

Global analysis. Exercises 1

Find the Jacoby matrices and the Jacobians (when exist) of the following maps and say which of these maps are immersions, submersions and diffeomorphisms on their images.

$$1) x = 3\rho \cos t, \quad y = 4\rho \sin t, \quad (\rho, t) \in (0, 1) \times (0, 2\pi).$$

$$2) x = \cos u \cos v, \quad y = \sin u \cos v, \quad z = \sin v, \quad (u, v) \in (0, 2\pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

$$3) x = \sqrt{\rho} \cos \varphi, \quad y = \sqrt{\rho} \sin \varphi, \quad z = \rho, \quad (\rho, \varphi) \in (0, +\infty) \times (0, 2\pi).$$

$$4) x = \frac{2u}{1+u^2+v^2}, \quad y = \frac{2v}{1+u^2+v^2}, \quad (u, v) \in \mathbb{R} \times \mathbb{R}.$$