Global analysis. Exercises 4

1) Which of the following distributions on $\mathbb{R}^3\backslash\{(0,0,0)\}$ are involutive:

- the distribution generated by the vector fields $X = x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+z\frac{\partial}{\partial z}$, $Y = \frac{\partial}{\partial x}+\frac{\partial}{\partial y}$;
- the distribution generated by the vector fields $X = xyz\frac{\partial}{\partial x} + y^2\frac{\partial}{\partial y}$, $Y = x\frac{\partial}{\partial x} + (z+y)\frac{\partial}{\partial z}$.

2) Prove that any 1-dimensional distribution is involutive.

3) Find the dimensions of the spaces $\bigotimes^r V$ and $\bigwedge^r V$ if $\dim V = n$.

4) Prove that $\bigotimes^2 V = S^2 V \bigoplus \bigwedge^2 V$

5) Let V be a vector space, $A \in \bigotimes^r V$, $e_1, ..., e_n$ and $e'_1, ..., e'_n$ bases of V and B the transition matrix from the first basis to the second one. Find the relation between the components $A^{i_1 \dots i_r}$ and $A^{i'_1 \dots i'_r}$ of the tensor A in these bases.

6) Let $A \in \bigotimes^3 V$. Prove that $\mathrm{Sym}(\mathrm{Sym} A) = \mathrm{Sym} A$, $\mathrm{Alt}(\mathrm{Alt} A) = \mathrm{Alt} A$, $\mathrm{Sym}(\mathrm{Alt} A) = 0$, $\mathrm{Alt}(\mathrm{Sym} A) = 0$.