Dobrý den!

25. října 2009
2. přednáška

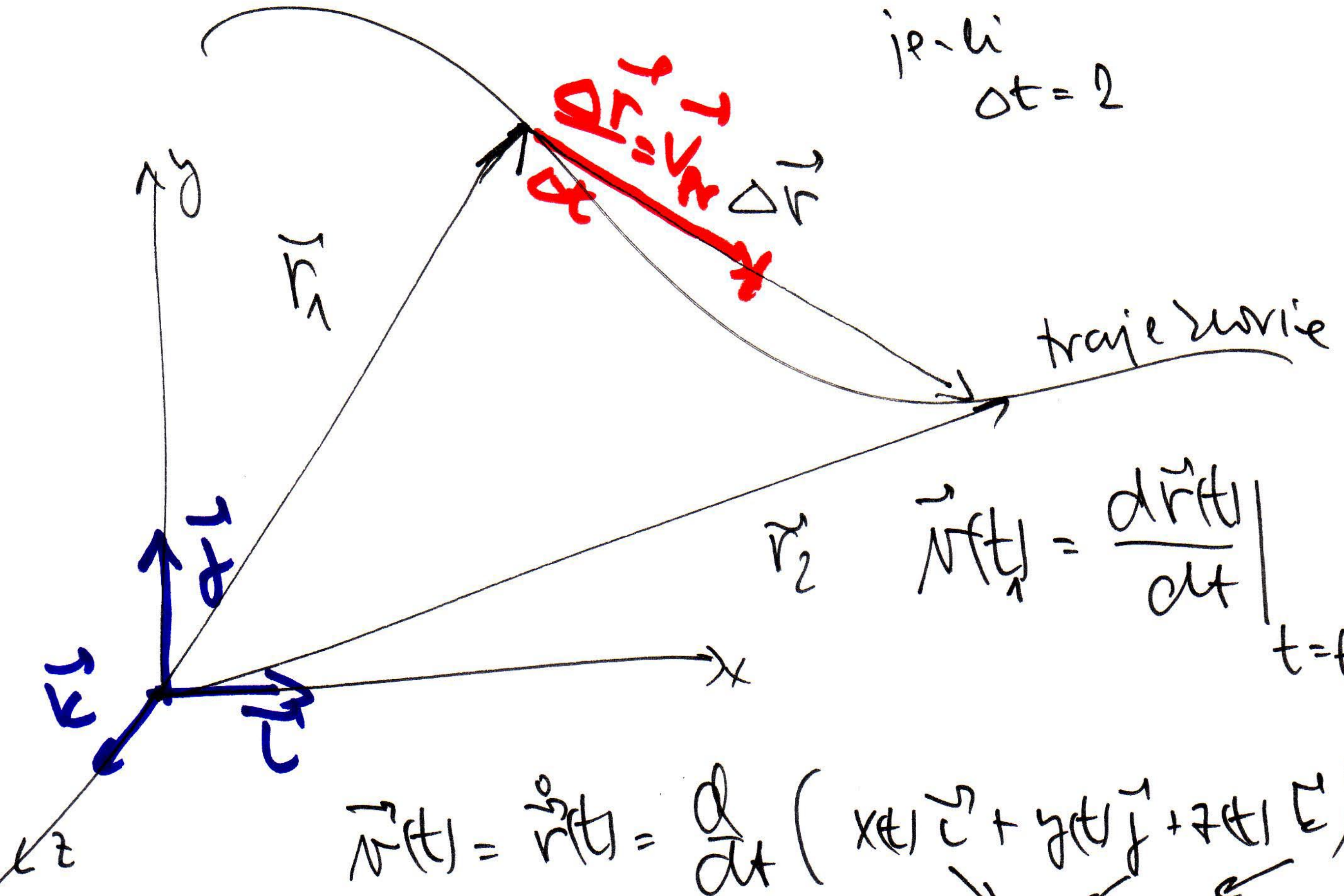
Minulé opakování:

- → vektory (. . . .)
- → polohový vektor $\vec{r}(t) = \left(x(t);\ y(t);\ z(t) \right)$
- → průměrná rychlost

$$\vec{v}_{pr}(\Delta t) = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} \qquad m/s$$

- → okamžitá rychlost $\boxed{\lim_{\Delta t \to 0}}$ $\vec{v}_{ok}(\Delta t) = \frac{d\vec{r}}{dt}$

je-li $\Delta t = 2$

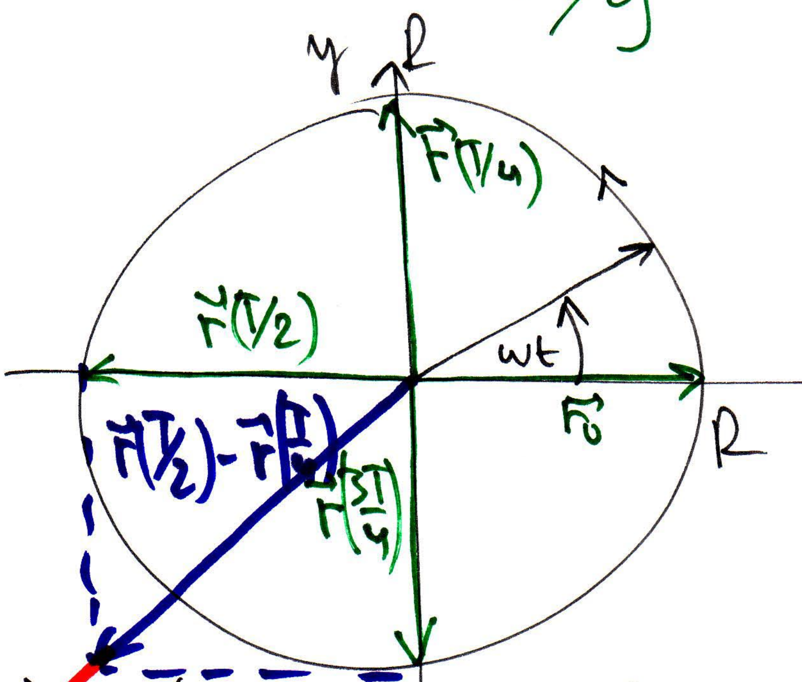


$$\vec{v}(t_1) = \left. \frac{d\vec{r}(t)}{dt} \right|_{t=t_1}$$

$$\vec{v}(t) = \dot{\vec{r}}(t) = \frac{d}{dt}\left(x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k} \right) =$$

konstantní

$$= \left(\dot{x}(t)\,\vec{i} + \dot{y}(t)\,\vec{j} + \dot{z}\,\vec{k} \right) = \left(\dot{x}(t);\ \dot{y}(t);\ \dot{z}(t) \right)$$

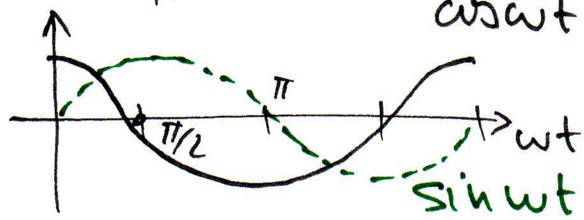


$\omega = \frac{2\pi}{T}$ … konst

$x = R\cos\omega t$

$y = R\sin\omega t$

$z = 0$



$\vec{r}(0) = (R, 0, 0)$ m $\quad \vec{r}(T/4) = (0, R, 0)$

$\vec{r}(T/2) = (-R, 0, 0) \quad \vec{r}(3T/4) = (0, -R, 0) \quad \vec{r}(T) = (R, 0, 0)$

určení rychlosti

i) průměrná rychlost od $t_1 = T/4$ do $t_2 = T/2$

$\Delta t = t_2 - t_1 = \dots$

$$\vec{v}_{pr}(\langle t_1, t_2 \rangle) = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{\Delta t} =$$

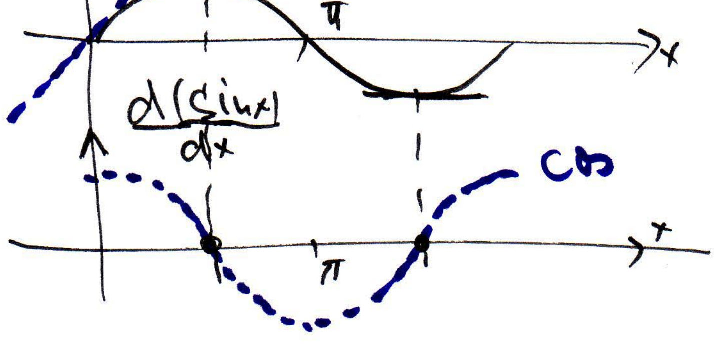
je-li $T = 2$ s pak $\Delta t = \frac{T}{4} = \frac{1}{2}$ s

ii) okamžitá rychlost

$$\vec{v}(t) = \dot{\vec{r}}(t) = (-R\omega\sin\omega t, R\omega\cos\omega t, 0)$$

$$|\vec{v}(t)| = \sqrt{R^2\omega^2(\sin^2\omega t + \cos^2\omega t)}$$

$$\underline{\underline{v(t) = R\omega}}$$

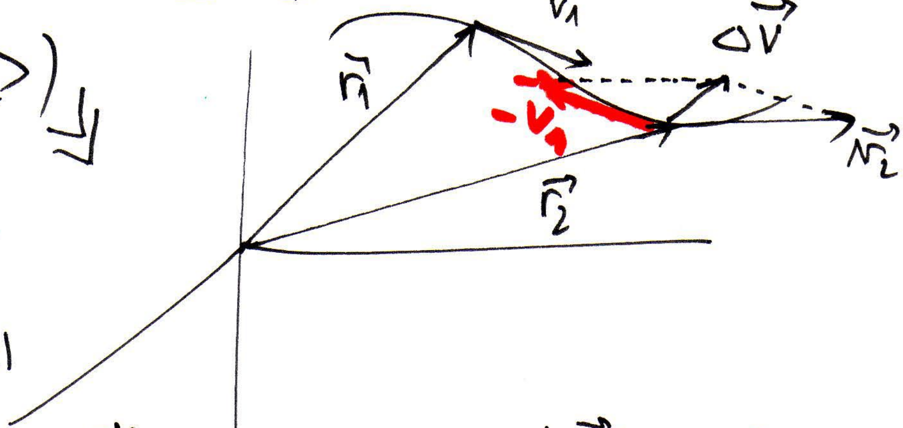


Zrychlení

průměrné

$$\vec{a}_{pr}(\Delta t) = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1} \qquad [m \cdot s^{-2}]$$

$$\Gamma(\Delta t) = (\langle t_1, t_2 \rangle)$$

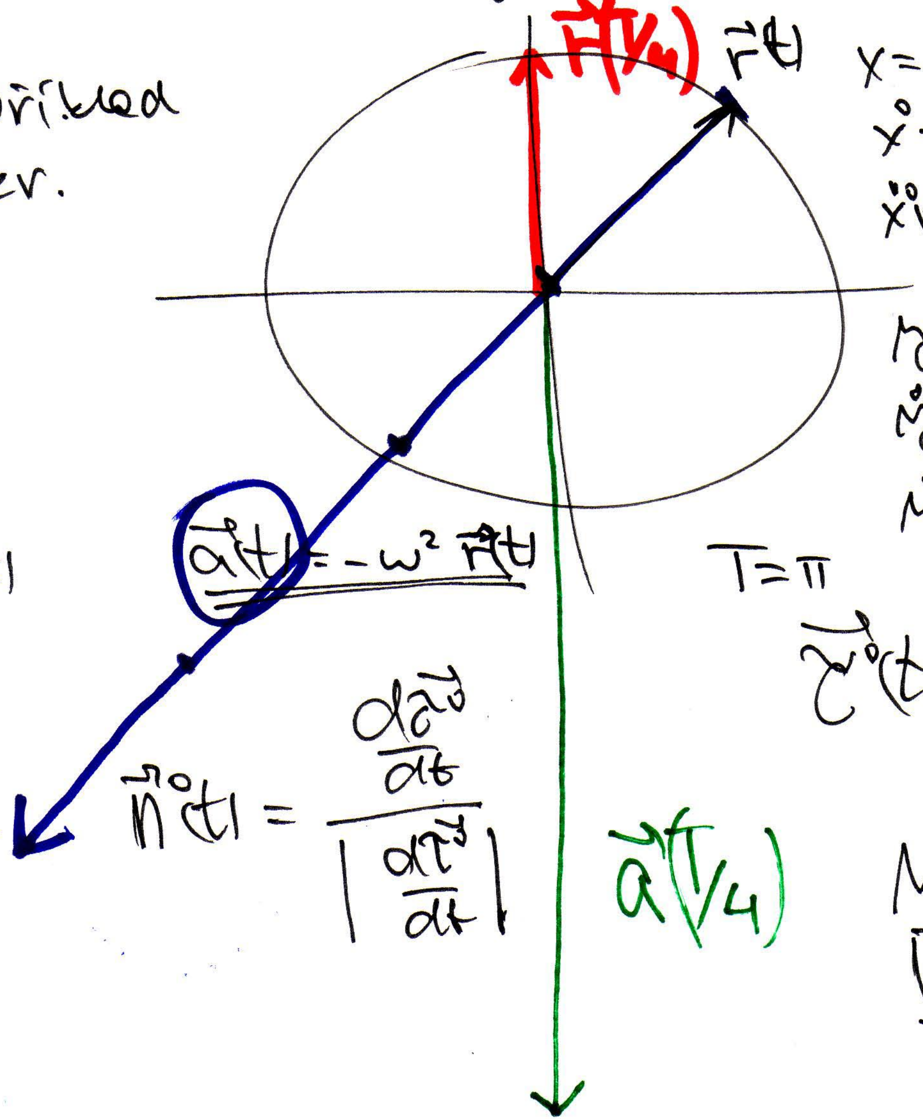


okamžité zr. $\vec{a}(t)$

$$\vec{a}(t) = \lim_{\Delta t \to 0} \vec{v}_{pr}(\Delta t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt} = \dot{\vec{v}}(t)$$

$$\vec{a}(t) = \left(\dot{v}_x(t);\ \dot{v}_y(t);\ \dot{v}_z(t)\right) = \left(\ddot{x}(t), \ddot{y}(t), \ddot{z}(t)\right)$$

Příklad pokr.



$x = R\cos\omega t$

$\dot{x} = -R\omega \sin\omega t$

$\ddot{x}(t) = -R\omega^2 \cos\omega t$

$y = R\sin\omega t$

$\dot{y}(t) = R\omega \cos\omega t$

$\ddot{y}(t) = -R\omega^2 \sin\omega t$

$\vec{r}(t)$

$$\vec{a}(t) = -\omega^2 \vec{r}(t)$$

$T = \pi$

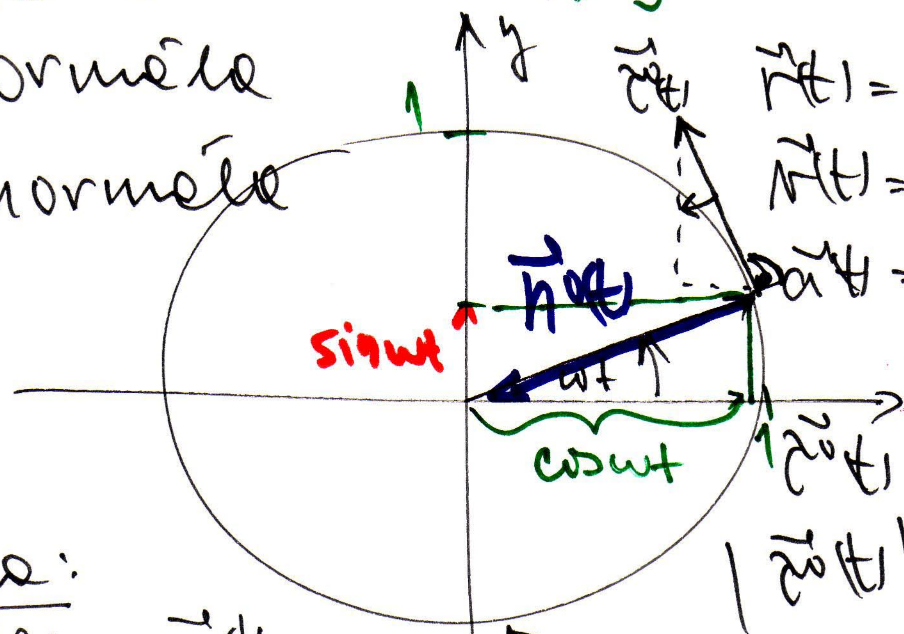
$$\vec{\tau}^{\,0}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|}$$

$$\vec{n}^{\,0}(t) = \frac{\frac{d\vec{\tau}^{\,0}}{dt}}{\left|\frac{d\vec{\tau}^{\,0}}{dt}\right|}$$

normála?

$\vec{n}^{\,0}(t) = ?$

tecne
normale
binormale



$\vec{r}(t) = R(\cos\omega t, \sin\omega t, 0)$

$\vec{v}(t) = R\omega(-\sin\omega t, \cos\omega t, 0)$

$\vec{a}(t) = -\omega^2 \vec{r}(t)$

$\vec{\tau}^0(t) = (-\sin\omega t, \cos\omega t, 0)$

$|\vec{\tau}^0(t)| = \sqrt{\sin^2 + \cos^2} = 1$

ucne:

$$\vec{\tau}^0(t) = \frac{\vec{v}(t)}{|v(t)|} = \frac{1}{R\omega}\left[R\omega(-\sin\omega t; \cos\omega t, 0)\right] = (-\sin\omega t, \cos\omega t, 0)$$

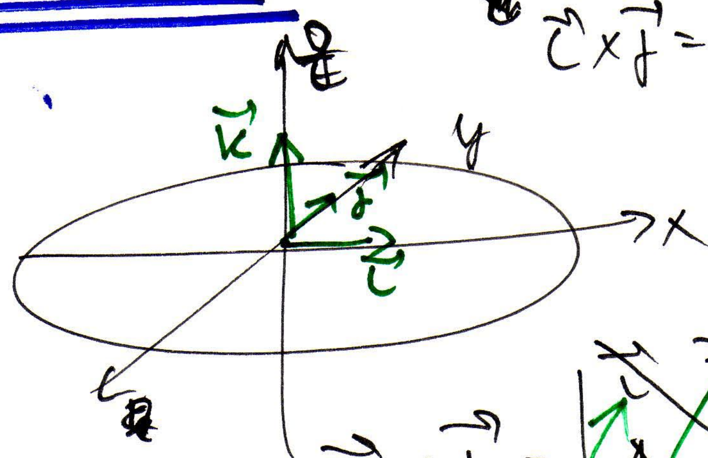
normale:

$$\vec{n}^0(t) = \frac{\frac{d\vec{\tau}^0}{dt}}{\left|\frac{d\vec{\tau}^0}{dt}\right|} = \frac{\dot{\vec{\tau}}^0(t)}{|\dot{\vec{\tau}}^0(t)|} = \frac{\omega(-\cos\omega t, -\sin\omega t, 0)}{\omega} =$$

$$\vec{n}^0(t) = -(\cos\omega t; \sin\omega t; 0)$$

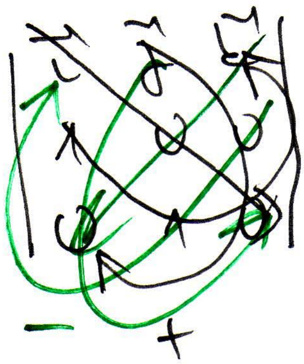
binormale:

$\vec{i} \times \vec{j} = \vec{k}$



$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_z)\vec{j} + (a_x b_y - a_y b_x)\vec{k}$$

$\vec{i} \times \vec{j} =$

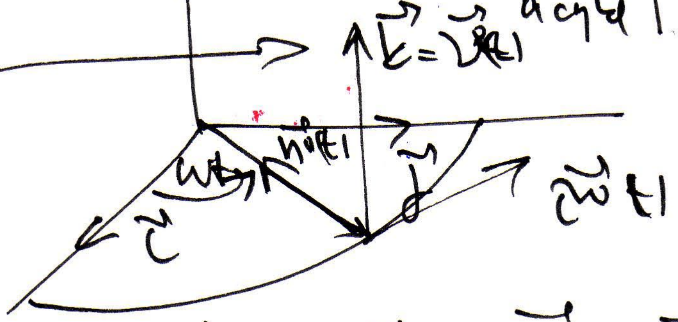


$x \to y$ $\quad \vec{i} \to \vec{j}$

$y \to z$ $\quad \vec{j} \to \vec{k}$

$z \to x$ $\quad \vec{k} \to \vec{i}$

$$\vec{k} = \vec{i} \times \vec{j}\Big|_{a\,\text{cykl}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1)$$



$$\vec{i} \times \vec{j} = \vec{k} \qquad \vec{j} \times \vec{k} = \vec{i}$$

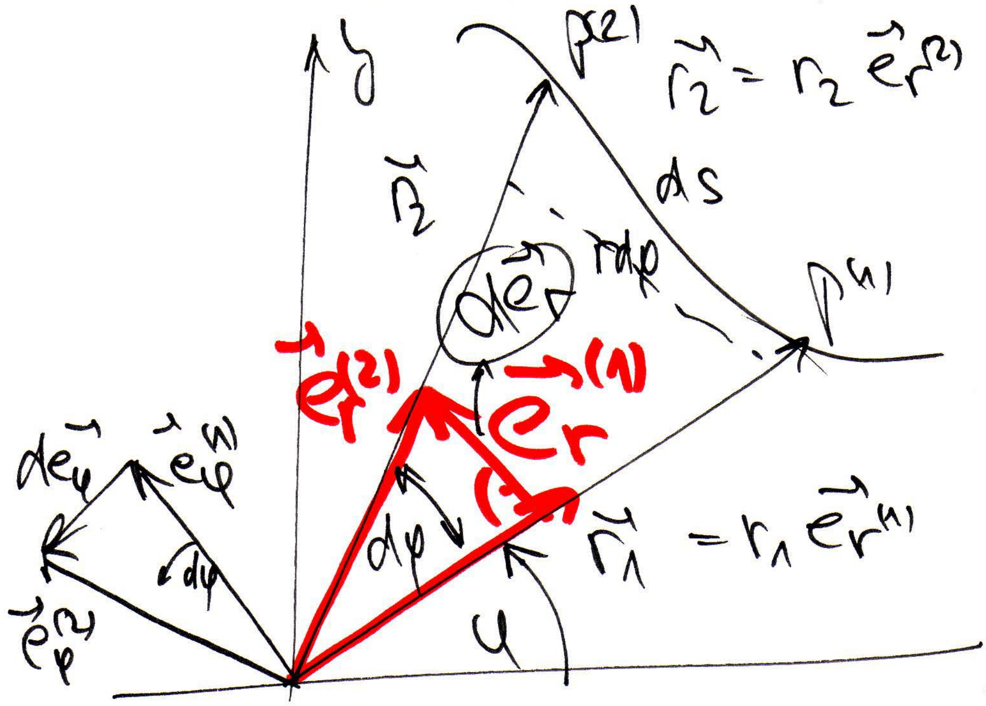
$$|\vec{a} \times \vec{b}| = c = |\vec{a}|\,|\vec{b}| \sin \sphericalangle(\vec{a},\vec{b})$$

$$\vec{\nu}^0(t) = \vec{c}^0(t) \times \vec{n}^0(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin\omega t & \cos\omega t & 0 \\ -\cos\omega t & -\sin\omega t & 0 \end{vmatrix} =$$

$$= \sin^2\omega t\,\vec{k} + \cos^2\omega t\,\vec{k} = \vec{k}$$

DÚ ditto pro „šroubovaný" pohyb

[odbočka, nepovinná, rozširujúca (ostatný)]



$(\vec{i}, \vec{j}, \vec{k}) \to$

$(\vec{e}_r, \vec{e}_\varphi, \vec{e}_z)$ $(\vec{k})$

$$d\vec{e}_r = \vec{e}_r^{(2)} - \vec{e}_r^{(1)} = d\varphi\ \vec{e}_\varphi$$

$$d\vec{e}_\varphi = \vec{e}_\varphi^{(2)} - \vec{e}_\varphi^{(1)} = d\varphi\ (-\vec{e}_r)$$

$$d\vec{e}_r = d\varphi\ \vec{e}_\varphi \qquad d\vec{e}_\varphi = -d\varphi\ \vec{e}_r$$

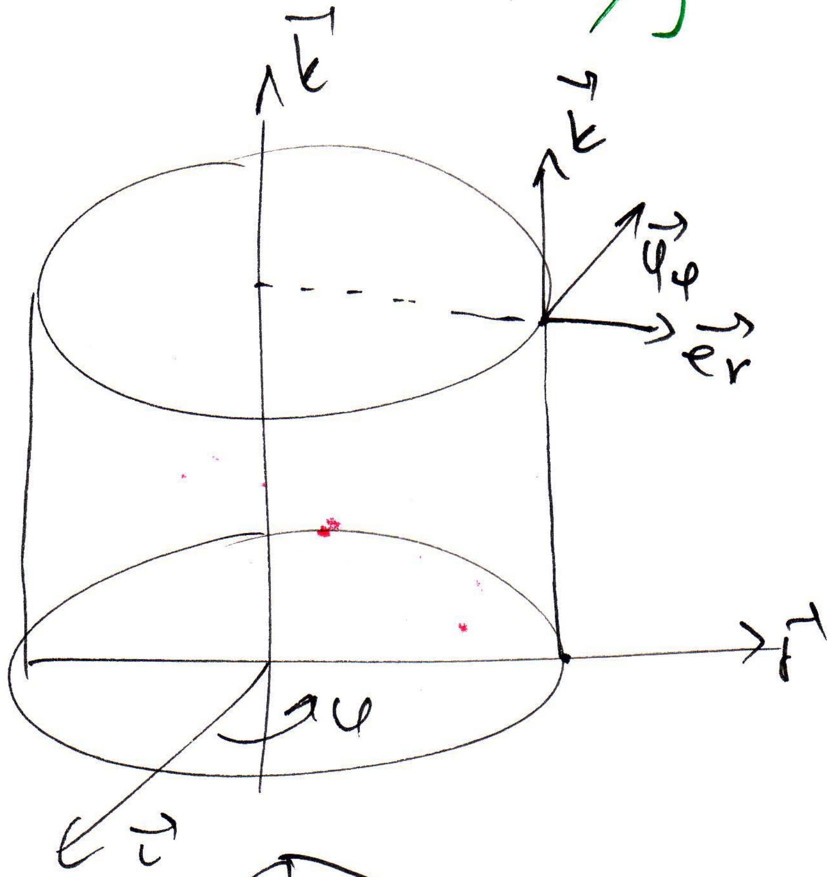
$$\mathring{\vec{e}}_r = \frac{d\vec{e}_r}{dt} = \frac{d\varphi}{dt}\vec{e}_\varphi \qquad \frac{d\vec{e}_\varphi}{dt} = \mathring{\vec{e}}_\varphi = -\frac{d\varphi}{dt}\vec{e}_r$$

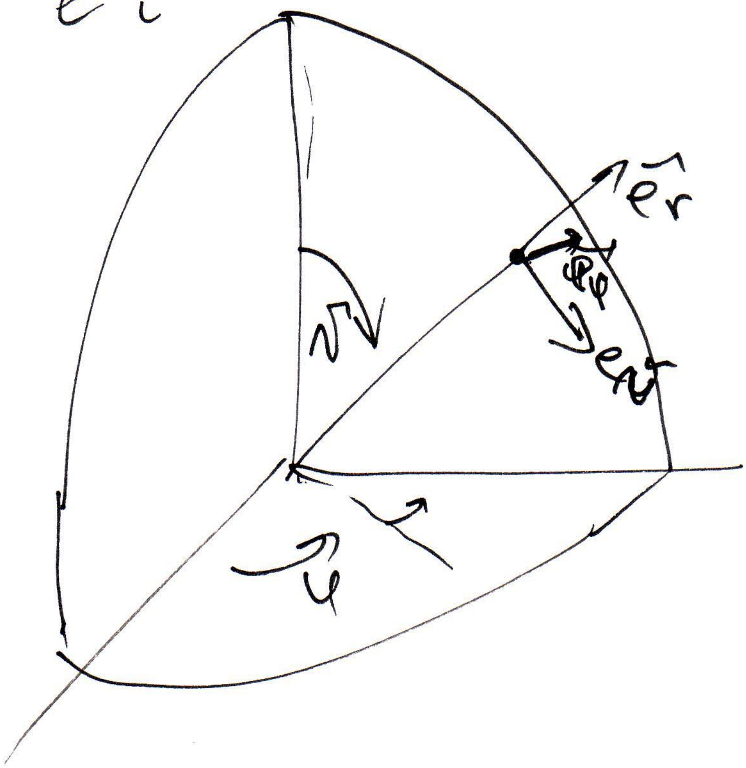
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\vec{e}_r) = \mathring{r}\vec{e}_r + r\mathring{\vec{e}}_r =$$

$$= \mathring{r}\vec{e}_r + r\mathring{\varphi}\vec{e}_\varphi$$

$$|\vec{v}(t)| = \sqrt{\mathring{r}^2 + (r\mathring{\varphi})^2}$$

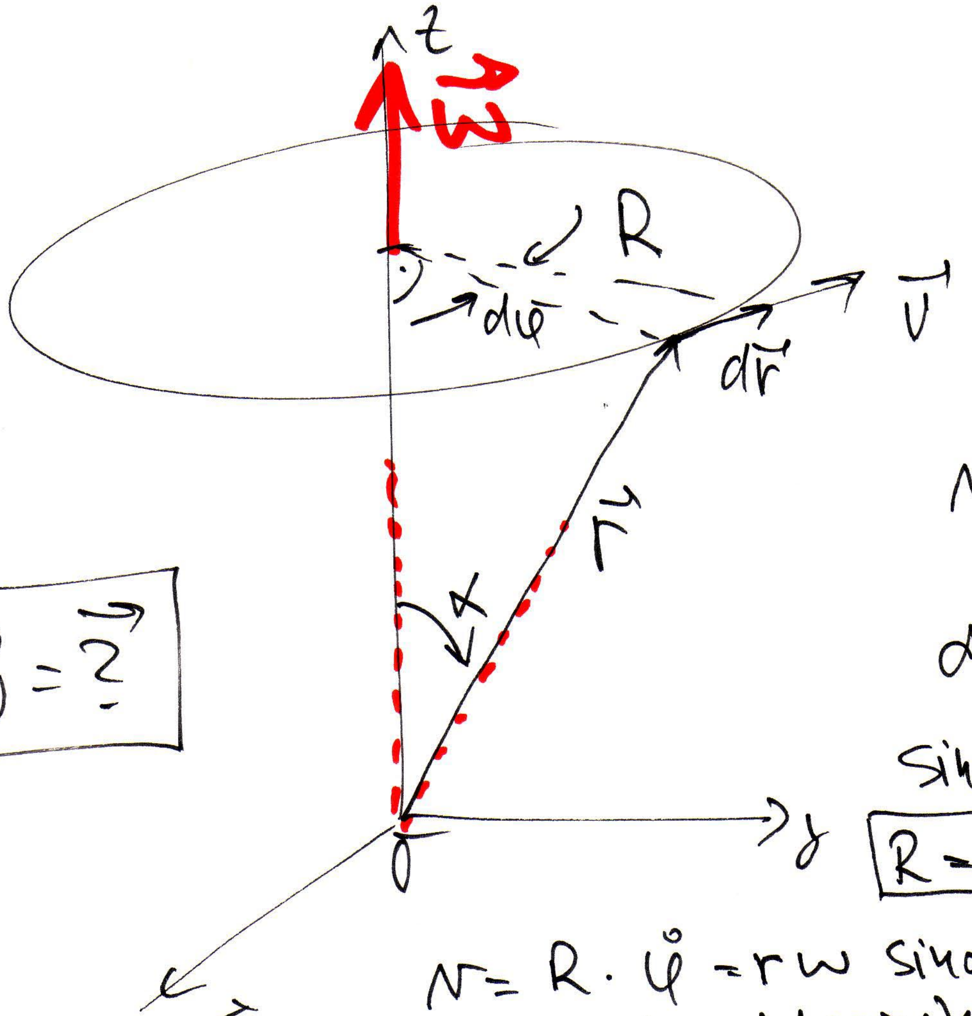
$$\vec{a}(t) = ?$$





konec odsozy

Zavést pojem <u>úhlová rychlost</u> $\vec{\omega} = \frac{d\vec{\varphi}}{dt}$



$$\boxed{\vec{\omega} = ?}$$

úhlová rychlost ω

$$v = \frac{dr}{dt} = R\frac{d\varphi}{dt}$$

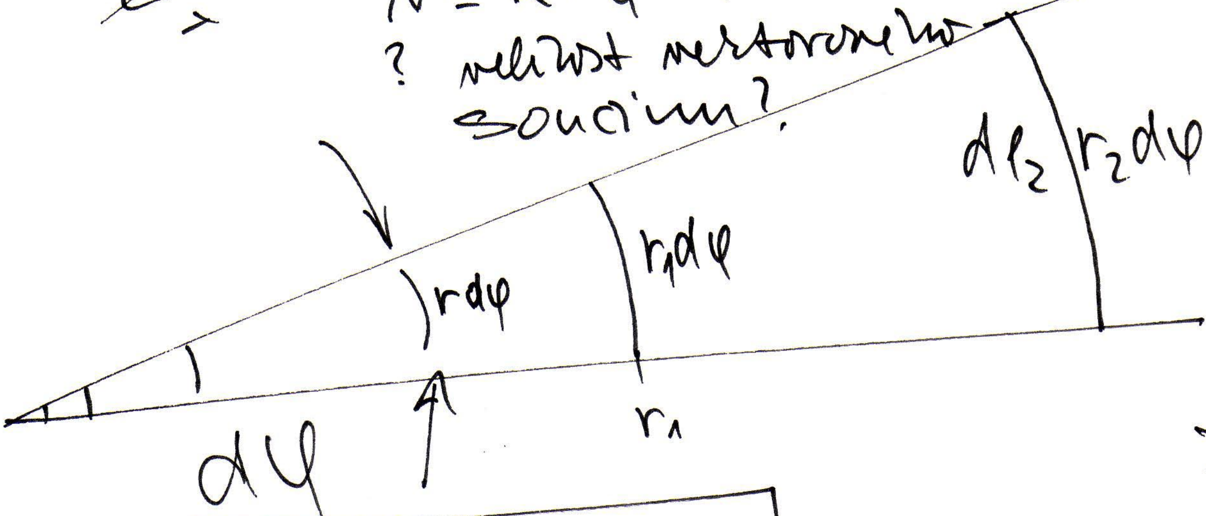
$$dr = R\,d\varphi$$

$$\sin\alpha = \frac{R}{r} \Rightarrow$$

$$\boxed{R = r\sin\alpha}$$

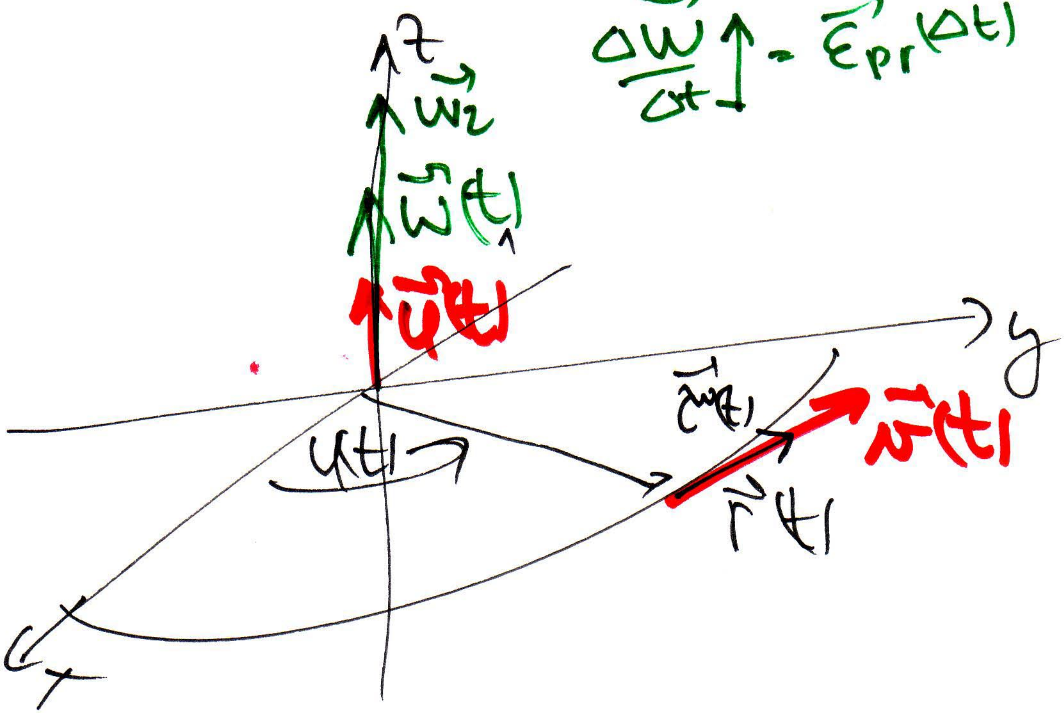
$$v = R\cdot\dot{\varphi} = r\omega\sin\alpha$$

? velikost vektorového součinu?



$$\boxed{\vec{v}(t) = \vec{\omega}(t) \times \vec{r}(t)} \qquad |\vec{v}(t)| = \omega r \sin\alpha$$

$$\frac{\Delta \vec{\omega}}{\Delta t} \uparrow = \vec{\varepsilon}_{pr}(\Delta t)$$



$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{\omega}(t) \times \vec{r}(t)\right) =$$

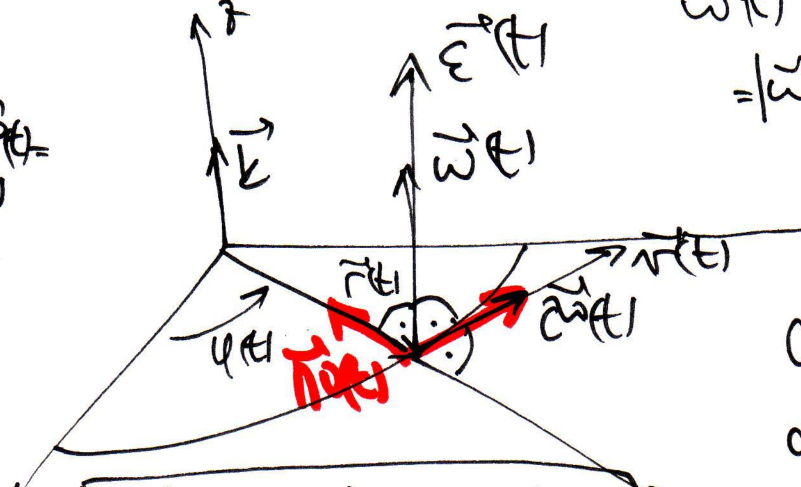
$$= \dot{\vec{\omega}}(t) \times \vec{r}(t) + \vec{\omega}(t) \times \dot{\vec{r}}(t) =$$

$\vec{\varepsilon}(t)$ úhlové zrychlení $= \frac{d\vec{\omega}}{dt} = \lim_{\Delta t \to 0} \vec{\omega}_{pr}(\Delta t) = \ldots\ldots$

$$\vec{\varepsilon}(t) \times \vec{r}(t) =$$

$$|\vec{\varepsilon}(t) \times \vec{r}(t)|\, \vec{\tau}(t) =$$

$$= \underbrace{\varepsilon(t)\, R\, \vec{\tau}(t)}_{\vec{a}_{t}(t)\ \text{tečné}}$$



$$\vec{\omega}(t) \times \dot{\vec{r}}(t) =$$

$$= |\vec{\omega}(t)|\,|\vec{v}(t)| \sin 90^\circ\, \vec{n}(t)$$

$$a_t(t) = R\,\varepsilon(t)$$

$$a_n(t) = \omega^2(t)\, R$$

$$= \frac{v^2(t)}{R}$$

$$\boxed{\vec{a}(t) = \vec{a}_t(t) + \vec{a}_n(t)}$$

$\vec{r}(t)$