Dobrý den!

13. přednáška
4. 11. 05

Opak.:

I. impulsová věta

a) $\vec{F}_V^{ext} = M\vec{a}_T$!

b) $\frac{d\vec{P}}{dt} = \vec{F}_V^{ext}$!

c)! ZZH

DÚ rozbor „písemky"

II. impulsová věta

$$\frac{d\vec{L}}{dt} = \vec{M}_V^{ext}$$!

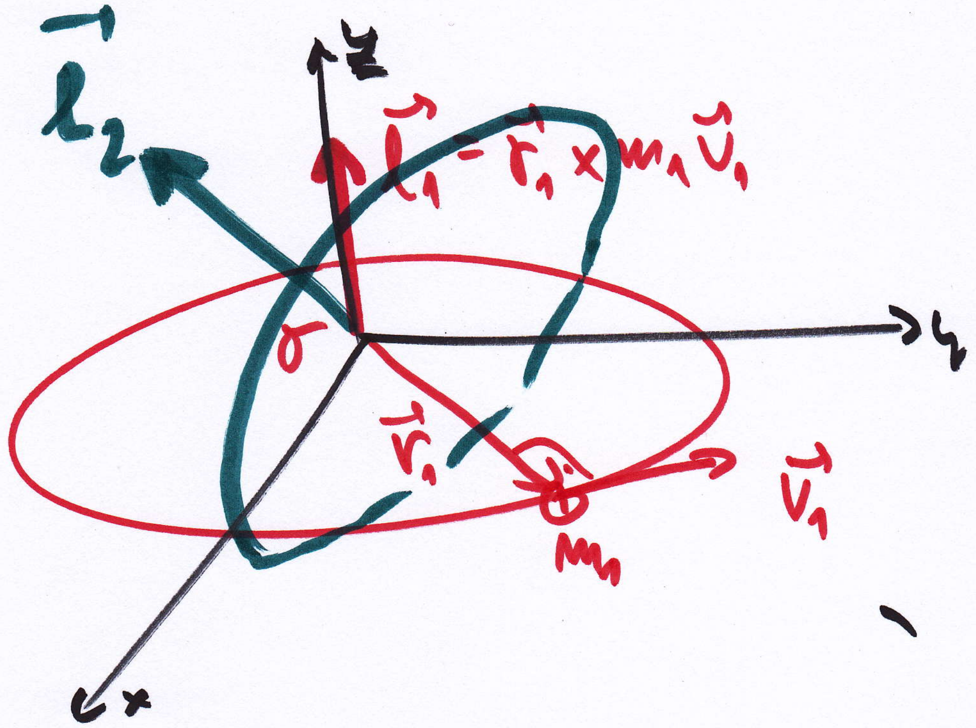
$\vec{L}$; $\vec{M}_V^{ext} = \sum \vec{r}_j \times \vec{F}_{j,V}^{ext}$ ... výsledný moment vnějších sil

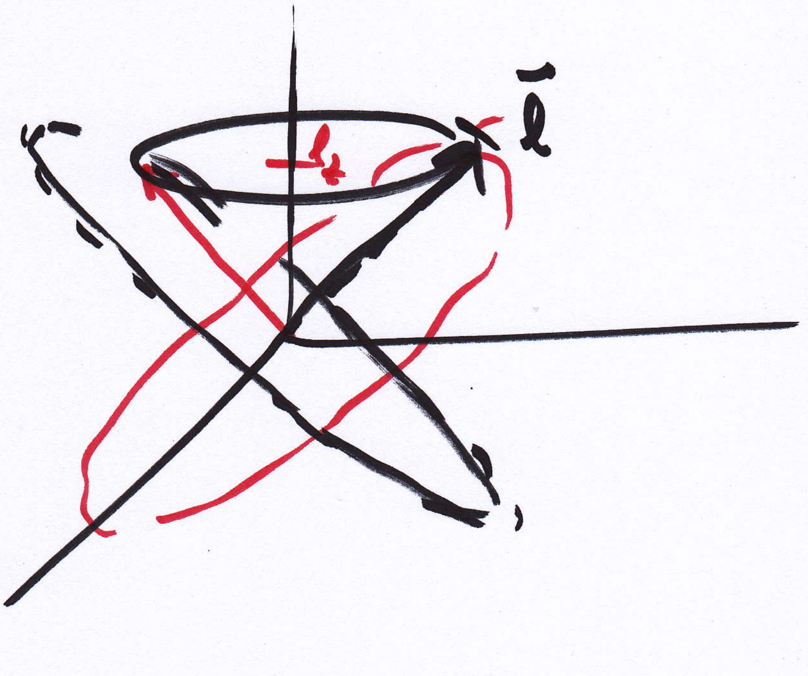
... výsledný moment hybnosti soustavy vzhledem k bodu O

$$\vec{L} = \sum_{i=1}^{N} \vec{l}_i = \sum_{i=1}^{N} \vec{r}_i \times m_i \vec{v}_i$$

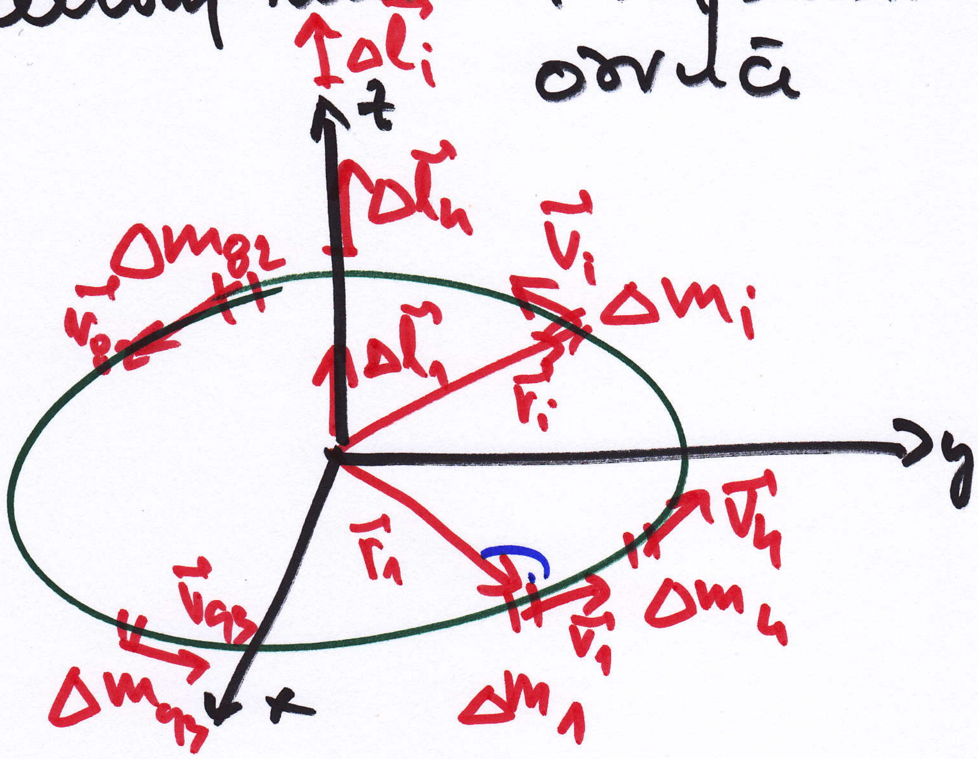
Podr. opak.:

Moment hybnosti





$\vec{L}$: celkový moment hybnosti obruče



$$\Delta \vec{l}_i = \vec{r}_i \times \Delta m_i \vec{v}_i$$

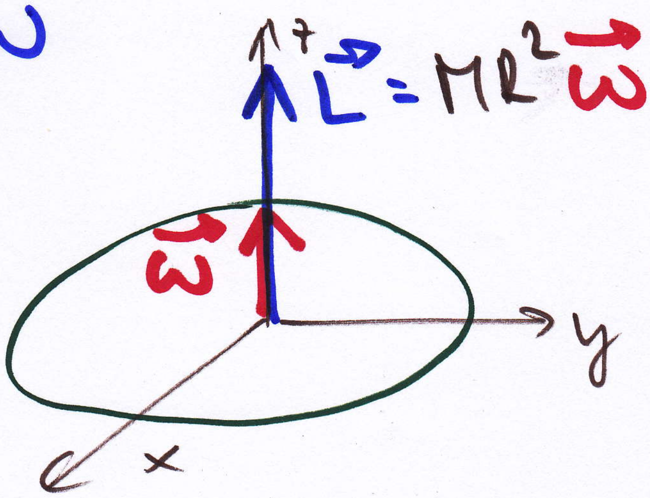
$$|\vec{L}| = \left|\sum_i \Delta \vec{l}_i\right| = \ldots = \sum_i |\Delta \vec{l}_i| =$$

$$= \sum_i |\vec{r}_i \times \Delta m_i \vec{v}_i| = \sum_i R\, \Delta m_i v_i \overbrace{\sin\sphericalangle(\vec{r}_i, \vec{v}_i)}^{1}$$

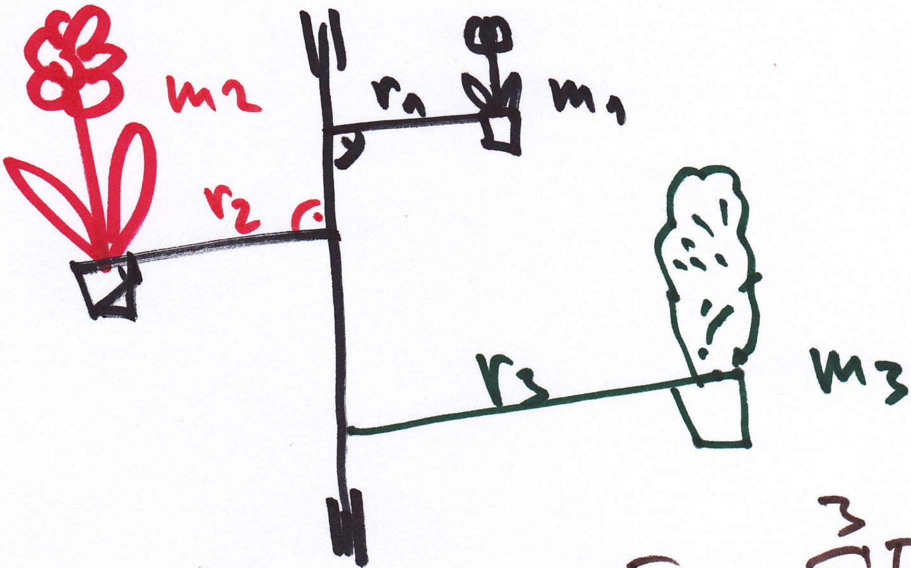
$$= R \sum_i \Delta m_i v_i = R \left(\sum_i \Delta m_i\right) R\omega$$

$$v_i = v = \text{konst} \qquad R\omega = v$$

$$|\vec{L}| = MR^2\omega$$



Energie rotujícího tuhého tělesa



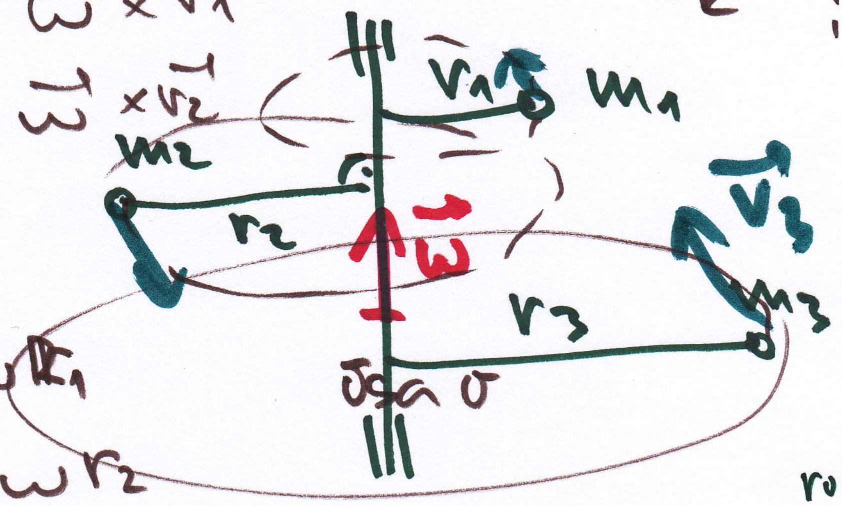
$\vec{v}_1 = \vec{\omega} \times \vec{r}_1$

$\vec{v}_2 = \vec{\omega} \times \vec{r}_2$

$\vdots$

$v_1 = \omega r_1$

$v_2 = \omega r_2$



$$E_k = \sum_{i=1}^{3} E_{k_i} = \sum_1^3 \frac{1}{2} m_i v_i^2$$

$$= \sum_i \frac{1}{2} m_i (\omega r_i)^2$$

$$= \frac{1}{2}\omega^2 \underbrace{\sum_{i=1}^{3} m_i r_i^2}_{I_o}$$

diskrétně rozložená hmotnost $\sum_i m_i r_i^2$

I ... moment setrvačnosti

$$E_k = \frac{1}{2} I_o \omega^2$$

$\int_M r^2 \, dm$ spojitě rozložená hmotnost

$\rho = \frac{dm}{dV}$

$\sigma = \frac{dm}{dS}$

$\tau = \frac{dm}{dl}$

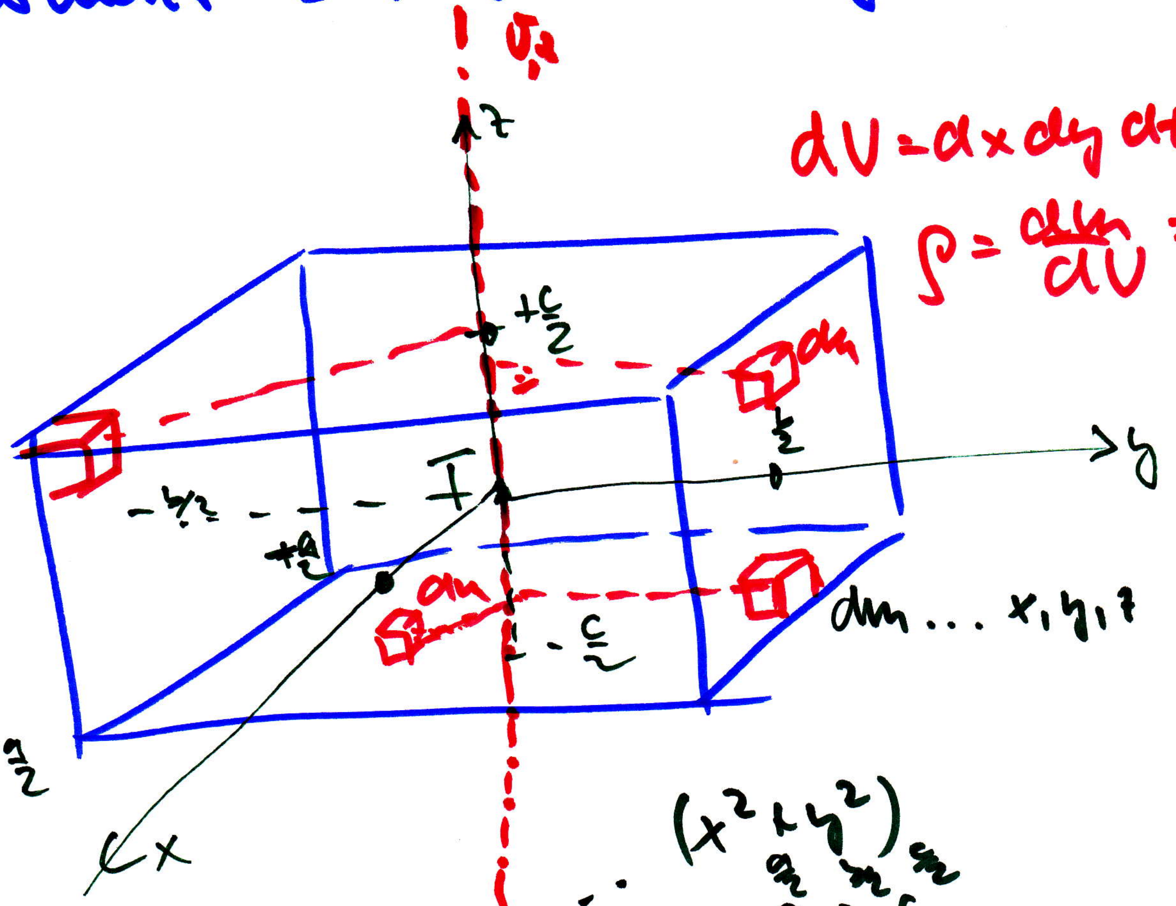
$$\boxed{E_k = \frac{1}{2}\left(\iiint_V \rho r^2 \, dV\right)\omega^2}$$

$dm = \rho \, dV$

1) Moment setrvačnosti desky:

$dV = dx\,dy\,dz$

$\rho = \frac{dm}{dV} = \text{konst.}$



$$I_{o,z} = ? = \iiint\limits_{V_{desky}} r^2 \rho \, dV = \rho \underset{(x)}{\int_{-\frac{a}{2}}^{\frac{a}{2}}} \underset{(y)}{\int_{-\frac{b}{2}}^{\frac{b}{2}}} \underset{(z)}{\int_{-\frac{c}{2}}^{\frac{c}{2}}} (x^2+y^2)\, dz\, dy\, dx$$

$$= \rho \left[\iiint x^2\, dz\, dy\, dx + \iiint y^2\, dz\, dy\, dx \right]$$

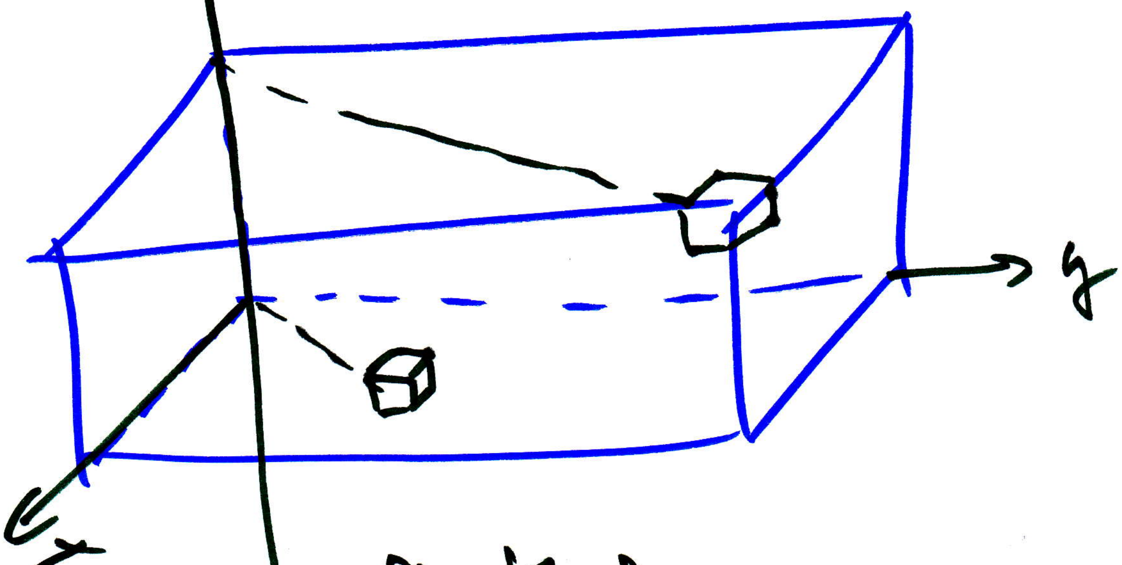
$$\rho \left(\left[\frac{x^3}{3}\right]_{-\frac{a}{2}}^{\frac{a}{2}} \underbrace{\left[y\right]_{-\frac{b}{2}}^{\frac{b}{2}}}_{b} \underbrace{\left[z\right]_{-\frac{c}{2}}^{\frac{c}{2}}}_{c} + \left[\frac{y^3}{3}\right]_{-\frac{b}{2}}^{\frac{b}{2}} \underbrace{\left[x\right]_{-\frac{a}{2}}^{\frac{a}{2}}}_{a} \underbrace{\left[z\right]_{-\frac{c}{2}}^{\frac{c}{2}}}_{c} \right)$$

$$= \rho \frac{1}{3} \left(\frac{a^3bc}{4} + \frac{b^3ac}{4} \right) = \frac{1}{12} \underbrace{\rho\, abc}_{m} (a^2+b^2)$$

$$\left(\left(\frac{a}{2}\right)^3 - \left(-\frac{a}{2}\right)^3 \right) = \frac{a^3}{2^3} + \frac{a^3}{2^3} = \frac{a^3}{2^2}$$

$$I_{o,z} = \frac{1}{12} M (a^2+b^2)$$

$$I_\sigma = \rho \iiint_V r^2 \, dV$$



$$I_0 = \rho \underset{(x)}{\int_0^a} \underset{(y)}{\int_0^b} \underset{(z)}{\int_0^c} (x^2+y^2)\,dz\,dy\,dx$$

$$= \rho\left(\left[\frac{x^3}{3}\right]_0^a [y][z] + \left[\frac{y^3}{3}\right] \dots\right) =$$

$$= \frac{1}{3}\rho\left(a^3bc + ab^3c\right) =$$

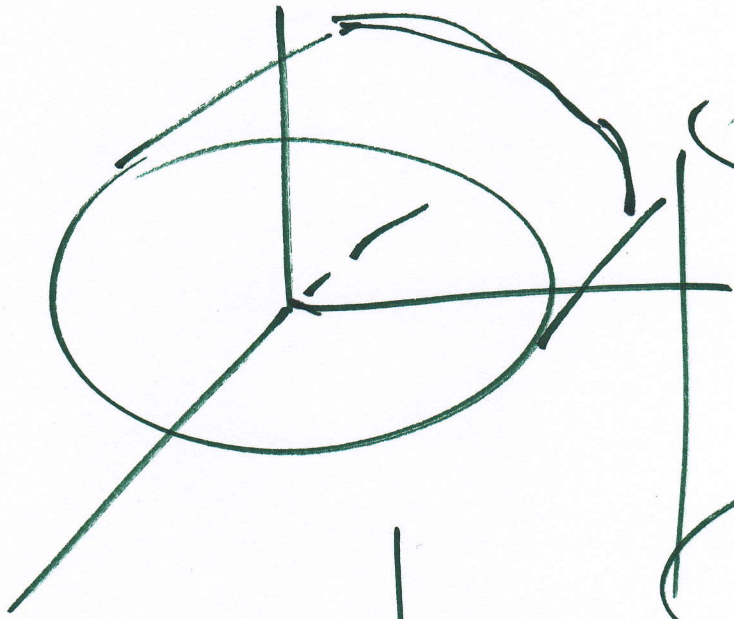
$$= \frac{1}{3}\rho\, abc\left(a^2+b^2\right) = \frac{1}{3} M\left(a^2+b^2\right)$$

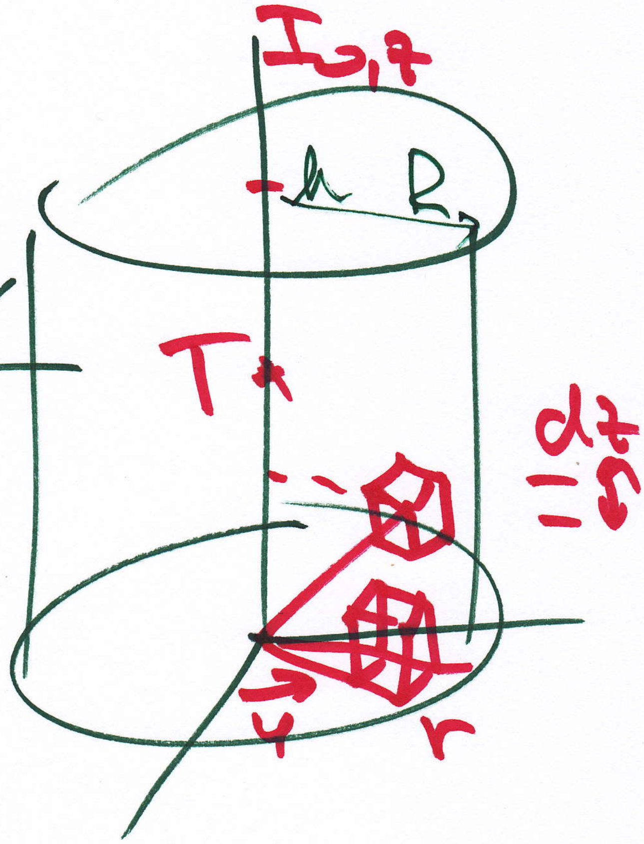
$$\boxed{I_\sigma = I_{\sigma T} + m d^2}$$

d vzdálenost σ a σ_T

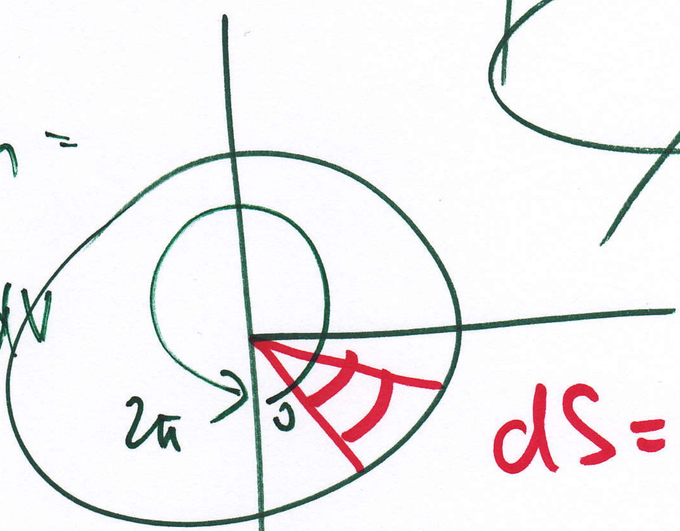
Steinerova věta

Moment setrvačnosti válce





$$I_O = \int_M r^2\, dm = \iiint_V r^2 \rho\, dV$$



$$dS = r\,dr\,d\varphi$$

$$dV = r\,dr\,d\varphi\,dz$$

$$I_{O,z} = \rho \iiint_{V_{válce}} r^2\, dV = \rho \underset{(z)}{\int_0^h} \underset{(\varphi)}{\int_0^{2\pi}} \int_0^R \underbrace{r^2\, r}_{r^3}\, dr\, d\varphi\, dz$$

$$= \rho\, [z]_0^h\, [\varphi]_0^{2\pi} \left[\frac{r^4}{4}\right]_0^R = \rho\, h\, 2\pi \frac{R^4}{\cancel{4}_2} = \underbrace{\rho\, \underbrace{\pi R^2 \cdot h}_{V}}_{M} \frac{R^2}{2}$$

$$\boxed{I_{O,z} = \frac{1}{2} M R^2}$$ válce

Dráhový účinek síly:

$$\int \vec{F} \cdot d\vec{r}$$

Časový účinek síly

$$\int \vec{F}(t)\, dt$$

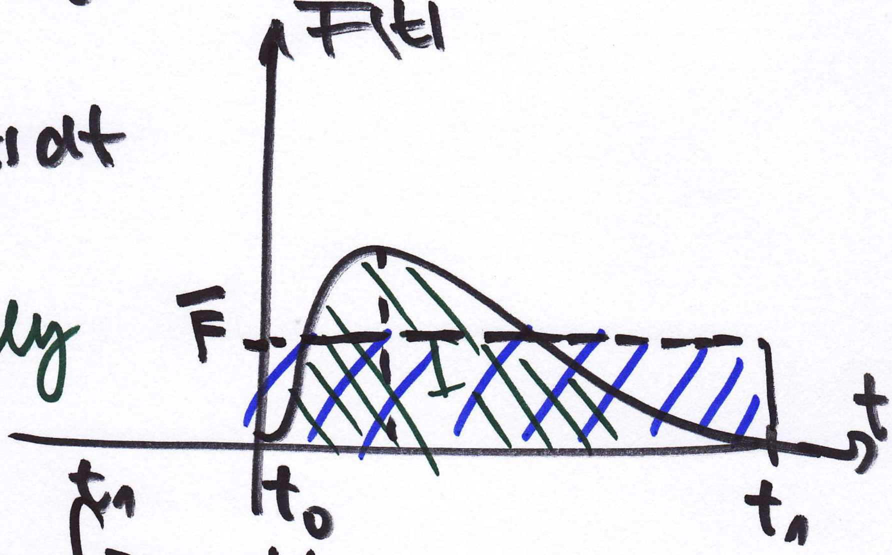
II. NPZ

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow$$

$$\int_{t_1}^{t_2} \vec{F}\, dt = \int d\vec{p} = \Delta\vec{p} = \vec{p}(t_2) - \vec{p}(t_1)$$

$$\vec{I} = \int \vec{F}(t)\, dt$$

↗ impuls síly



$$\overline{F}\,(t_1 - t_0) = \int_{t_0}^{t_1} F(t)\, dt$$

$$\overline{F} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} F(t)\, dt$$