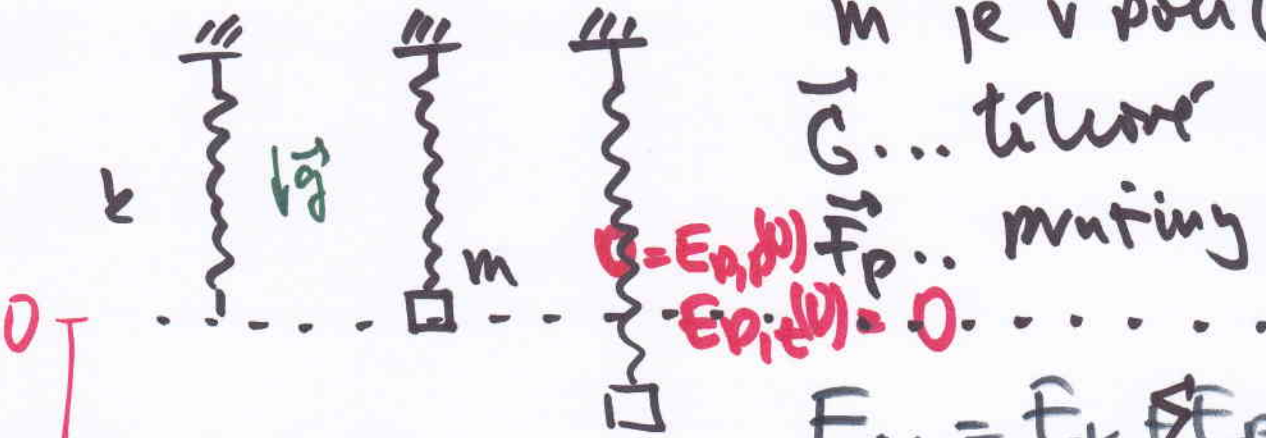
23. 10. 09

10. prednáška

Dobrý deň!

Pr.:



m je v poli(ach).

$\vec{G}$ ... tiažové $E_{p,t}$

$\vec{F}_p$ .. pružiny $E_{p,p}$

$$E_M = E_k + \sum E_p = E_k + E_{p,p} + E_{p,t}$$

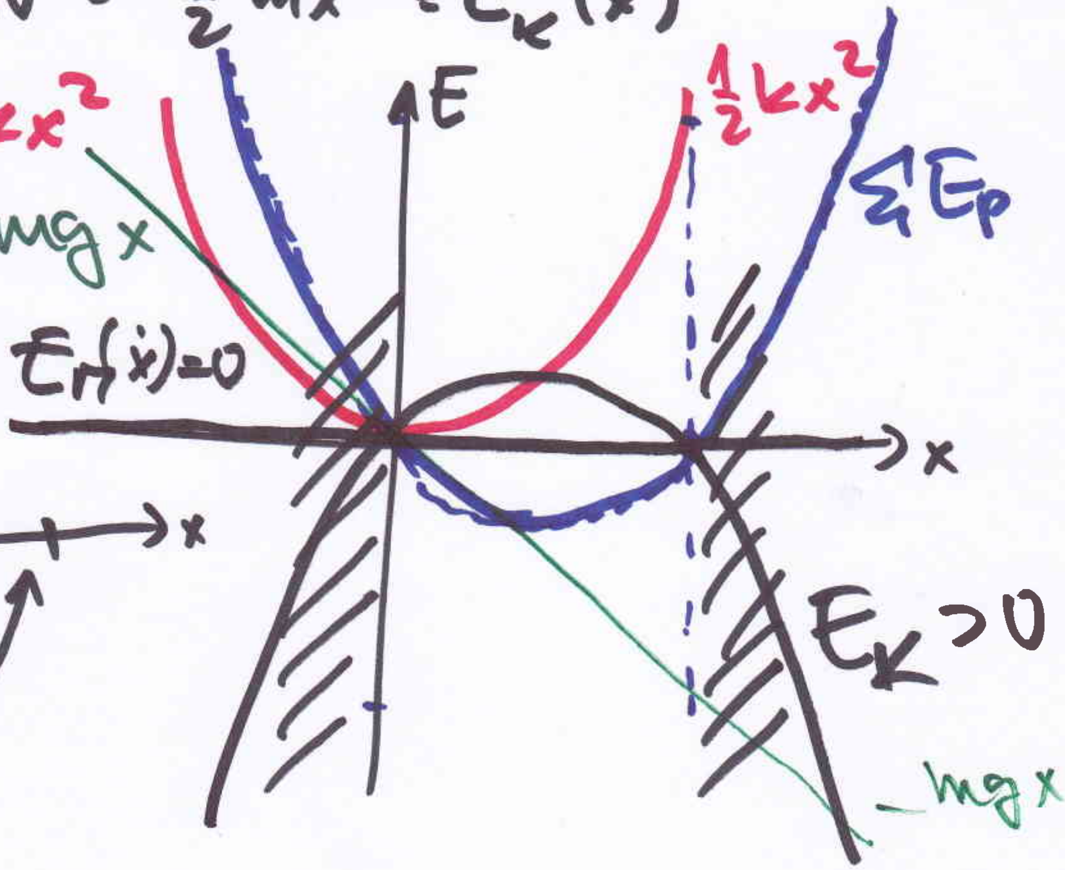
$$\boxed{\Delta E_M = W_{\vec{F}_j,\, I \to II}}$$

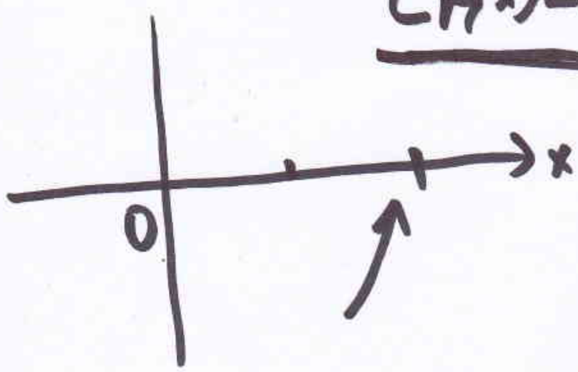
? $E_M = \text{konšt}$?

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{x}^2 = E_k(\dot{x})$$

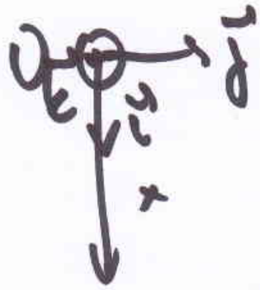
$$E_{p,p}(x) = \frac{1}{2} k x^2$$

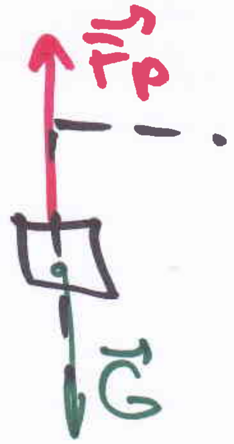
$$E_{p,t}(x) = -mgx$$





odbočka





$$\vec{F}_V = m\vec{a}$$

$$\vec{F}_p + \vec{G} = m\vec{a}$$

Počáteční podmínky:

$$\vec{v}(t=0) = \vec{v}_0 = (0,0,0)$$

$$\vec{r}_0 = \vec{r}(t=0) = (0,0,0)$$

x:

$$-kx + mg = m\ddot{x}$$

$$m\ddot{x} + kx = mg$$

$$\boxed{\ddot{x} + \frac{k}{m}x = g}$$

$$\ddot{x}_0 + \omega^2 x_0 = 0$$

$$x_0(t) = A\cos(\omega t + \alpha)$$

Předp.:

$$x(t) = x_0(t) + C \qquad C = ?$$

$$\dot{x}(t) = \dot{x}_0 \; ; \; \ddot{x} = \ddot{x}_0$$

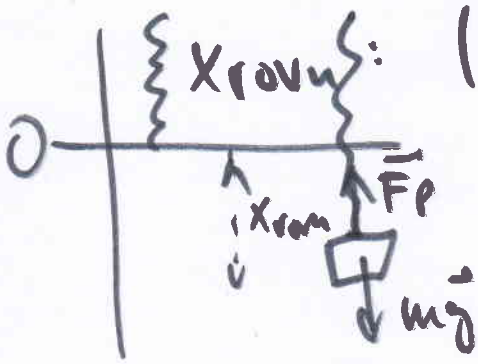
$$\ddot{x}(t) + \omega^2 x = g \Rightarrow$$

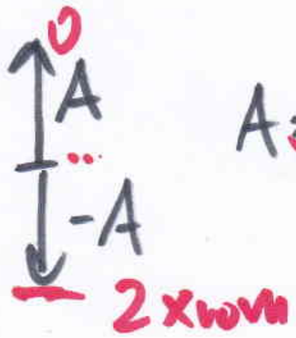
$$\ddot{x}_0 + \omega^2(x_0 + C) = g \Rightarrow \underbrace{\ddot{x}_0 + \omega^2 x_0}_{0} + \omega^2 C = g$$

$$\boxed{C = \frac{g}{\omega^2}}$$

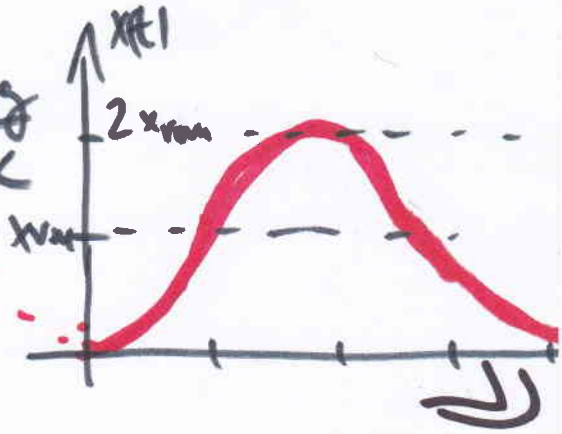
TF ODHAD STUDENTA:

$$x_{rovn}: \quad |\vec{F}_p| = |\vec{G}| \Rightarrow x_{rovn} = \frac{mg}{k}$$





$$A = \frac{mg}{k}$$



teory

$$x(t) = x_0(t) + C = A\sin(\omega t + \alpha) + \frac{g}{\omega^2}$$

P.p.:

$$x(0) = 0 = A\sin\alpha + \frac{g}{\omega^2}$$

$$\dot{x}(0) = 0 \Rightarrow A\omega\cos(\underbrace{\omega\cdot 0 + \alpha}_{\pi/2}) = 0$$

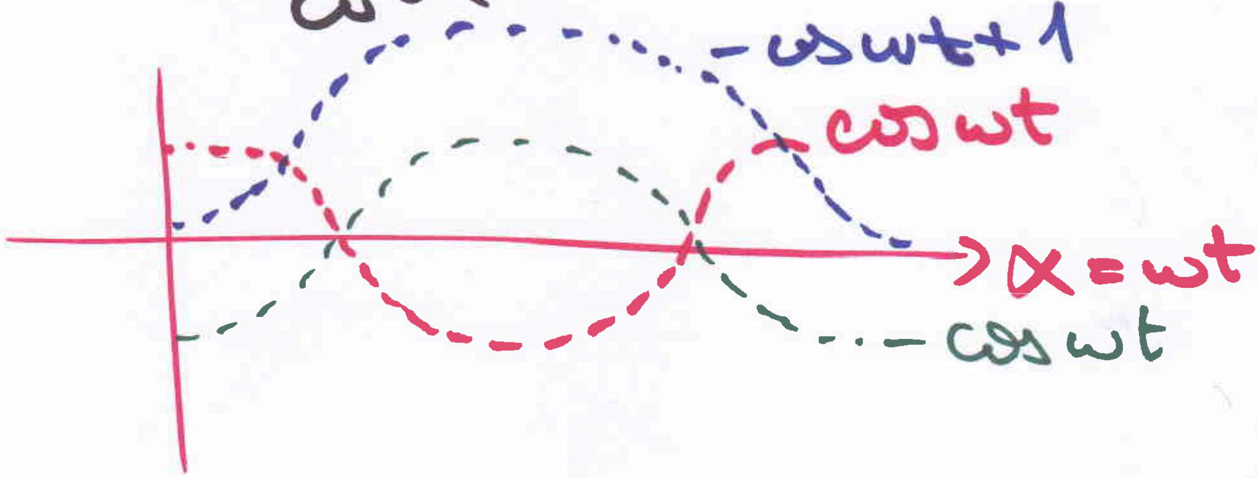
$$0 = A\sin\frac{\pi}{2} + \frac{g}{\omega^2}$$

$$A = -\frac{g}{\omega^2}$$

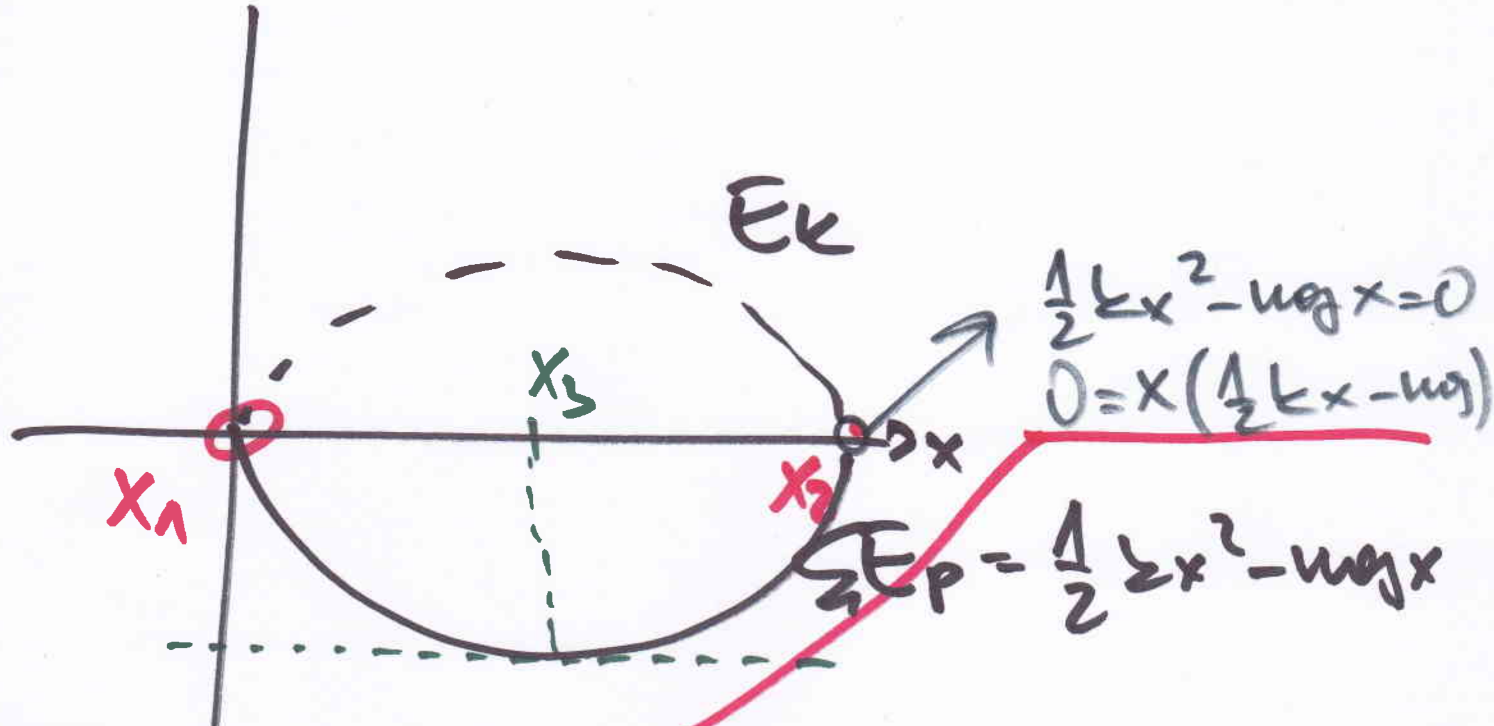
$$x(t) = -\frac{g}{\omega^2}\sin(\omega t + \pi/2) + \frac{g}{\omega^2}$$

$$= -\frac{g}{\omega^2}\cos\omega t + \frac{g}{\omega^2}$$

$$= \frac{g}{\omega^2}(1 - \cos\omega t)$$



Hledání „význačných" bodů



x_1, x_2: body obratu ($E_k = 0$)

$x_1 = 0$

$\frac{1}{2}kx_2 = mg \Rightarrow x_2 = \frac{2mg}{k}$

x_3: $\frac{d}{dx}\left(\Sigma E_p\right)\Big|_{x_3} = 0 \ \dots \ \frac{d}{dx}\left(\frac{1}{2}kx^2 - mgx\right) =$

$\Rightarrow kx_3 - mg = 0$

$x_3 = \frac{mg}{k}$

Známe „kuchařku", jak od $\vec{F}_k$ (síly) přejít k poli (potenciálnímu)

$\vec{F}_k \rightarrow E_p$

$$E_p$$

a)

$$W_{\vec{F}_k} = \int_{I}^{II} \vec{F}_k \cdot d\vec{r} = -\Delta E_p = E_{p,I} - E_{p,II}$$

b) referenční hladina a hodnota E_p v ní

? Existuje „kuchařka" i na:

$E_p \rightarrow \vec{F}_k$

c)

$$dE = \frac{\partial E}{\partial x} dx + \frac{\partial E}{\partial y} dy + \frac{\partial E}{\partial z} dz$$

$$\frac{\partial^2 E}{\partial x \partial y} = \frac{\partial^2 E}{\partial y \partial x} \qquad \frac{\partial^2 E_y}{\partial y \partial z} = \frac{\partial^2 E_z}{\partial z \partial y} \text{ a cyklí}$$

$$\int dE = E_2 - E_1$$

Souvislost s $\vec{F}_k$:

$$\int \vec{F}_k \cdot d\vec{r} = E_1 - E_2 = -\int_{I}^{II} dE$$

$$-dE = \vec{F}_k \cdot d\vec{r}$$

$$-\left(\frac{\partial E}{\partial x}dx + \frac{\partial E}{\partial y}dy + \frac{\partial E}{\partial z}dz\right) = \left(\overset{E}{\frac{\partial}{\partial x}}; \overset{E}{\frac{\partial}{\partial y}}; \overset{E}{\frac{\partial}{\partial z}}\right) \cdot (dx, dy, dz)$$

$$= -\vec{\nabla} E \cdot d\vec{r}$$

nabla: $\vec{\nabla} = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\right) =$

$$= \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\int_{I}^{II} \vec{F}_k \cdot d\vec{r} = \int_{I}^{II} (-\vec{\nabla} E) \cdot d\vec{r} = -\int_{I}^{II} dE = E_I - E_{II}$$

$$\oint \vec{F}_k \cdot d\vec{r} = 0 \quad \ldots\ldots \quad \boxed{\vec{F}_k = -\mathrm{grad}\, E_p}$$

$$\vec{F}_k = -\vec{\nabla} E_p$$

uzavřená křivka

$$I = II : E_1 - E_{1=2} = 0$$

$$\boxed{\mathrm{rot}\, \vec{F}_k = 0}$$

$$\vec{\nabla} \times (-\vec{\nabla} E) = ? = -\vec{\nabla} \times \vec{\nabla} E = \vec{0}$$

$$\oint \vec{F}_k \cdot d\vec{r} = \vec{0} \quad .. \quad \mathrm{rot}\,\vec{F}_k = \vec{0} \quad .. \quad \vec{\nabla} \times \vec{F}_k = \vec{0}$$

$$\vec{\nabla} \times (-\vec{\nabla} E) = \vec{0}$$

$$-\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial E}{\partial x} & \frac{\partial E}{\partial y} & \frac{\partial E}{\partial z} \end{vmatrix} = \vec{0}.$$

$$\vec{i}\left(\frac{\partial^2 E}{\partial y \partial z} - \frac{\partial^2 E}{\partial z \partial y}\right) + \vec{j}\left(\frac{\partial^2 E}{\partial z \partial x} - \frac{\partial^2 E}{\partial x \partial z}\right) +$$

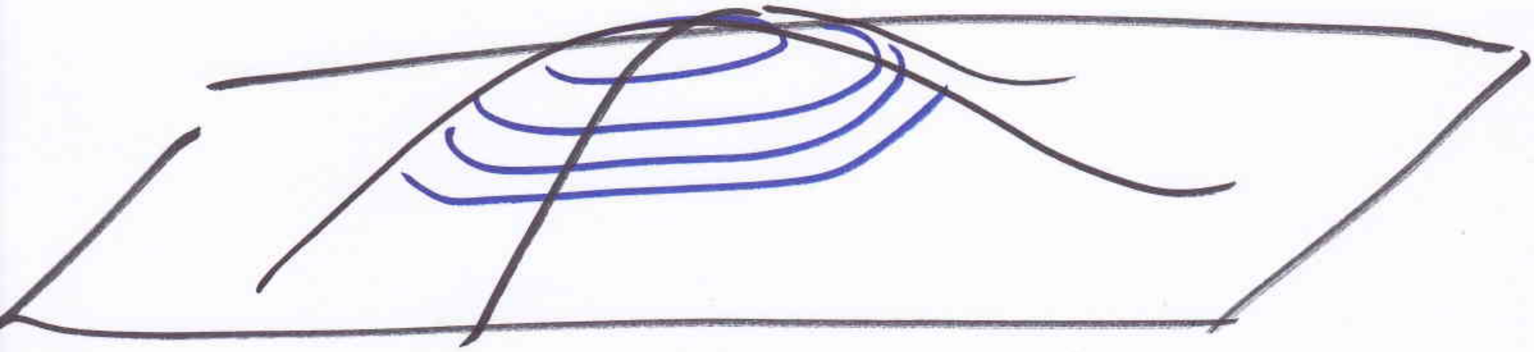
$$\vec{k}\left(\frac{\partial^2 E}{\partial x \partial y} - \frac{\partial^2 E}{\partial y \partial x}\right) = \vec{0} = (0, 0, 0)$$

Test „konzervativnosti"

$$\vec{F}_{k?} = \left(-\frac{\partial E}{\partial x}; -\frac{\partial E}{\partial y}; -\frac{\partial E}{\partial z}\right) = (F_{k,x}; \ldots; \ldots)$$

$$\frac{\partial F_{k,x}}{\partial y} = \frac{\partial F_{k,y}}{\partial x} \quad .. \quad \frac{\partial F_{k,x}}{\partial z} = \frac{\partial F_{k,z}}{\partial x} \quad \ldots$$

Trodimenzionalna „nazdla" (gradient) $\vec{\nabla}$



$\vec{\nabla}\varphi(x,y)$

$$\boxed{d\varphi = (\vec{\nabla}\varphi)\cdot d\vec{r}}$$

$$\left(\frac{\partial\varphi}{\partial x}\vec{i} + \frac{\partial\varphi}{\partial y}\vec{j} + \cancel{\frac{\partial\varphi}{\partial t}\vec{k}}\right)\cdot\left(dx\,\vec{i} + dy\,\vec{j}\right)$$

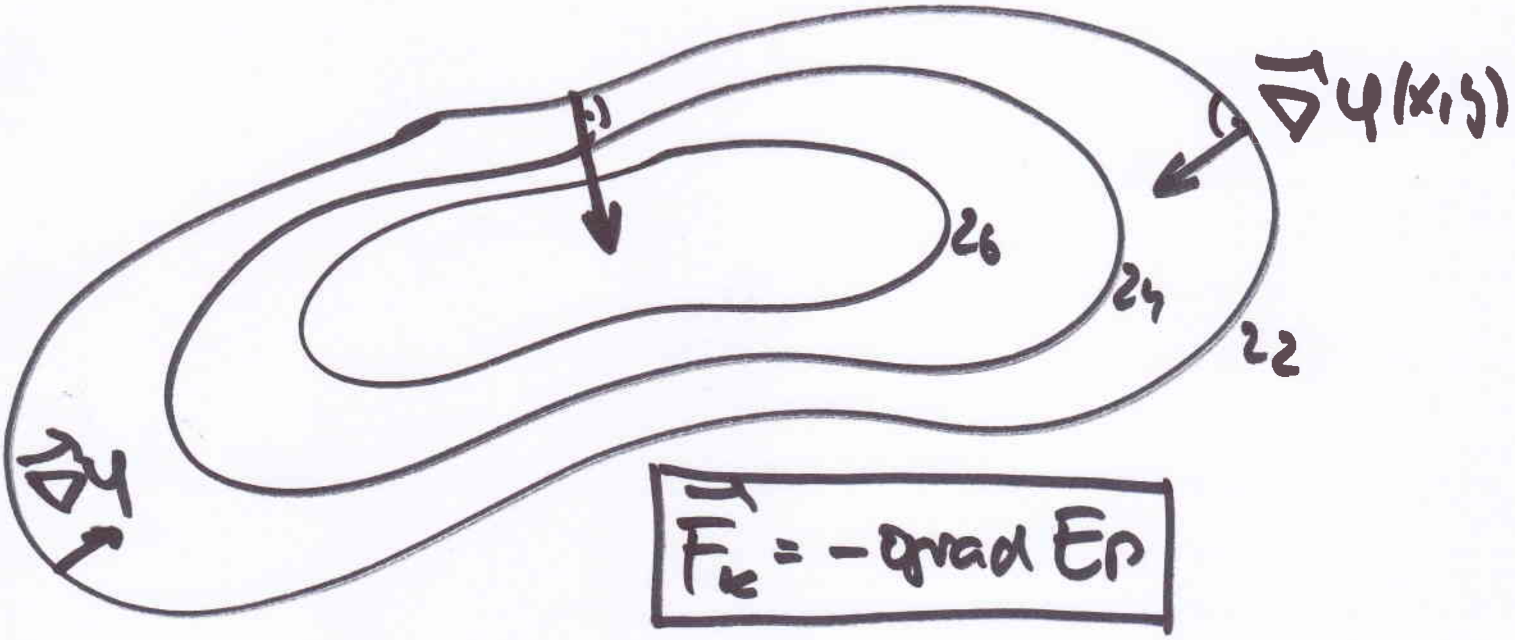
$$d\varphi = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy$$

a) $d\varphi = 0 \Rightarrow$
$d\vec{r} \perp \vec{\nabla}\varphi(x,y)$

b) $(d\varphi)_{max}$: $d\vec{r} \uparrow\uparrow \vec{\nabla}\varphi(x,y)$

$$(d\varphi)_{max} = |\vec{\nabla}\varphi|\,|d\vec{r}| = (\nabla\varphi)\, dr$$

$$\left(\frac{d\varphi}{dr}\right)_{max} = |\vec{\nabla}\varphi|$$



$$\boxed{\vec{F}_k = -\mathrm{grad}\, E_p}$$

Příklady

a) $E_{p,t} = mgy + U$ (< 18 J)

$$\vec{G} = \left(-\frac{\partial E}{\partial x}, -\frac{\partial E}{\partial y}, -\frac{\partial E}{\partial t}\right) =$$

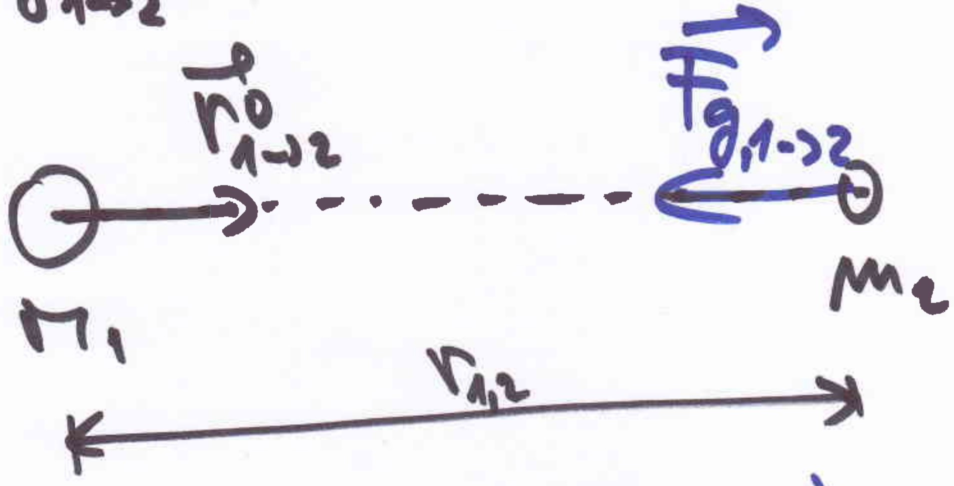
$$= (0, -mg, 0)$$

b) $E_{p,p} = \frac{1}{2} k x^2$

$$\vec{F}_p = (-kx, 0, 0)$$

Gravitační pole

$$\vec{F}_{g_{1\to2}} = -\varkappa \frac{m_1 m_2}{r^2} \vec{r}^{\,0}_{1\to2}$$



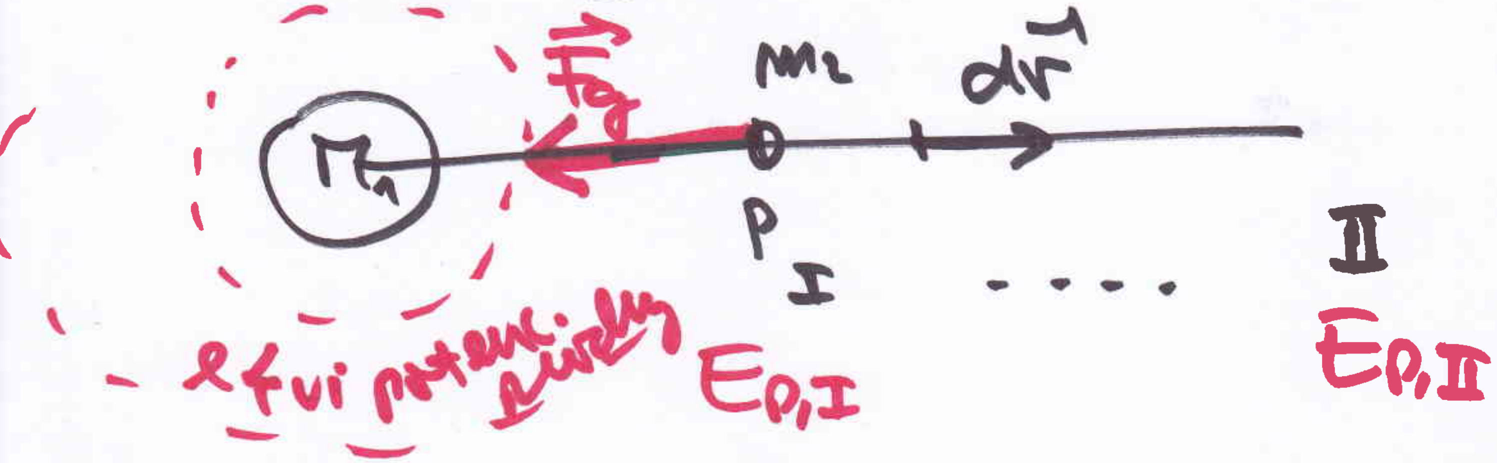
Intenzita (gravitačního) pole:

$$\vec{K}_g = \frac{\vec{F}_g}{m_2}$$

Potenciální energie gravitačního pole

$$E_{p,g} = ?$$

$$! \quad \Delta E_{p,g} = -\int_{I}^{II} \vec{F}_g \cdot d\vec{r} \quad !$$



$$-\int_{I}^{II} \vec F_g \cdot d\vec r = -\int_{I}^{II} |\vec F_g|\,|d\vec r| \cos 180^\circ =$$

$$= -\left(+\varkappa\, m_1 m_2\right)(-1)\int_{I}^{II} \frac{dr}{r^2}$$

$$= \varkappa\, m_1 m_2 \underbrace{\int_{r_I}^{r_{II}} r^{-2}\,dr}_{\left[-\frac{1}{r}\right]} = \varkappa\, m_1 m_2 \left[\frac{1}{r}\right]_{r_{II}}^{r_I}$$

$E_{p,II} - E_{p,I} =$

$$= \varkappa\, m_1 m_2 \left(\frac{1}{r_I} - \frac{1}{r_{II}}\right)$$

podle J.S. (ne určitě)

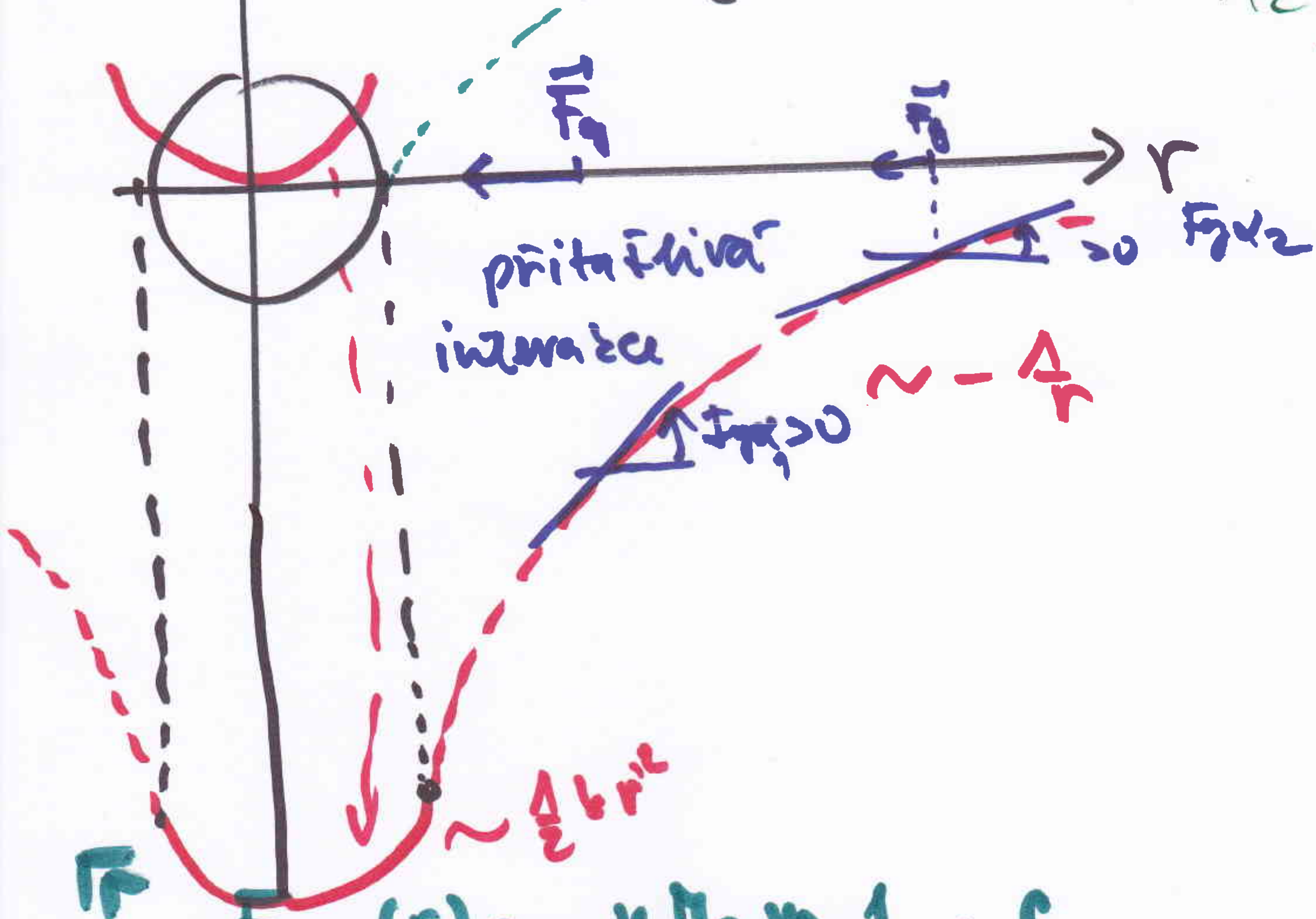
$r_{II} \to \infty$

Pak: $E_{p,g}(\infty) - E_{p,g}(r) = \varkappa\, m_1 m_2 \left(+\frac{1}{r_I}\right)$

volena 0 → $\underline{\underline{E_{p,g}}}$

$$E_{p,g}(r) = -\varkappa\, M_1 m_2 \frac{1}{r}$$

$E_{p,g}(r) = -\underbrace{\kappa M_1 m_2}_{k} \frac{1}{r}$ $\quad E_{p,g}(r = R_Z) = 0$



$$E_{p,g}(r) = -\kappa M_Z m \frac{1}{r} + C$$

$$0 = -\kappa \frac{M_Z m}{R_Z} + C$$

$$C = \kappa \frac{M_Z m}{R_Z}$$

$$E_{p,g}(r)^{>R_Z} = -\kappa \frac{M_Z m}{r} + \kappa \frac{M_Z m}{R_Z}$$

$$E_{p,g}(h) = -\kappa \frac{M_Z m}{R_Z + h} + \kappa \frac{M_Z m}{R_Z}$$

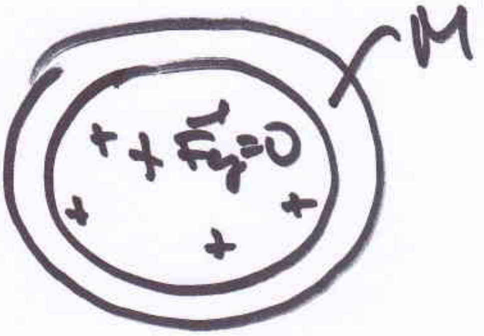
$$\frac{h}{R_Z} \ll 1 \quad = -\kappa \frac{M_Z m}{1 + \frac{h}{R_Z}} + \kappa \frac{M_Z m}{R_Z}$$

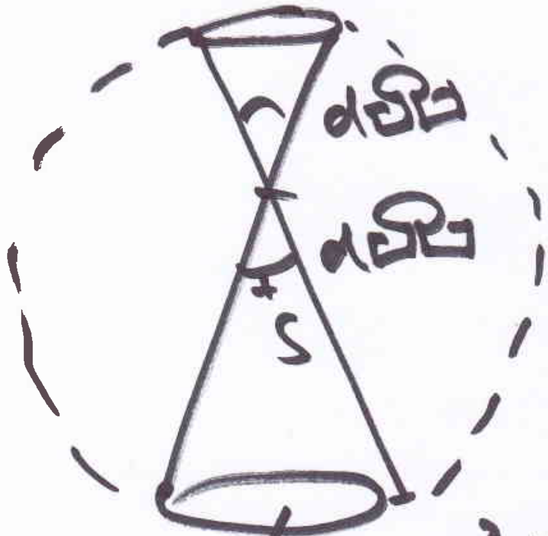
DÚ HRW

podn. gravitační pole: „slupkový teorém“

$dS_1 = r_1^2\, d\Omega$

3D



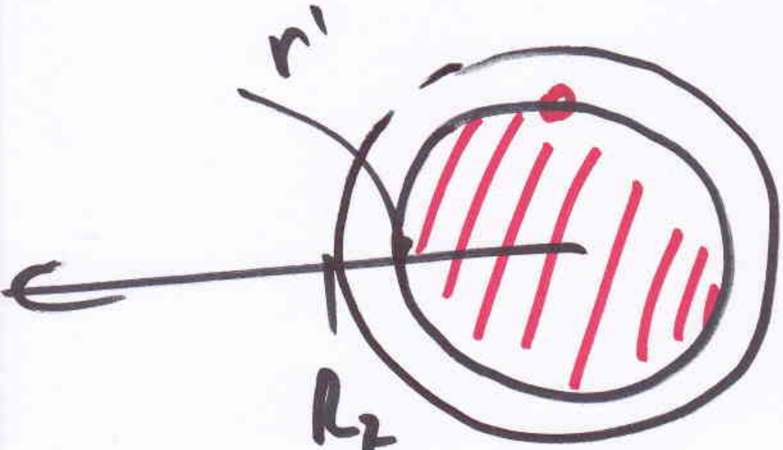


$dS_2 = r_2^2\, d\Omega$

$\sigma = \frac{dm}{dS}$

$dS_1 = \dots\, dm_1$

$dS_2 \dots\dots\, dm_2$



$F_g(r < R_z) = ?$ prafine

$$\kappa \frac{M_z' m}{r'^2} = \kappa \frac{M_z}{R_z^3} m\, r' = k\, r'$$

$$\left.\begin{aligned} M_z &= \rho_z \cdot \tfrac{4}{3}\pi R_z^3 \\ M_z' &= \rho_z \cdot \tfrac{4}{3}\pi r'^3 \end{aligned}\right\} \Rightarrow M_z'(r' < R_z) = \frac{M_z}{\cancel{\frac{4}{3}\pi} R_z^3} \cdot \cancel{\tfrac{4}{3}\pi}\, r'^3$$

$$\boxed{M_z'(r') = \frac{M_z}{R_z^3}\, r'^3}$$

nepovinné:

Elektrostatické pole:

• Síla:

$+Q \quad \vec{r}^{\,0}_{1,2} \quad \vec{F}_{C\,1\to 2} \quad -q$

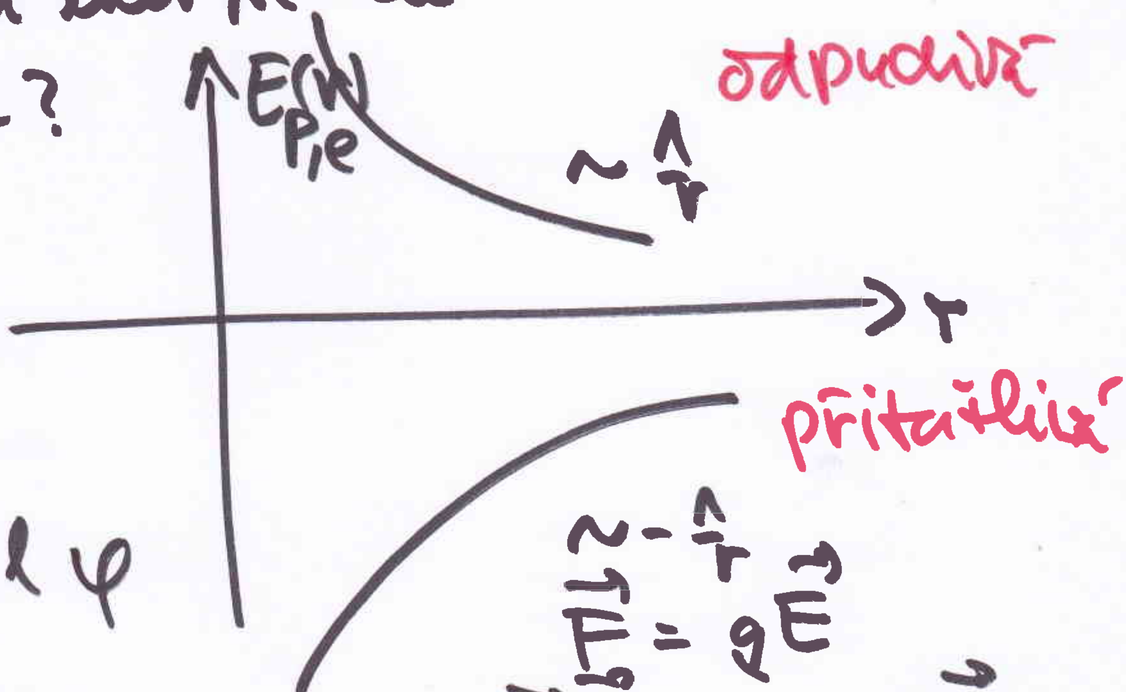
$$\vec{F}_{C\,1\to 2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_{1,2}^2}\,\vec{r}^{\,0}_{1\to 2}$$

• Intenzita

$$\vec{E} = \frac{\vec{F}}{q_{=+1}} \Rightarrow \vec{F}_C = Q\vec{E}$$

• Potenciální energie elektrost. pole:

$E_{p,e}(r) = ?$



• potenciál φ

$$\frac{E_{p,e}}{q_{=1}} = \varphi$$

$$\vec{F}_e = q\vec{E}$$

$$-\vec{\nabla}E = -\operatorname{grad}\vec{E}$$

$$= \frac{\Delta E_{p,e}}{q} = \boxed{\Delta\varphi = -\vec{\nabla}\vec{E}}$$