Dobrý den!

30. září 2009

3. přednáška

středa

úvod:

⌈ středoškolská „kinematika" velmi rychle & radostně během 10 minut ⌋

☺ $\vec{r}(t) \xrightarrow{\text{derivace}} \vec{v}(t) = \dot{\vec{r}}(t) \xrightarrow{\text{derivace}} \vec{a}(t) = \dot{\vec{v}}(t)$

známe

derivace

$\vec{a}(t) = \ddot{\vec{r}}(t)$

☹ známe $\vec{a}(t) \xrightarrow{\text{integrace}} \vec{v}(t) \xrightarrow{\text{integrace}} \vec{r}(t)$

!! ⊕ počáteční podmínky !!

⌈ příklad diference

$$y = x^2 \quad \ldots \quad dy = 2x\,dx \quad \ldots \quad \frac{dy}{dx} = 2x = y'$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} \Rightarrow d\vec{r} = \vec{v}(t)\,dt \Rightarrow \int \vec{v}(t)\,dt = \int d\vec{r}$$

$$\int_{t=0}^{t=t} \vec{v}(t')\,dt' = \int_{\vec{r}_0=\vec{r}(0)}^{\vec{r}(t)} d\vec{r} = \int_{(r_{0x},r_{0y},r_{0z})}^{(x(t),y(t),z(t))} \left(dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}\right)$$

skalár

1 vektorová rovnice = 3 skalární

$$\vec{a} = \vec{b} \Rightarrow a_x = b_x,\quad a_y = b_y,\quad a_z = b_z$$

$$\int_0^t \left(v_x(t')\,\vec{i} + v_y(t')\,\vec{j} + v_z(t')\,\vec{k} \right) dt' =$$

$$= \underbrace{\int_0^t v_x(t')\,\vec{i}\,dt'} + \underbrace{\int_0^t v_y(t')\,\vec{j}\,dt'} + \underbrace{\int_0^t v_z(t')\,\vec{k}\,dt'}$$

$$= \underbrace{\int_{x_0}^{x(t)} dx\,\vec{i}} + \underbrace{\int_{y_0}^{y(t)} dy\,\vec{j}} + \underbrace{\int_{z_0}^{z(t)} dz\,\vec{k}}$$

tedy

$$\underbrace{\int_{x_0}^{x(t)} dx = \int_0^t v_x(t')\,dt'}$$

$$\underbrace{\int_{y_0}^{y(t)} dy = \int_0^t v_y(t')\,dt'}$$

$$\underbrace{\int_{z_0}^{z(t)} dz = \int_0^t v_z(t')\,dt'}$$

$$\int_0^{x(t)} dx = \underbrace{x(t) - x_0 = \int_0^t v_x(t')\,dt'}$$

$$\boxed{\begin{aligned} x(t) &= \int_0^t v_x(t')\,dt' + x_0 \\ y(t) &= \int_0^t v_y(t')\,dt' + y_0 \\ z(t) &= \int_0^t v_z(t')\,dt' + z_0 \end{aligned}}$$

DÚ
pro
$\vec{a}(t)$ a $\vec{v}(t)$
$\left(v_x(t) = \int_0^t a_x(t')\,dt' + v_{0x}\right)$

pr. ①

$x(t) = 0$

$y(t) = -\frac{g}{2}t^2 + h$

$z(t) = 0$

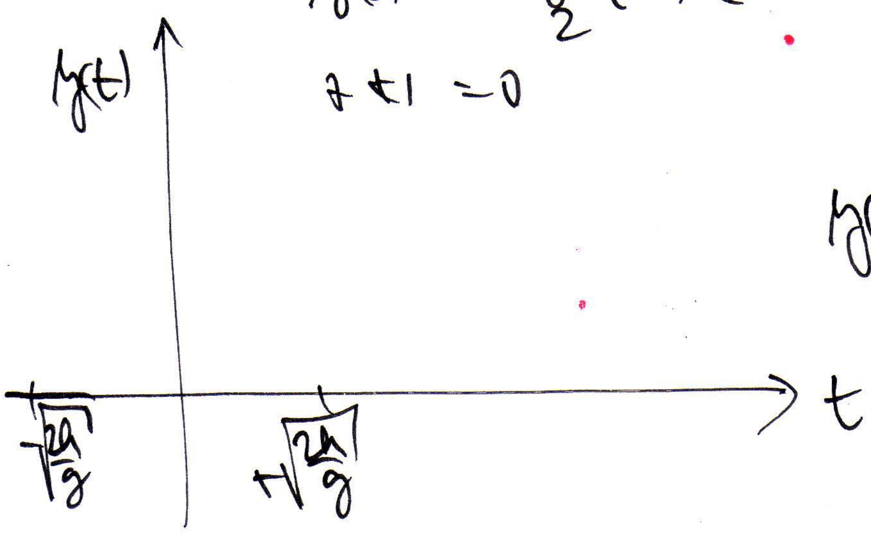
poč. podmínky

$x(0) = ? = 0$ $\qquad \dot{x}(0) = 0$

$y(0) = ? = h$ $\qquad \dot{y}(0) = -gt$

$z(0) = 0$ $\qquad \dot{z}(0) = 0$

$y(t) = -\frac{g}{2}t^2 + h$



Nová verze – 4./8 stránky

Mějme

i) $y(t) = -\frac{g}{2}t^2 + h$ graf fce: ?:

$y - n = (x - m)^2$

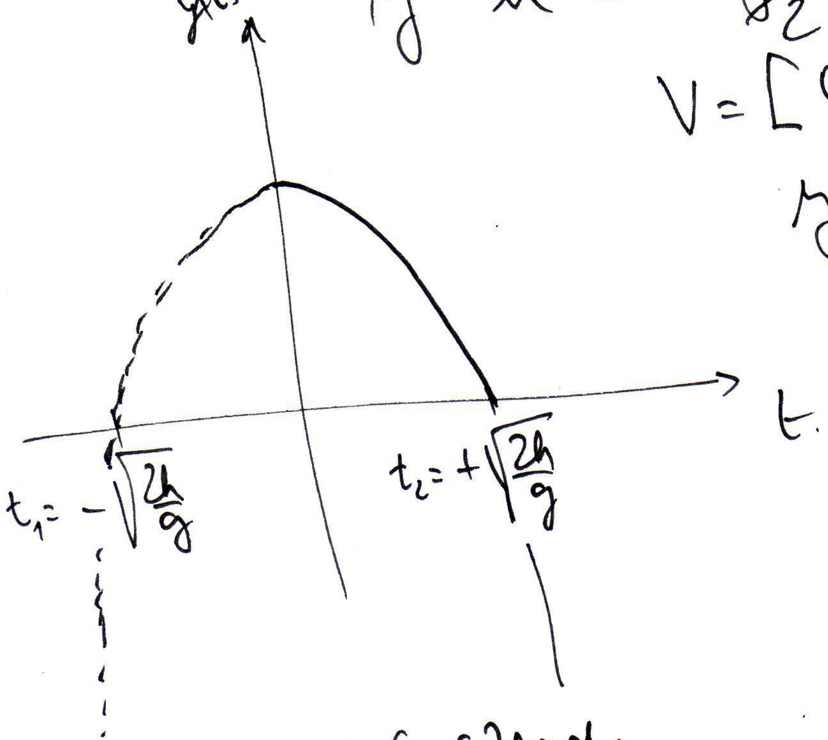
$V = [m, n]$

$y - h = -\frac{g}{2}t^2$

$V = [0, h]$

$y(t_{1,2}) = 0$

$t_{1,2} = ?\ \pm\sqrt{\frac{2h}{g}}$



druhý příklad:

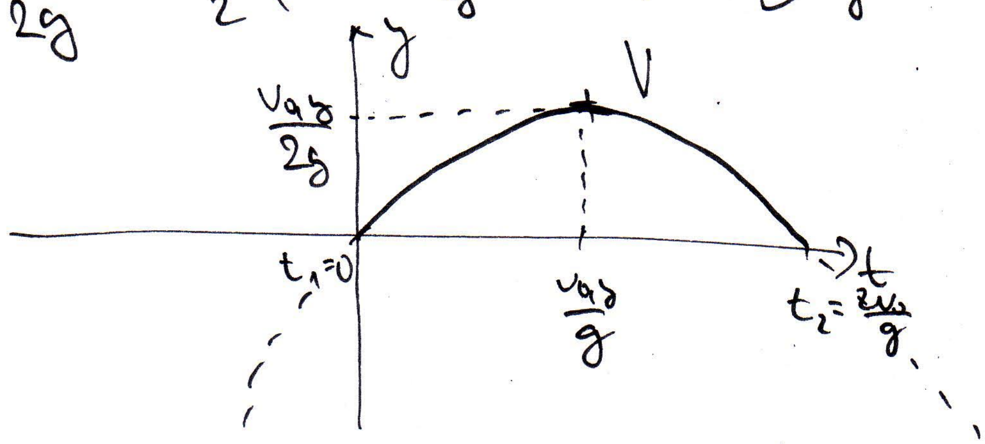
ii) $y(t) = -\frac{g}{2}t^2 + v_{0,y}t = -\frac{g}{2}\left[t^2 - \frac{2v_{0,y}t}{g}\right] =$

$= -\frac{g}{2}\left[\left(t - \frac{v_{0,y}}{g}\right)^2 - \frac{v_{0,y}^2}{g^2}\right] =$

$= -\frac{g}{2}\left(t - \frac{v_{0,y}}{g}\right)^2 + \frac{v_{0,y}}{2g}$

tedy

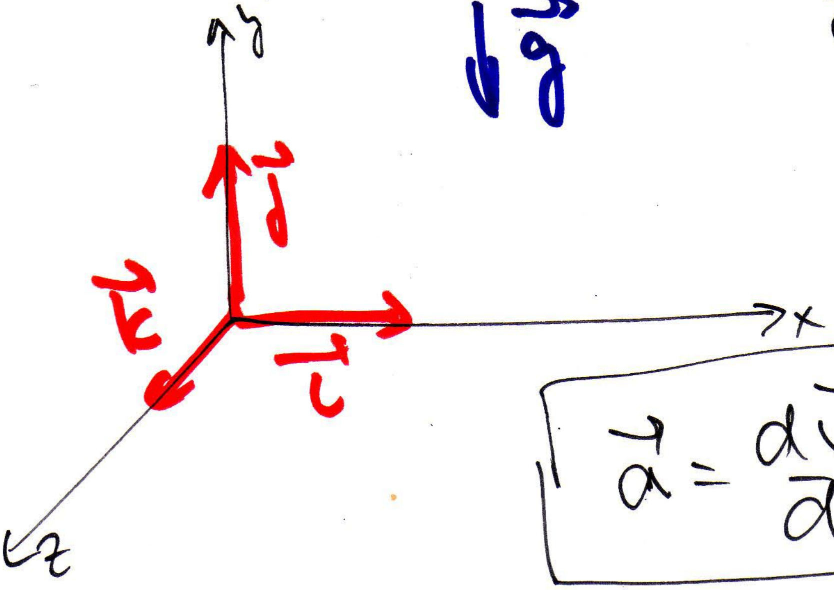
$y - \frac{v_{0,y}}{2g} = -\frac{g}{2}\left(t - \frac{v_{0,y}}{g}\right)^2$ $V = \left[\frac{v_{0,y}}{g} ; \frac{v_{0,y}}{2g}\right]$



Pohyb HmB s $\boxed{\vec{a} = (0, -g, 0)}$

$\downarrow \vec{g}$

$$\vec{g} = (\vec{g}\cdot\vec{i}\,;\ \vec{g}\cdot\vec{j}\,;\ \vec{k}\cdot\vec{g}) = (0,\ g\cdot\cos 180^\circ,\ 0) = (0, -g, 0)$$



$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}} \qquad \vec{r}(t) = \boxed{\vec{v}(t) = \frac{d\vec{r}}{dt}}$$

$$a_x = 0 = \frac{dv_x(t)}{dt} \Rightarrow v_x(t) = v_x = A = \frac{dx(t)}{dt} \Rightarrow x(t) = At + D$$

$$a_y = -g = \frac{dv_y(t)}{dt} \Rightarrow v_y(t) = -gt + B = \frac{dy(t)}{dt} \Rightarrow y(t) = -\frac{g}{2}t^2 + Bt + F$$

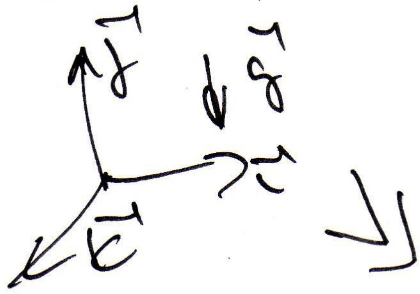
$$a_z = 0 = \frac{dv_z(t)}{dt} \Rightarrow v_z(t) = v_z = C = \frac{dz(t)}{dt} \Rightarrow z(t) = Ct + G$$

$$\vec{r}(t) = \left(At + D;\ -\frac{g}{2}t^2 + Bt + F;\ Ct + G\right)$$

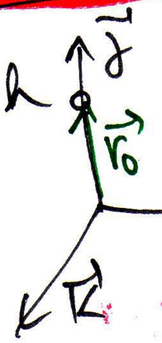
$$\boxed{\begin{aligned}(A, B, C) &= \vec{v}_0 \\ (D, F, G) &= \vec{r}_0\end{aligned}}$$

m/s $(\vec{v}(0))$ r !

m $(\vec{r}(0))$



VOLNÝ PÁD



$\vec a = \vec g$

$\vec v_0 = (0, 0, 0) = (A, B, C)$

$\vec r_0 = (0, h, 0) = (D, F, G)$

$\vec r(t) = \left(At + D;\ -\frac{g}{2}t^2 + Bt + F;\ Ct + G\right)$

$\vec r(t) = \left(0;\ -\frac{g}{2}t^2 + h;\ 0\right)$ $\quad \dot{\vec r}(t) = \vec v(t) = (0, -gt, 0)$

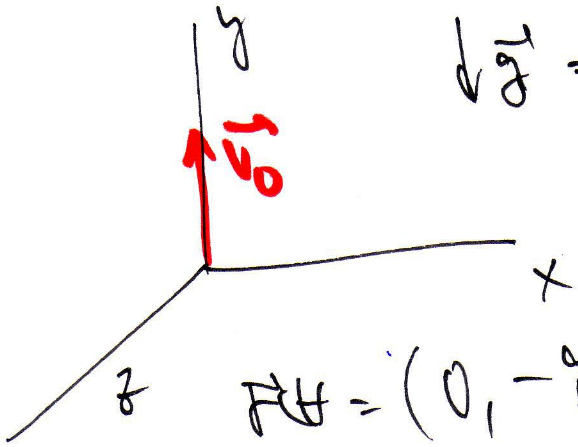
i) doba dopadu: $t_d = ?$ … $y(t_d) = 0$

$$-\frac{g}{2}t_d^2 + h = 0 \Rightarrow t_d = \pm\sqrt{\frac{2h}{g}}$$

SVISLÝ VRH

$\vec r_0 = (\overset{A}{0}, \overset{B}{0}, \overset{C}{0})$

$\vec g = \vec a$ $\quad \vec v_0 = (\underset{D}{0}, \underset{F}{v_0}, \underset{G}{0})$



$\vec r(t) = \left(0, -\frac{g}{2}t^2 + v_0 t;\ 0\right) \Rightarrow \dot{\vec r}(t) = (0, -gt + v_0, 0)$

doba výstupu $t_v = ?$ — doba dopadu $t_d = ?$

hledám extrém fce $y(t)$

$$y_{max} = \frac{dy(t)}{dt}\Big|_{t_v}$$

$\boxed{\dot y(t_v) = 0}$

$-g t_v + v_0 = 0 \Rightarrow \boxed{t_v = \frac{v_0}{g}}$

$7/8$

t dopadu: ?

$y(t_d) = 0$

$-\frac{g}{2}t_d^2 + v_0 t_d = 0 \rightarrow t_d\underbrace{\left(-\frac{g}{2}t_d + v_0\right)}_{0} = 0$

$\underline{\underline{t_{d,1} = 0}}$ $\underline{\underline{t_{d,2} = \frac{2v_0}{g}}}$

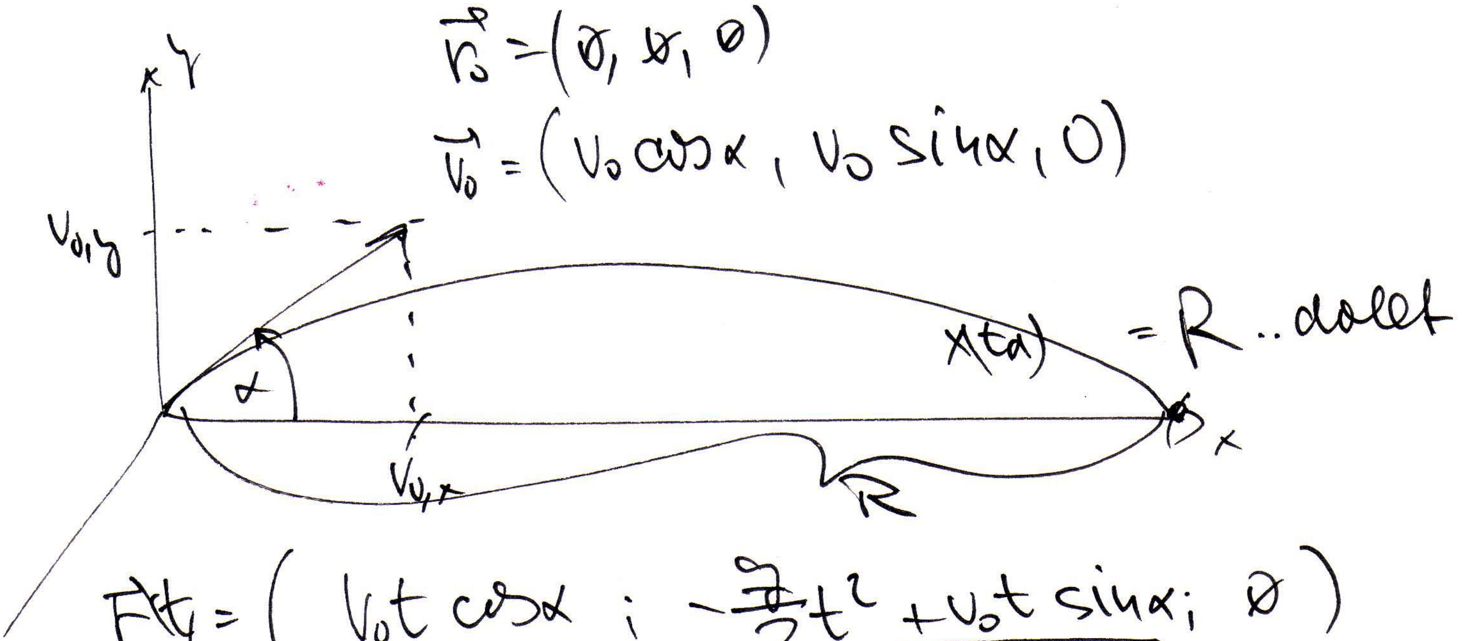
$y(t_d) = 0 = y(2t_v)$

VODOROVNÝ VRH, DÚ

ŠIKMÝ VRH, www.physics.fme.vutbr.cz

$\vec{r}_0 = (0, 0, 0)$

$\vec{v}_0 = (v_0 \cos\alpha, v_0 \sin\alpha, 0)$



$\vec{r}(t) = \left(v_0 t \cos\alpha \,;\, -\frac{g}{2}t^2 + v_0 t \sin\alpha \,;\, 0 \right)$

$t_v = ?$ $\dot{y}(t_v) = 0 \Rightarrow \boxed{t_v = \frac{v_0 \sin\alpha}{g}}$

$t_d = ?$ $y(t_d) = 0 \Rightarrow t_d = ; 2t_v) : -\frac{g}{2}t_d^2 + v_0 t_d \sin\alpha = 0$

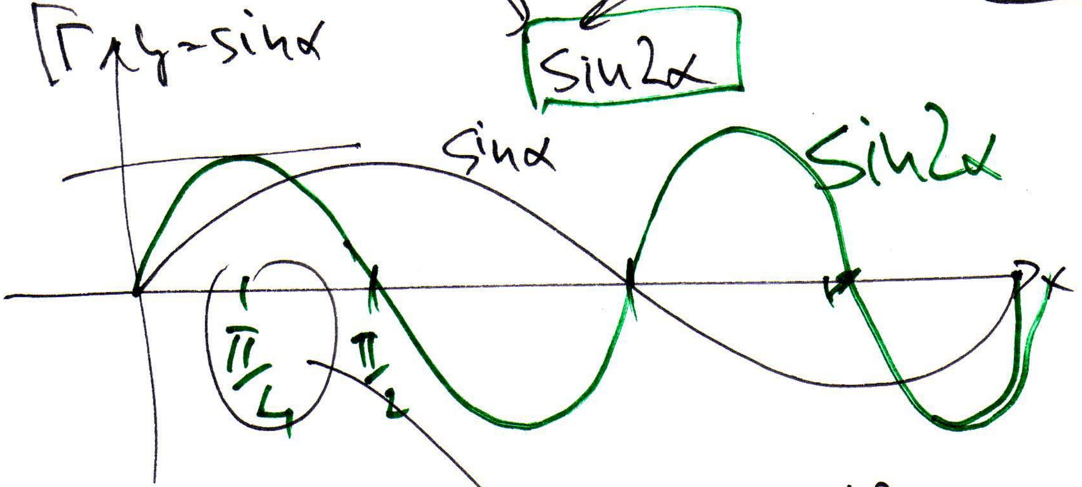
$\boxed{t_d = \frac{2v_0 \sin\alpha}{g}}$

Dolet R:

$$x(t_d) = x\left(\frac{2v_0\sin\alpha}{g}\right) = v_0\overbrace{\frac{2v_0\sin\alpha}{g}}^{t}\cos\alpha =$$

$$= 2\frac{v_0^2}{g}\underbrace{\sin\alpha\cos\alpha} = \boxed{\frac{v_0^2}{g}\sin 2\alpha}$$

$\boxed{\sin 2\alpha}$



$\Downarrow$

$R_{MAX} = ?$ $R(\alpha): \left.\frac{dR}{d\alpha}\right|_{\alpha_{max}} = 0$

$$\frac{dR}{d\alpha} = \frac{v_0^2}{g}\cos 2\alpha \cdot 2 \qquad \cos 2\alpha_M = 0$$

$2\alpha_M = \frac{\pi}{2}$ i ....

$$\boxed{\alpha_M = \frac{\pi}{4}}$$

i) maximální dolet

ii) symetrie R kolem $\pi/4$