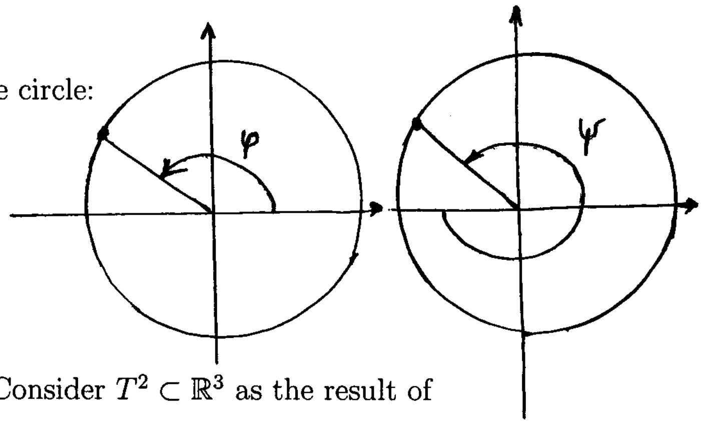
Global analysis. Exercises 2

1) Construct the following two atlases on the unite circle:

- using the stereographic projections;
- using the angles (see the picture).



Check if these atlases are equivalent.

2) Construct an atlas on the torus $T^2 = S^1 \times S^1$. Consider $T^2 \subset \mathbb{R}^3$ as the result of the rotation of the unit circle, show that the restrictions of the functions x, y, z to T^2 are smooth.

3) Show that on the union of two coordinate axels in $\mathbb{R}^2$ there is no atlas making this topological space (with the induced topology) into a smooth manifold.

4) Is there a structure of smooth manifolds on the following topological spaces (with the standard topology):

- a triangle;
- two triangles with one common vertix?

5) The graph of any C^r map $f : \mathbb{R}^n \to \mathbb{R}^m$ is a smooth manifold of the class C^r.

6) Show that two smooth atlases on a set M are equivalent if and only if their union is an atlas.

7) Show that two atlases on a set M are equivalent if and only if they define the same set of smooth functions.

8) Show that if two smooth manifolds are diffeomorphic, then they have equal dimensions (use the tangent map and the corresponding fact for isomorphic vector spaces).

9) Find the tangent space to the elepce $x^2 + \frac{y^2}{4} = 1$ at the point $(\frac{\sqrt{2}}{2}, \sqrt{2})$.

10) Find the relation between the basis tangent vectors to the circle at the point $(0, 1)$ corresponding to different coordinate systems from Exercise 1.

11) For two smooth manifolds M, N and points $x \in M, y \in N$ prove that

$$T_{(x,y)}M \times N = T_xM \oplus T_yN.$$