

Global analysis. Exercises 8

1) Find $\int_M \omega$ for

- $\omega = (x - y)dx + (x + y)dy$ and M is the segment AB with $A = (2, 3)$ and $B = (3, 5)$;
- $\omega = ydx + xdy$ and M is a quarter of the circle $g(t) = (R \cos t, R \sin t)$, $t \in (0, \frac{\pi}{2})$;
- $\omega = xdx + ydy + (x + y - 1)dz$ and M is the segment AB with $A = (1, 1, 1)$ and $B = (2, 3, 4)$.

2) Find

$$\int_M dx_3 \wedge dx_4 + x_1 x_3 dx_2 \wedge dx_4,$$

where $M = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1^2 + x_2^2 = 1, x_3^2 + x_4^2 = 1\}$ and the orientation is given by the parameterization $g(u, v) = (\cos u, \sin u, \cos v, \sin v)$, $(u, v) \in (0, 2\pi) \times (0, 2\pi)$.

3) Using the Stokes Theorem, find $\int_M \omega$, where

- $\omega = (x^2 + y^2)dx + (x^2 - y^2)dy$ and M is the circuit of the triangle with the vertexes $A = (0, 0)$, $B = (1, 0)$, $C = (0, 1)$;
- $\omega = (x^2 + y^2)dx + (x^2 - y^2)dy$ and M is the graph of the function $y = 1 - |1 - x|$, $x \in (0, 2)$, starting at $(0, 0)$;
- $\omega = xydy \wedge dx + yzdz \wedge dx + xzdx \wedge dy$ and M is the surface of the pyramid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 0$.