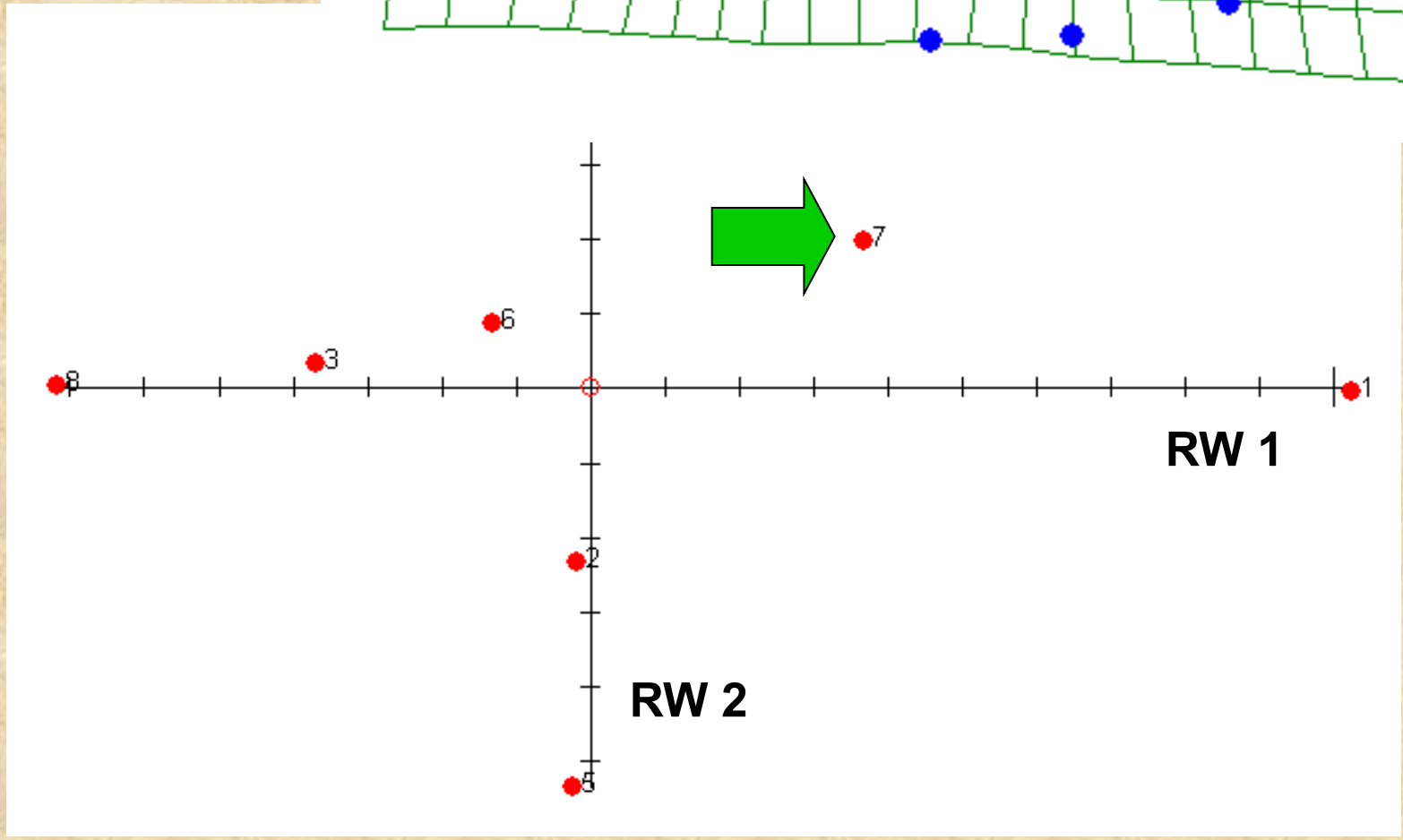
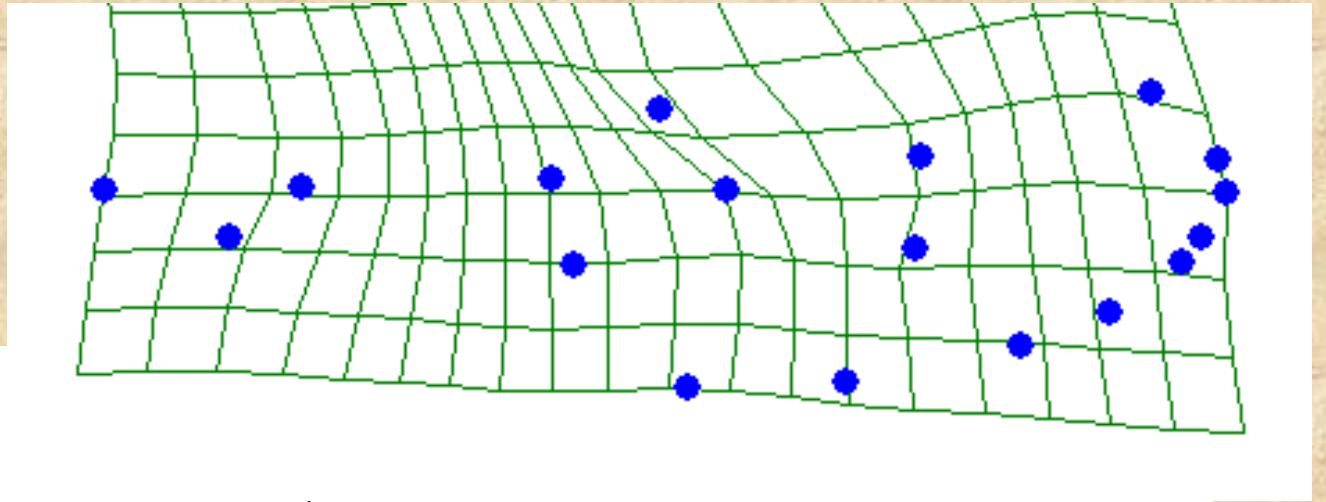
referenční objekt = *Pan*

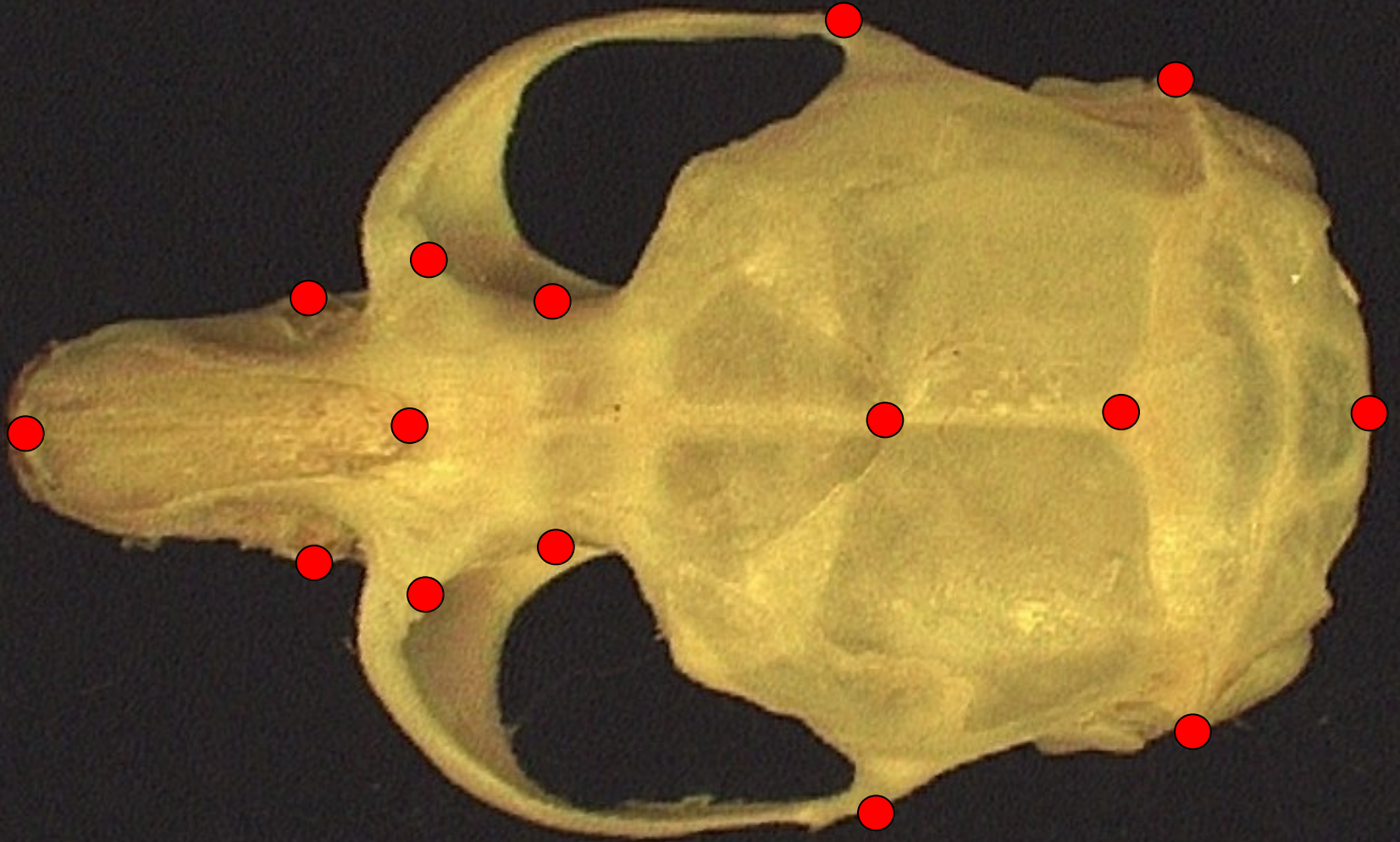
Více než 2 objekty:

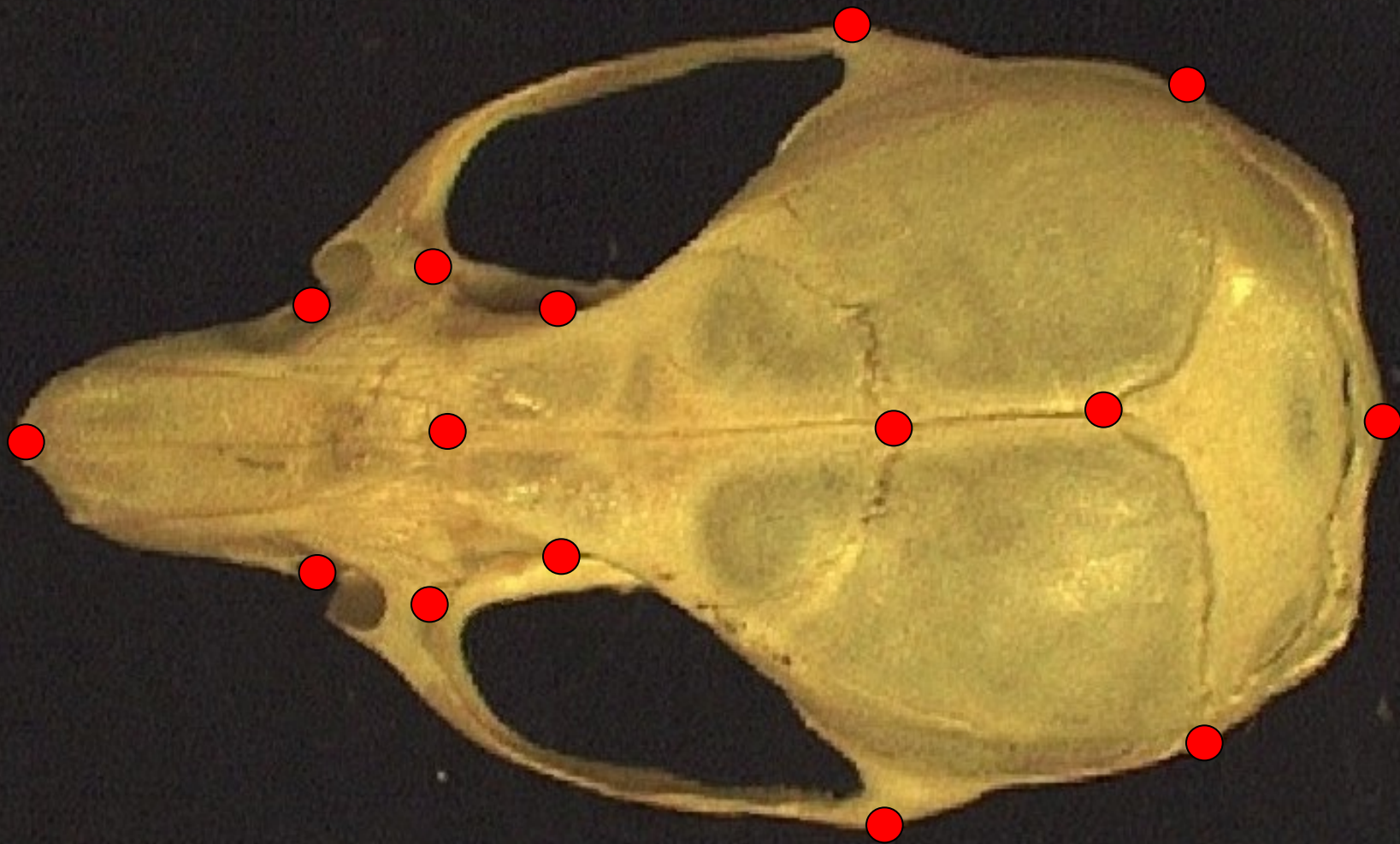
Relativní deformace, TPSRW (Thin-Plate Spline Relative Warps)











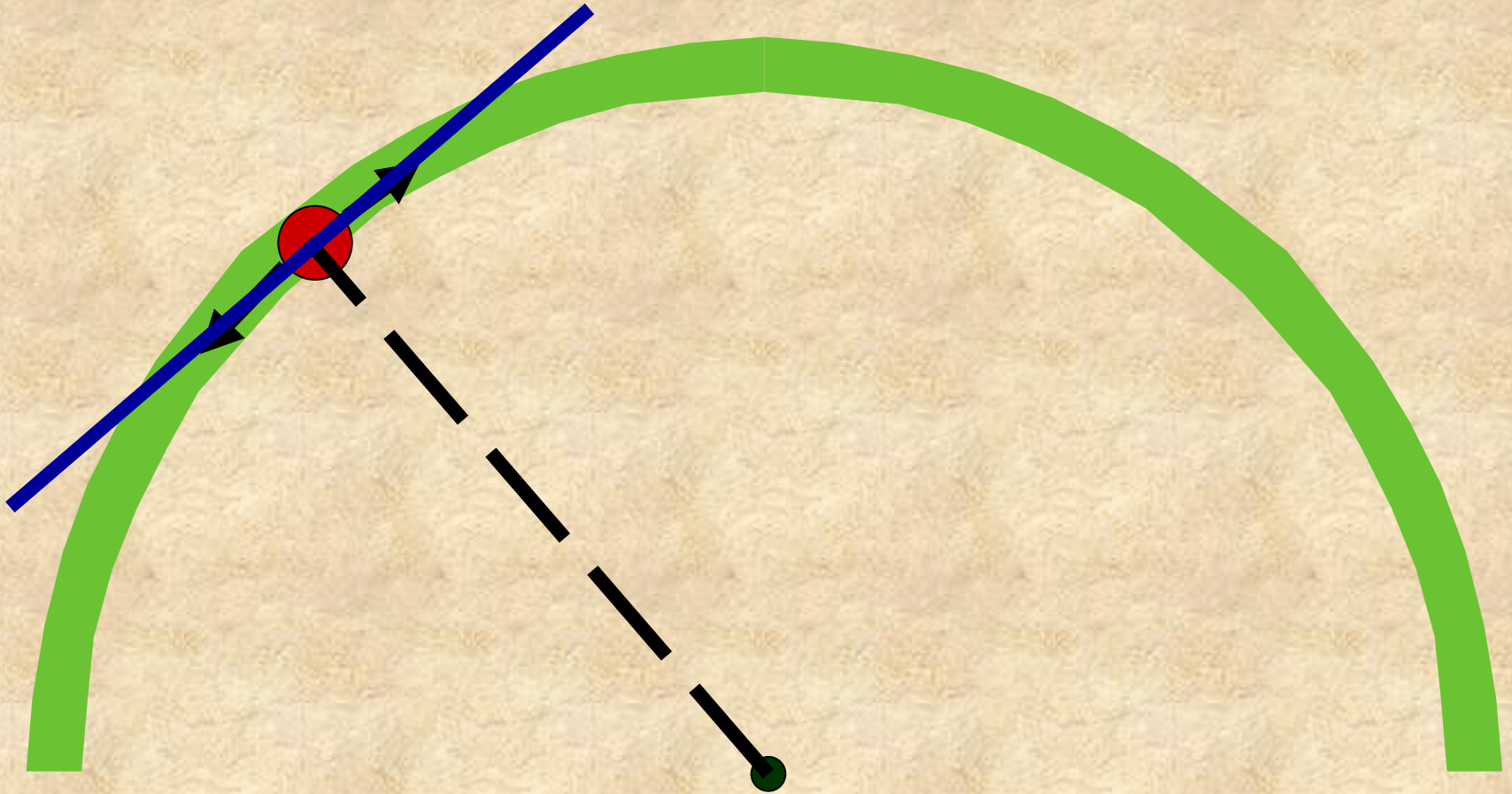


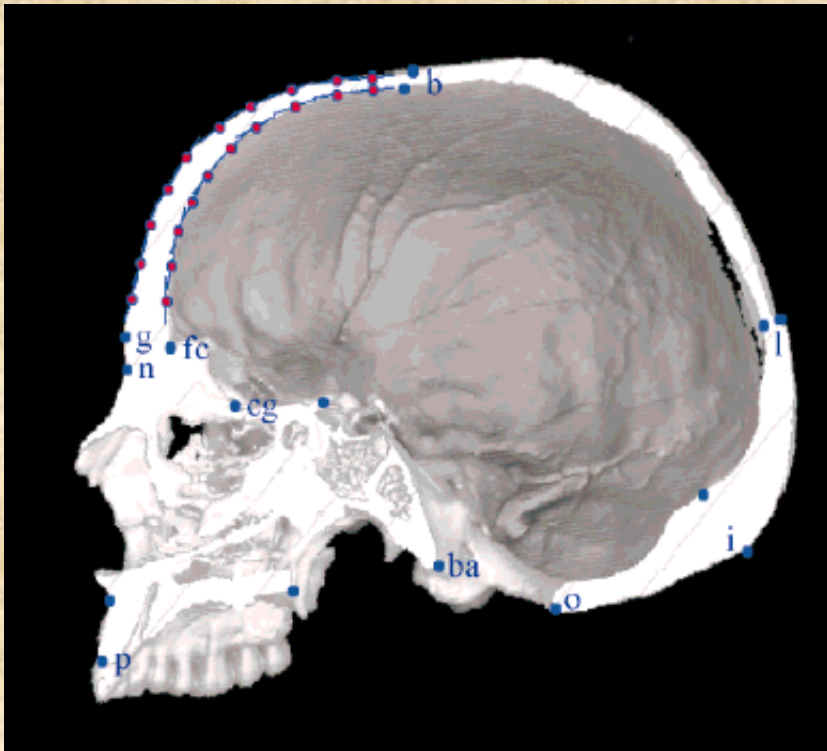
Software:

- **tpsDig: digitalizace bodů, měření rozměrů**
- **tpsSpln: Thin-Plate Spline**
- **tpsRelw: TPS Relative Warps**
- **tpsRegr: regrese na nezávislou proměnnou**
- **tpsPLS: parciální nejmenší čtverce (např. korelace 2 sad bodů)**
- **tpsSuper: deformace obrázků („unwarping“)**
- **tpsTree: analýza tvarových změn podél fylogenetického stromu**

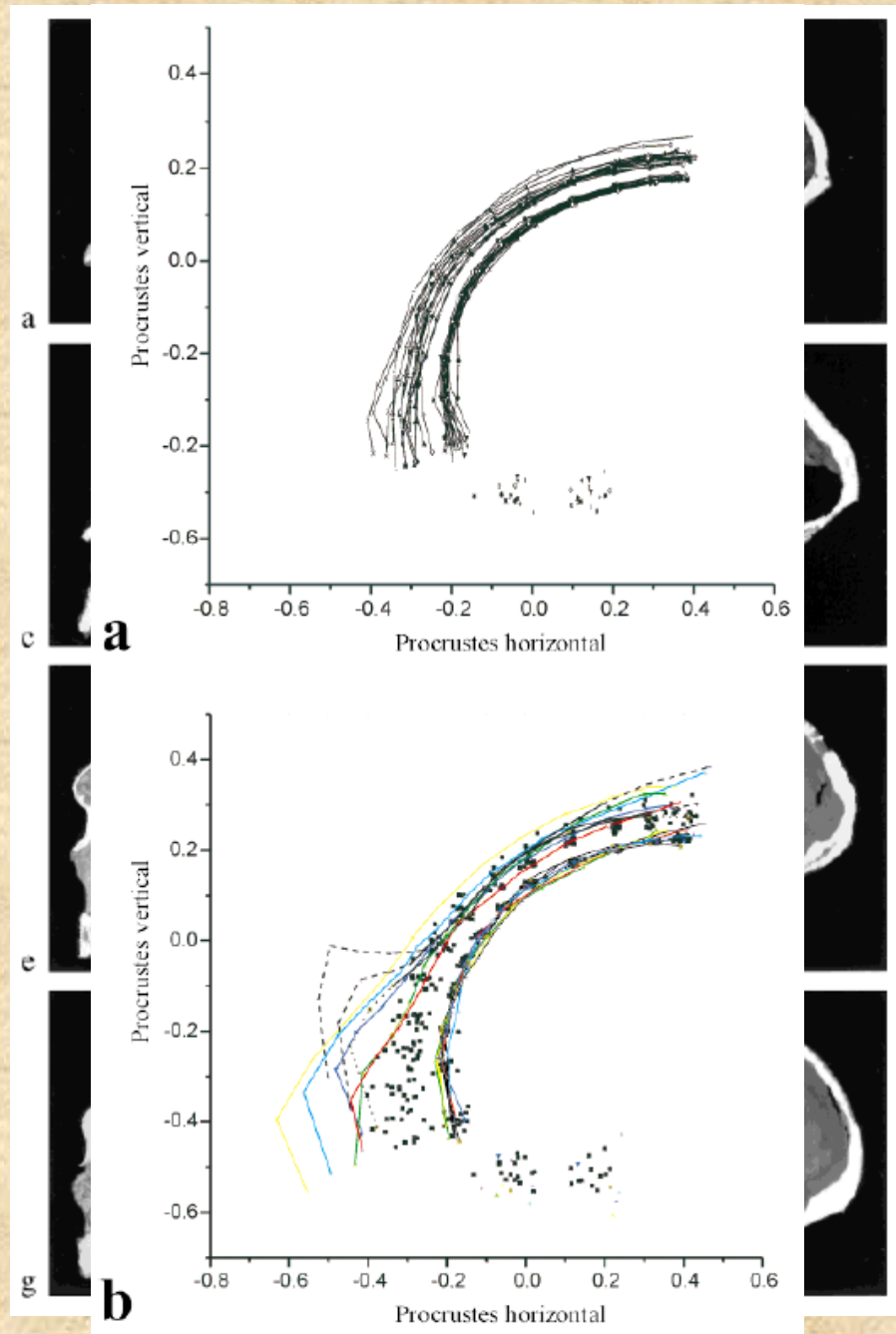
<http://life.bio.sunysb.edu/morph/>

Metody založené na význačných bodech bez landmarků – „sliding semilandmarks“

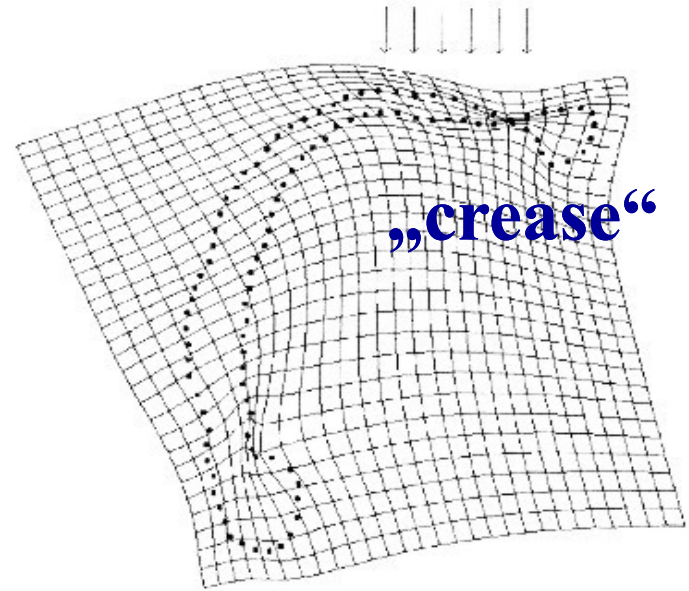




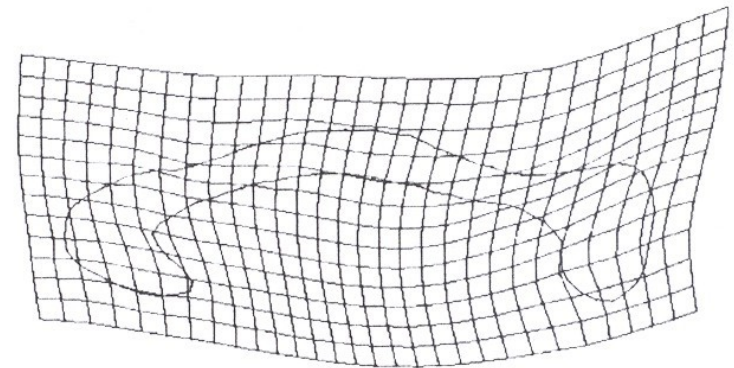
**Bookstein et al.,
Anat. Record (1999)**



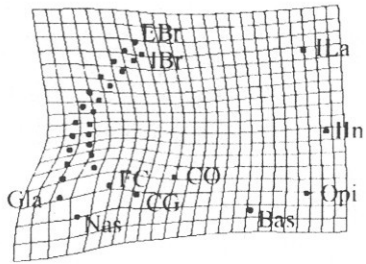
corpus callosum



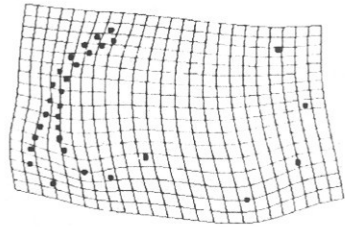
Extremum of Deformation Tensor:
Male → Female
(Extrapolated -2.42-Fold to a Crease)



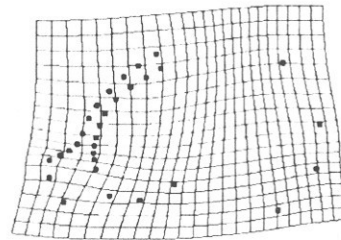
Deformation: Male → Female (x2)



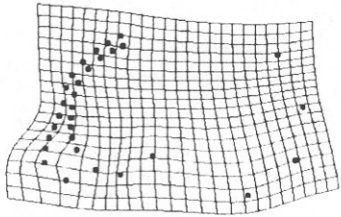
Modern *Homo* → Archaic *Homo*



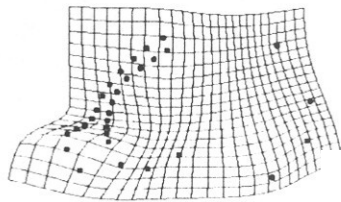
Archaic *Homo* → STS5



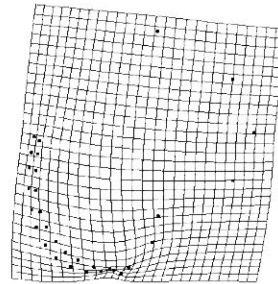
STS5 → *Pan*



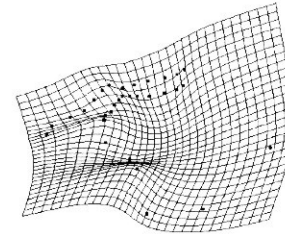
Modern *Homo* → STS5



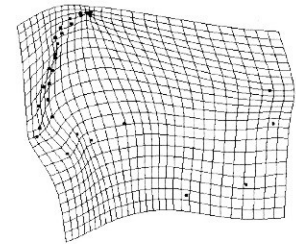
Modern *Homo* → *Pan*



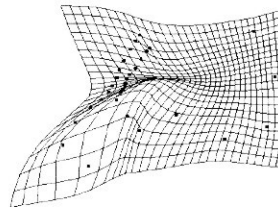
Modern *Homo* → Archaic *Homo*
(Positive Crease, Strain = 2.362)



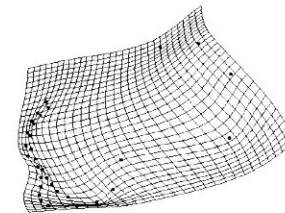
Archaic *Homo* → STS5
(Positive Crease, Strain = 1.377)



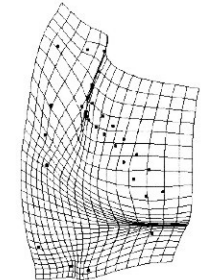
STS5 → *Pan*
(Negative Crease, Strain = 0.693)



Modern *Homo* → Archaic *Homo*
(Negative Crease, Strain = 0.701)



Archaic *Homo* → STS5
(Positive Crease, Strain = 0.553)



STS5 → *Pan*
(Positive Crease, Strain = 2.362)

<http://life.bio.sunysb.edu/morph/>