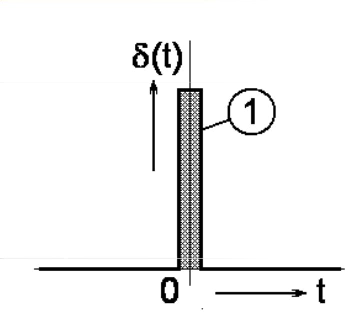
# SIGNÁLY A LINEÁRNÍ SYSTÉMY

prof. Ing. Jiří Holčík, CSc.

EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

# VI.
# KONVOLUCE
# &
# VZORKOVACÍ TEORÉM

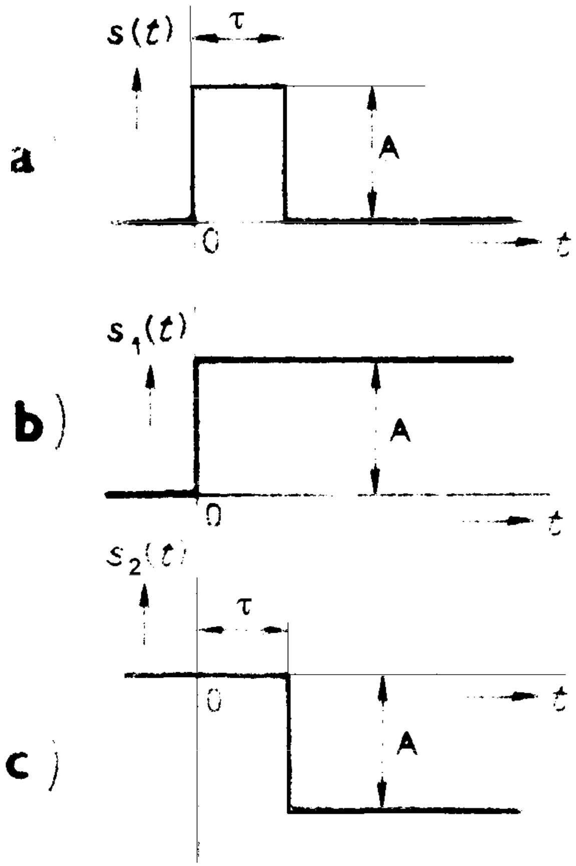
# SPEKTRUM DIRACOVA IMPULZU

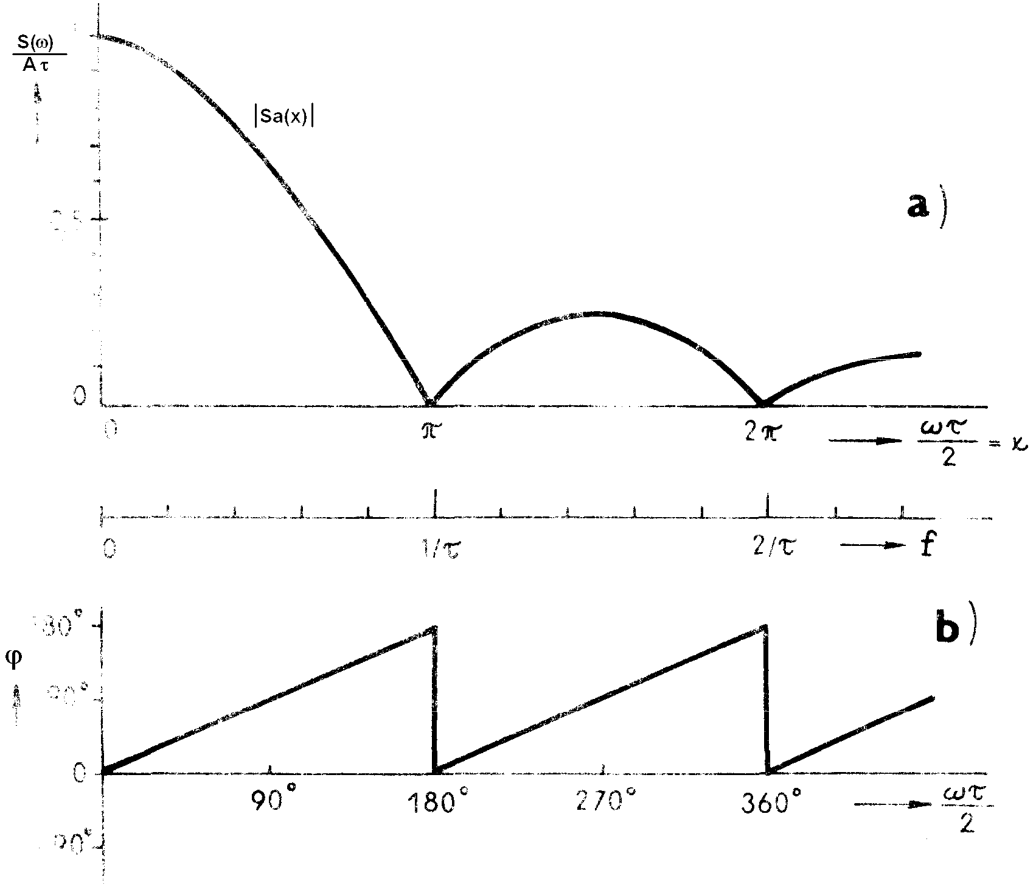



$\tau \rightarrow 0$

$A \rightarrow \infty$

$A.\tau = 1$



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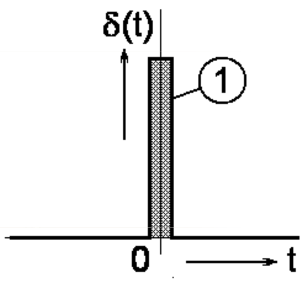


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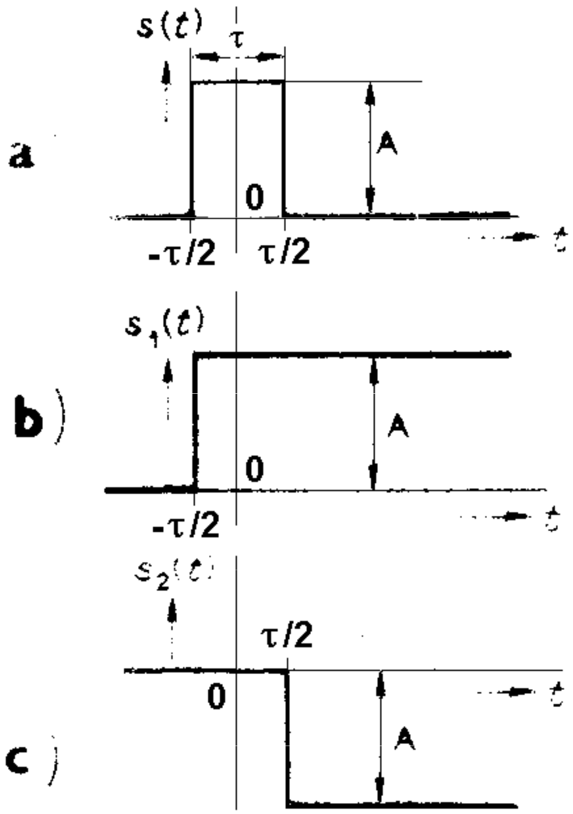

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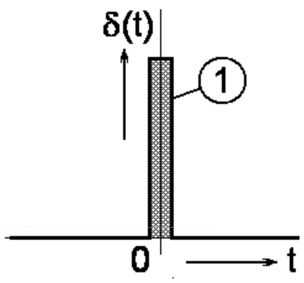


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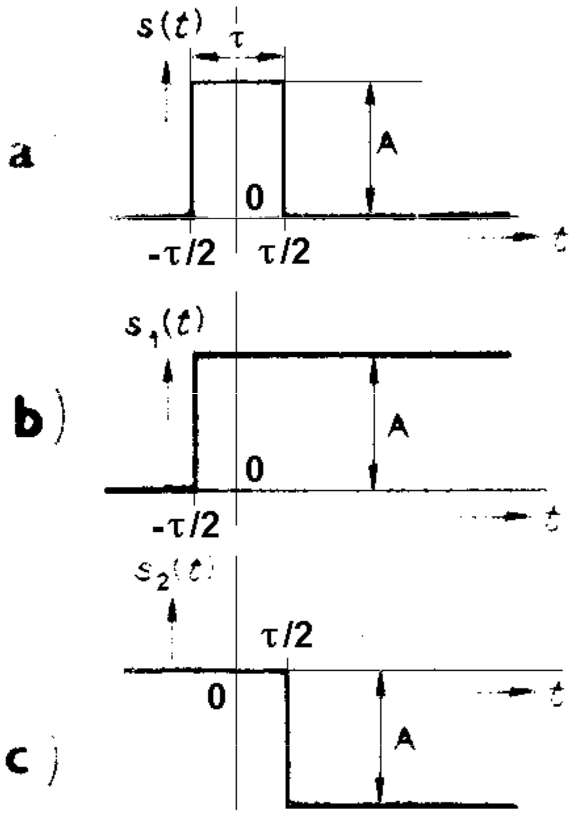
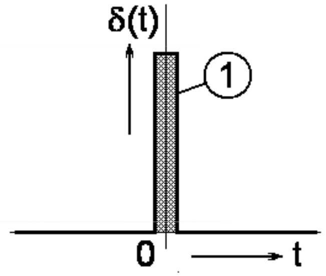



$$S(\omega) = A.\left(\frac{1}{j\omega}.e^{j\omega\tau/2} - \frac{1}{j\omega}.e^{-j\omega\tau/2}\right) =$$

$$= A.\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega} = \ldots =$$

$$= \frac{2A}{\omega}.\sin\frac{\omega\tau}{2} = \ldots = A.\tau.\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}.$$

# SPEKTRUM DIRACOVA IMPULZU

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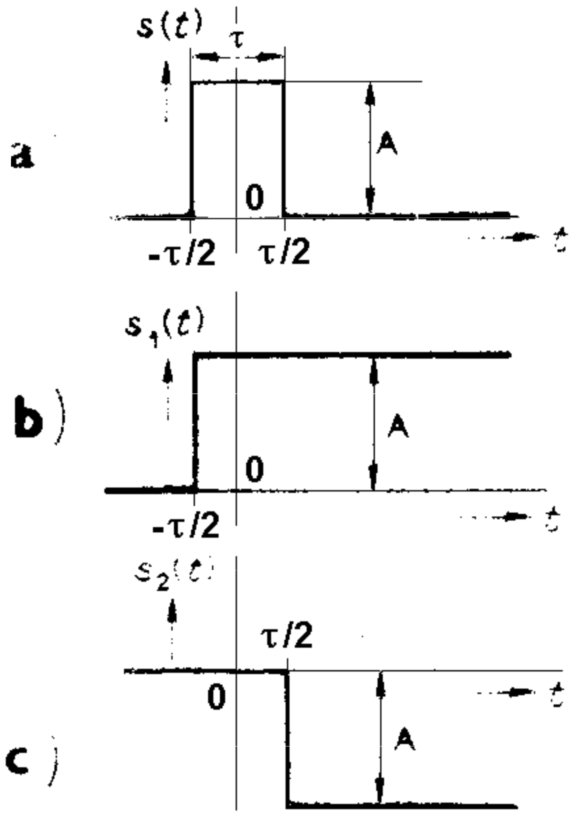


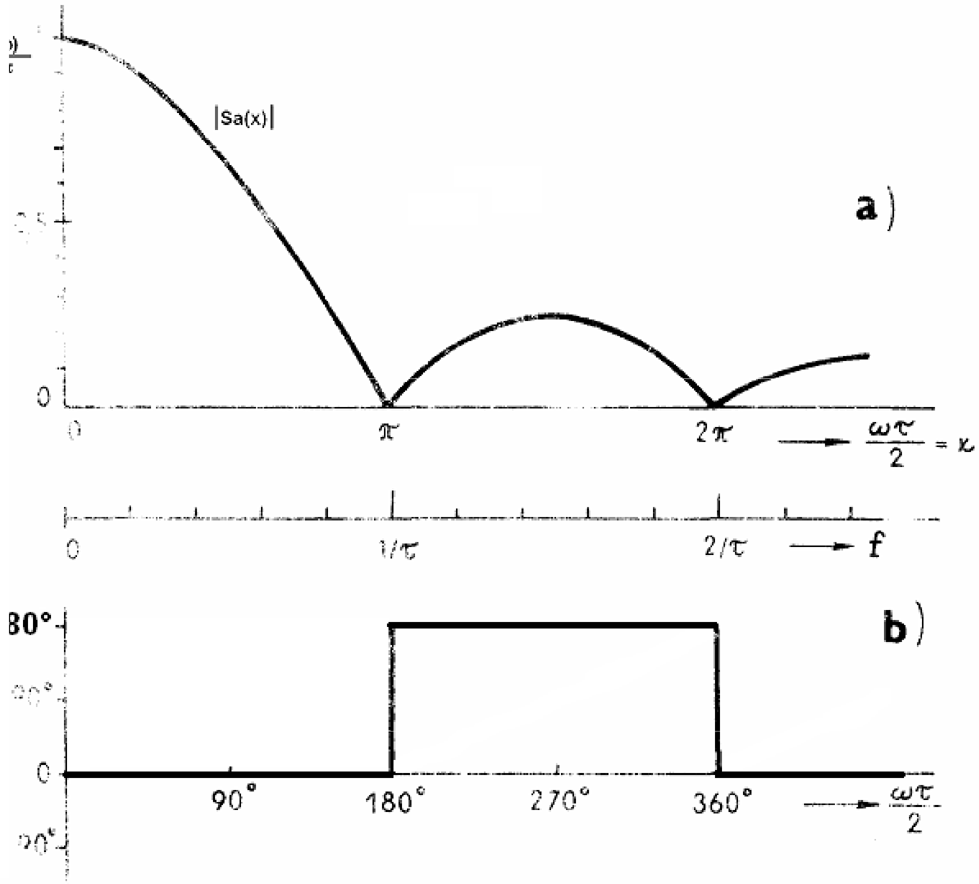

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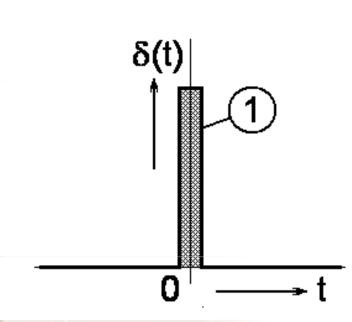
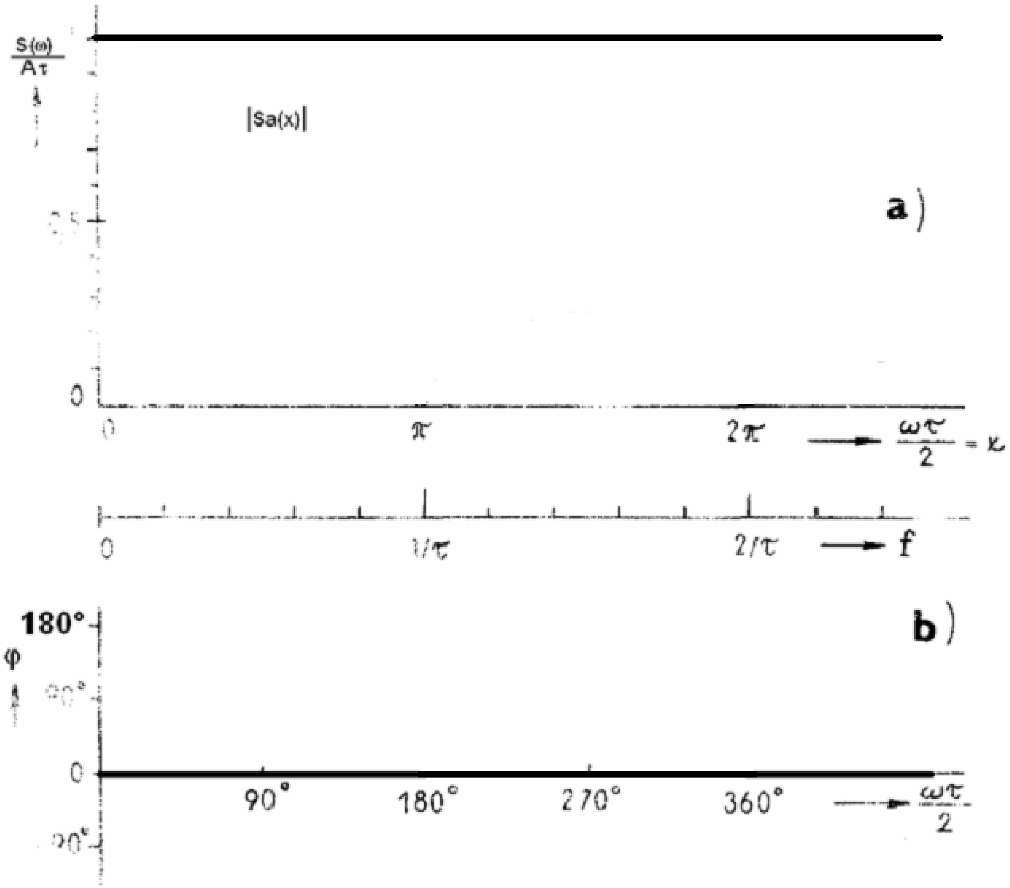
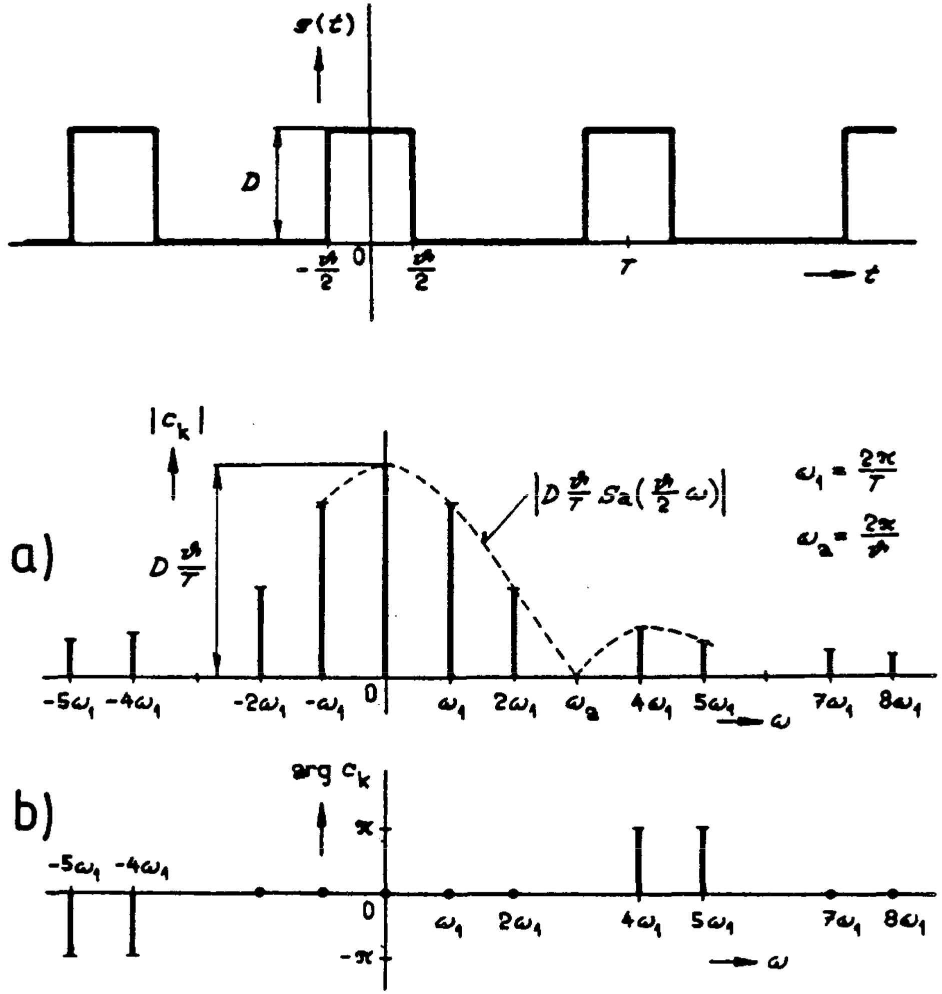



$$\tau \rightarrow 0$$
$$A \rightarrow \infty$$
$$A.\tau = 1$$



# SPEKTRUM PULZU DIRACOVÝCH IMPULZŮ



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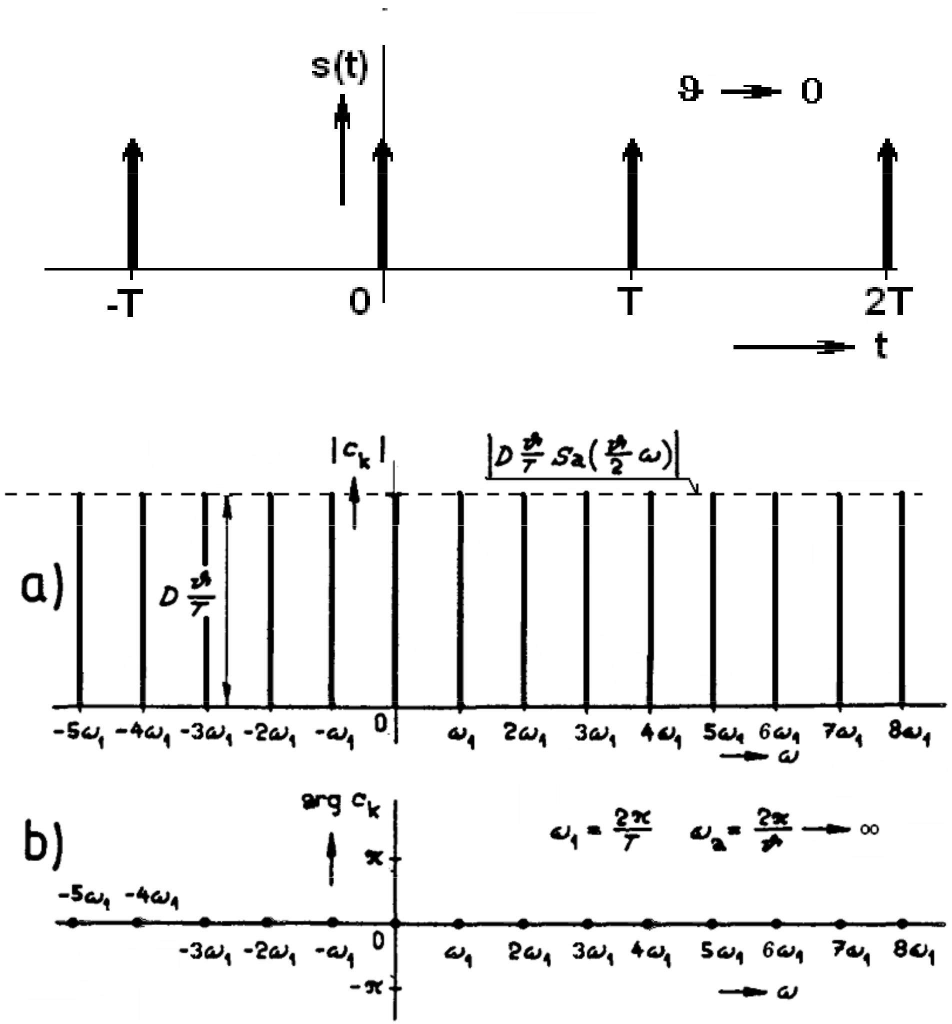


# KONVOLUCE

$$s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\tau).s_2(t-\tau).d\tau =$$

$$= \int_{-\infty}^{\infty} s_1(t-\tau).s_2(\tau).d\tau \qquad \approx S_1(\omega).S_2(\omega)$$

Důkaz:

$$s_1(t) * s_2(t) = \int_{-\infty}^{\infty} s_1(\tau).s_2(t-\tau).d\tau = \begin{vmatrix} x = t-\tau \\ \tau = t-x \\ d\tau = -dx \end{vmatrix} =$$

$$= -\int_{\infty}^{-\infty} s_2(x).s_1(t-x).dx = s_2(t) * s_1(t)$$

# KONVOLUCE

Distributivní zákon:

$$f_1(t)*\left[f_2(t)*f_3(t)\right]=\left[f_1(t)*f_2(t)\right]*f_3(t)$$

Asociativní zákon:

$$f_1(t)*\left[f_2(t)+f_3(t)\right]=f_1(t)*f_2(t)+f_1(t)*f_3(t)$$

# KONVOLUCE

## Zákon o posunu v čase

Je – li

$$f_1(t) * f_2(t) = c(t),$$

pak

$$f_1(t) * f_2(t - T) = c(t - T),$$

$$f_1(t - T) * f_2(t) = c(t - T)$$

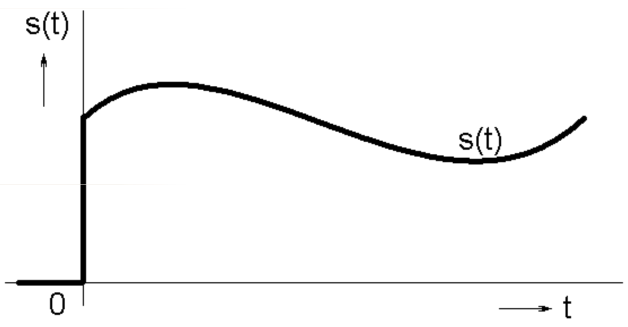
a

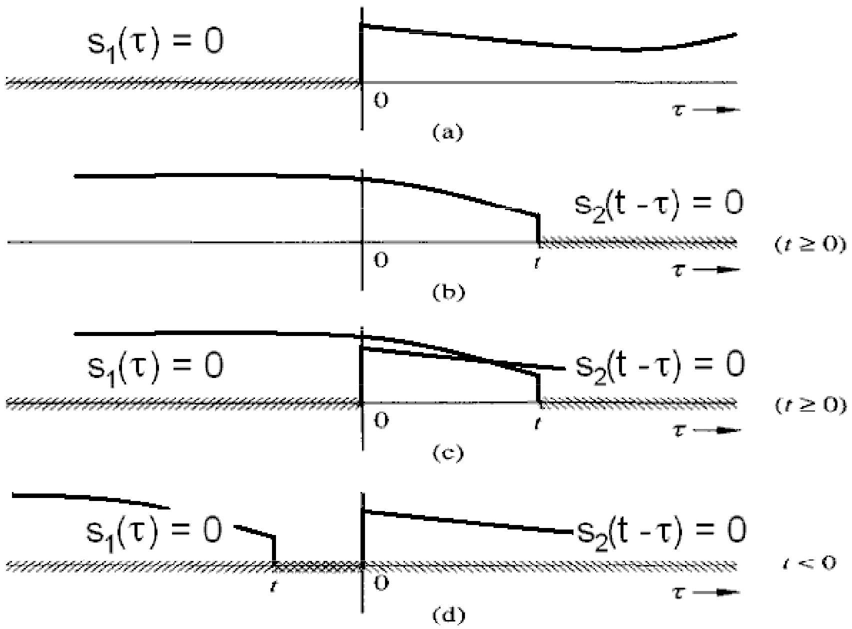
$$f_1(t - T_1) * f_2(t - T_2) = c(t - T_1 - T_2)$$

# KONVOLUCE

## Konvoluce kauzálních signálů:

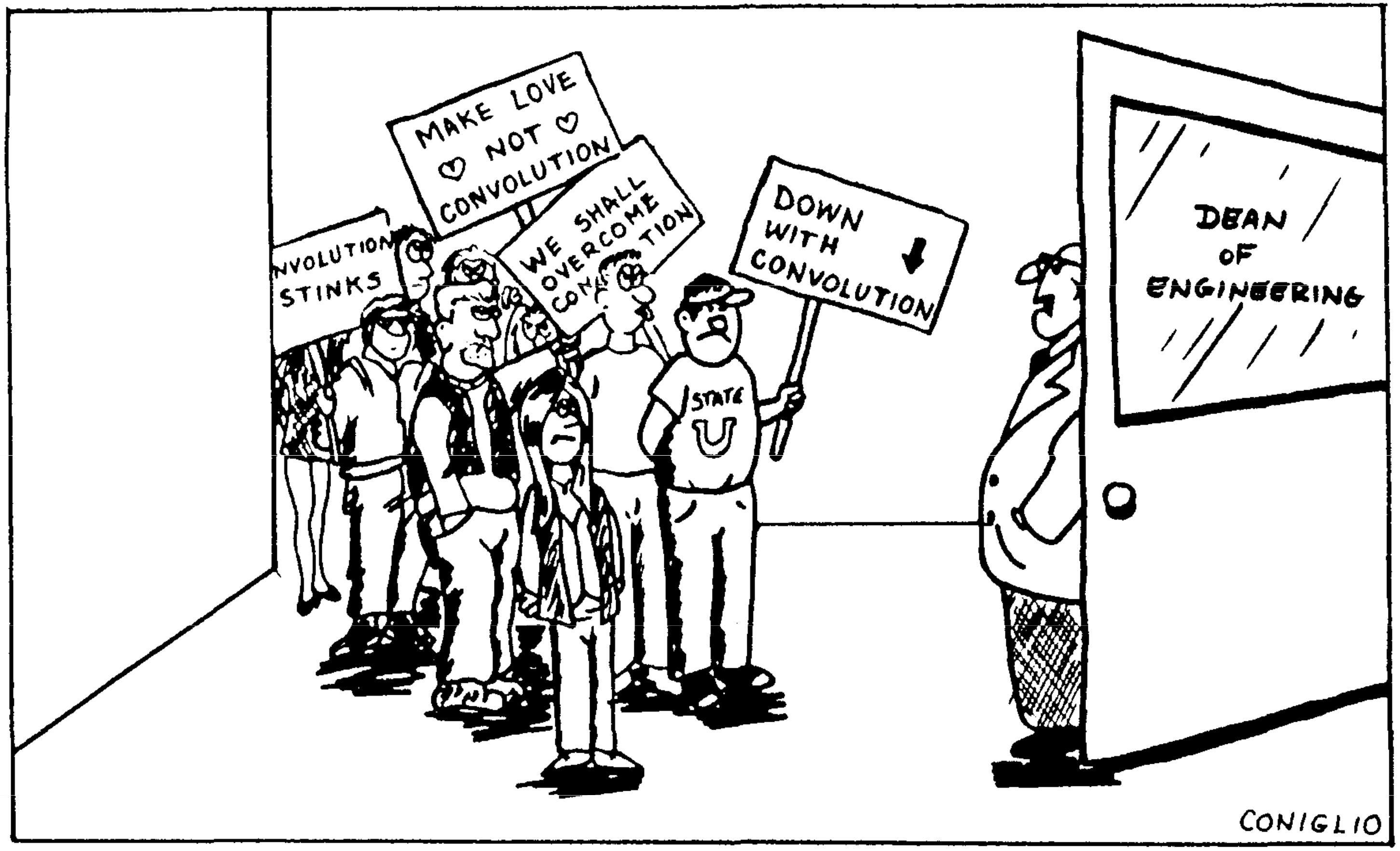
Pro kauzální signály platí s(t) = 0 pro t < 0



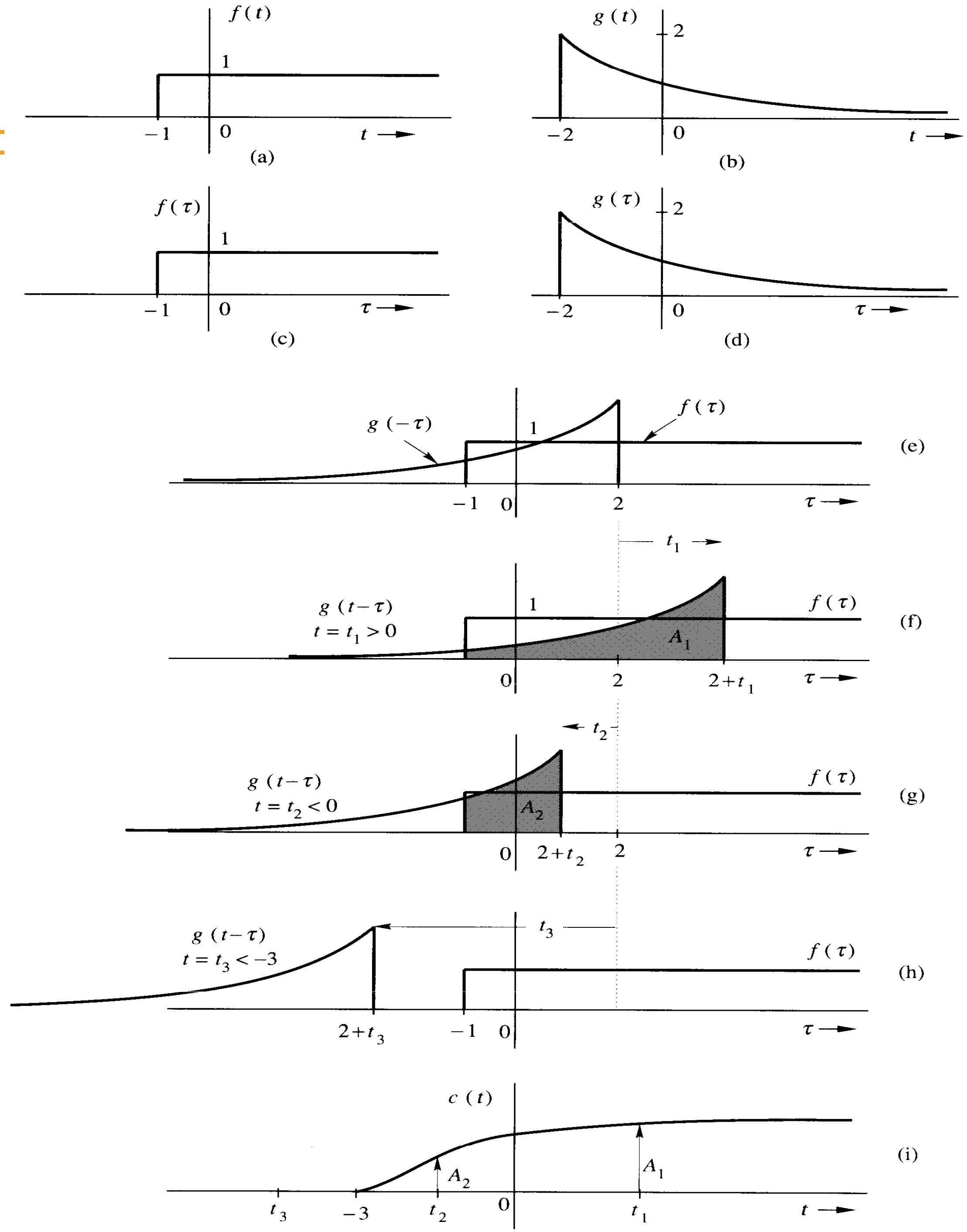

$$s_1(t) * s_2(t) = \int_0^t s_1(\tau).s_2(t-\tau).d\tau$$

# KONVOLUCE

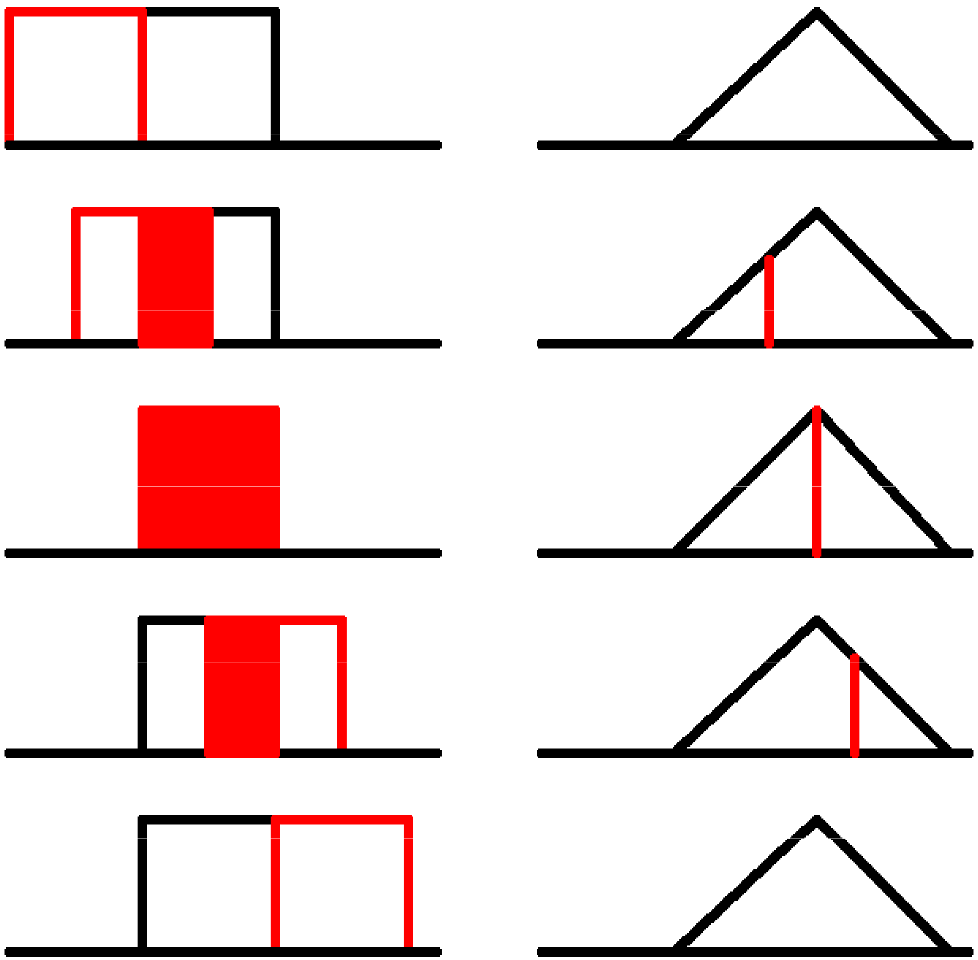



Convolution: its bark is worse than its bite!

# KONVOLUCE


f(t)
g(t)
(a)
(b)
f(τ)
g(τ)
(c)
(d)
g(−τ)
f(τ)
(e)
g(t−τ)
t = t1 > 0
A1
(f)
g(t−τ)
t = t2 < 0
A2
(g)
g(t−τ)
t = t3 < −3
(h)
c(t)
A2
A1
(i)

# KONVOLUCE



# KONVOLUCE

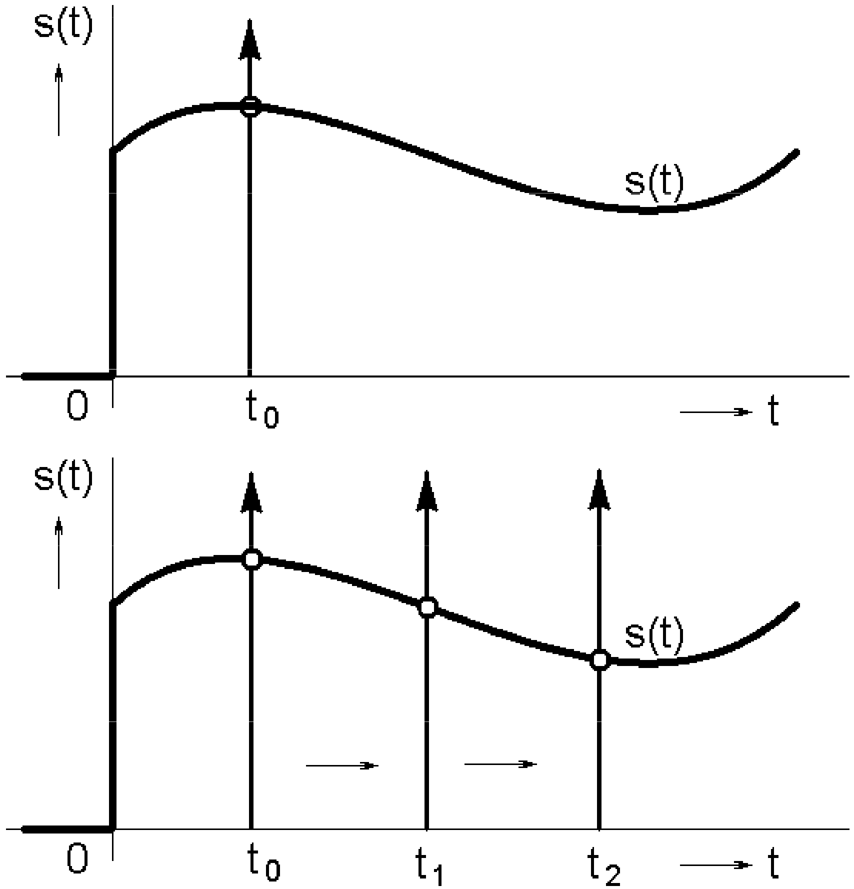
## signálu s jednotkovým impulsem

definice:

$$\int_{-\infty}^{\infty} s(t).\delta(t-t_0)dt = s(t_0)$$

konvoluce:

$$s(t)*\delta(t) = \int_{-\infty}^{\infty} s(t).\delta(t-\tau)d\tau = s(t)$$



# KONVOLUCE

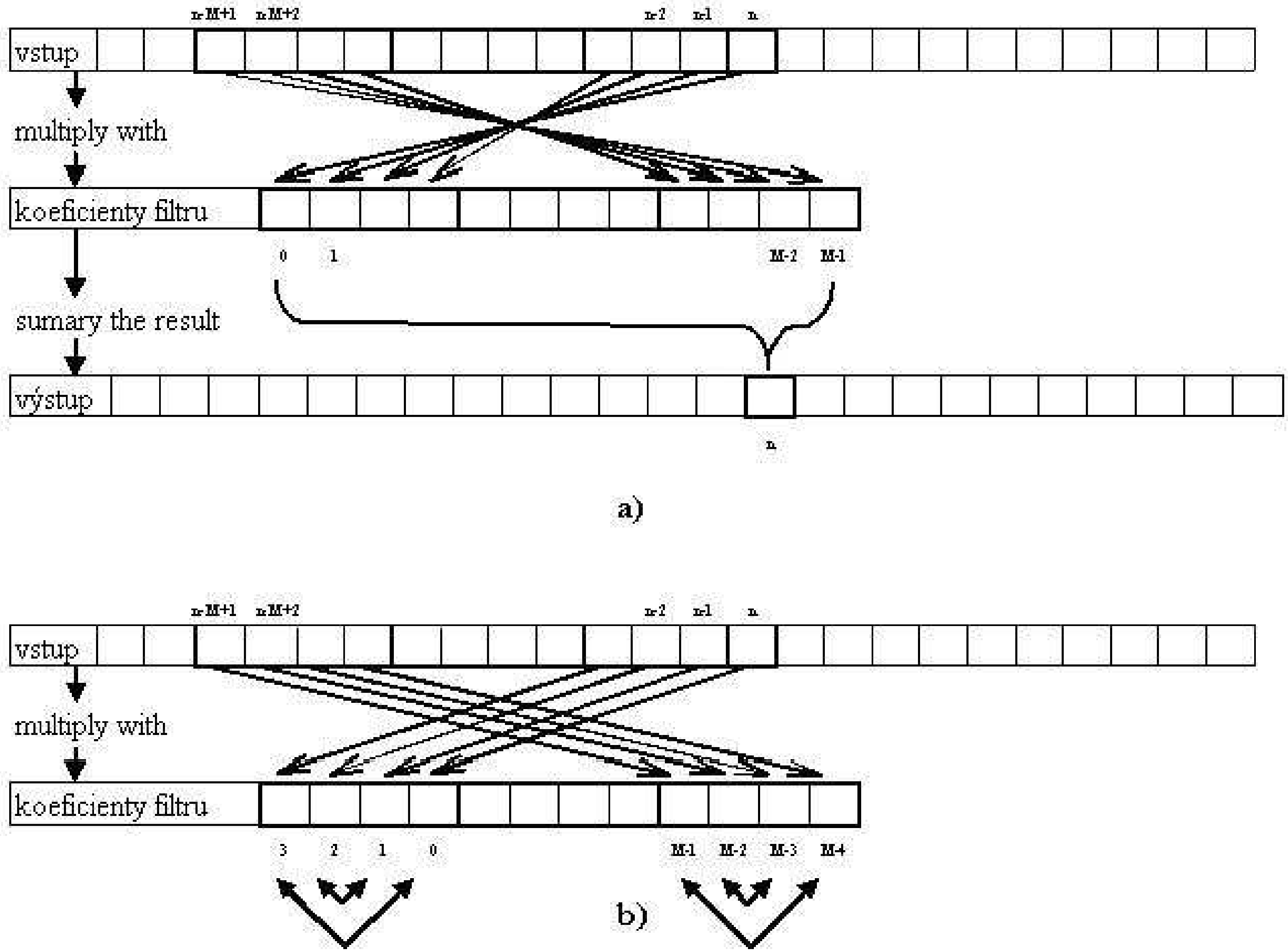
- spojité signály

$$s_1(t) * s_2(t) = \int_{-\infty}^{t} s_1(\tau).s_2(t-\tau).d\tau \approx S_1(\omega).S_2(\omega)$$

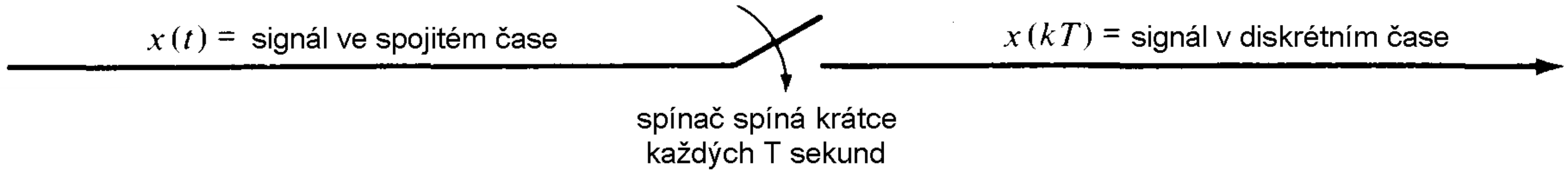
- diskrétní signály

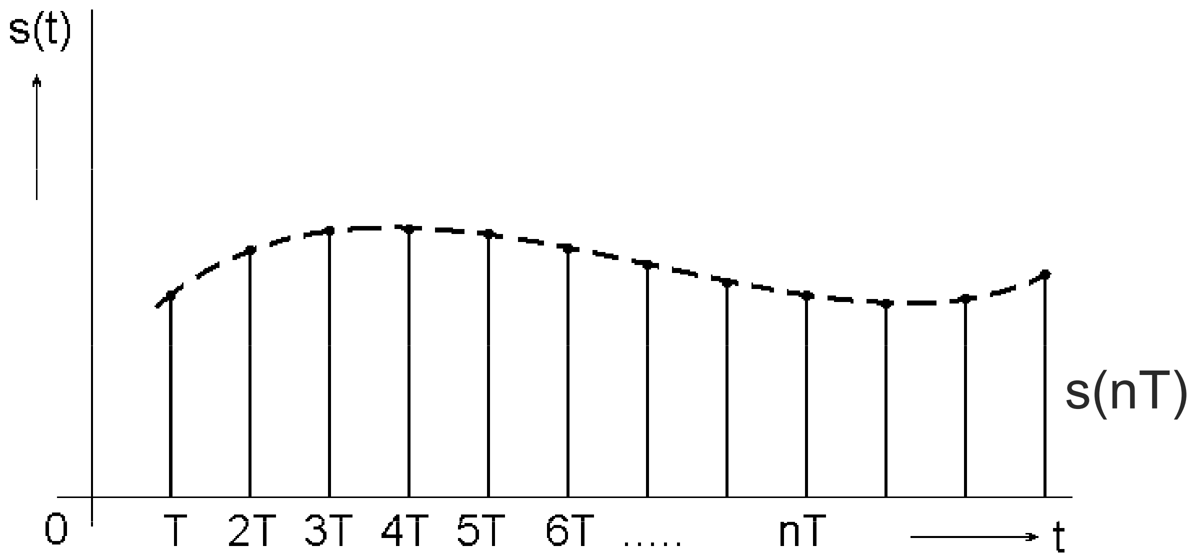
$$s_1(nT) * s_2(nT) = \sum_{i=0}^{n} s_1(iT).s_2(nT-iT) \approx S_1(z).S_2(z)$$

# DISKRÉTNÍ KONVOLUCE



# DISKRÉTNÍ SIGNÁL - VZORKOVÁNÍ

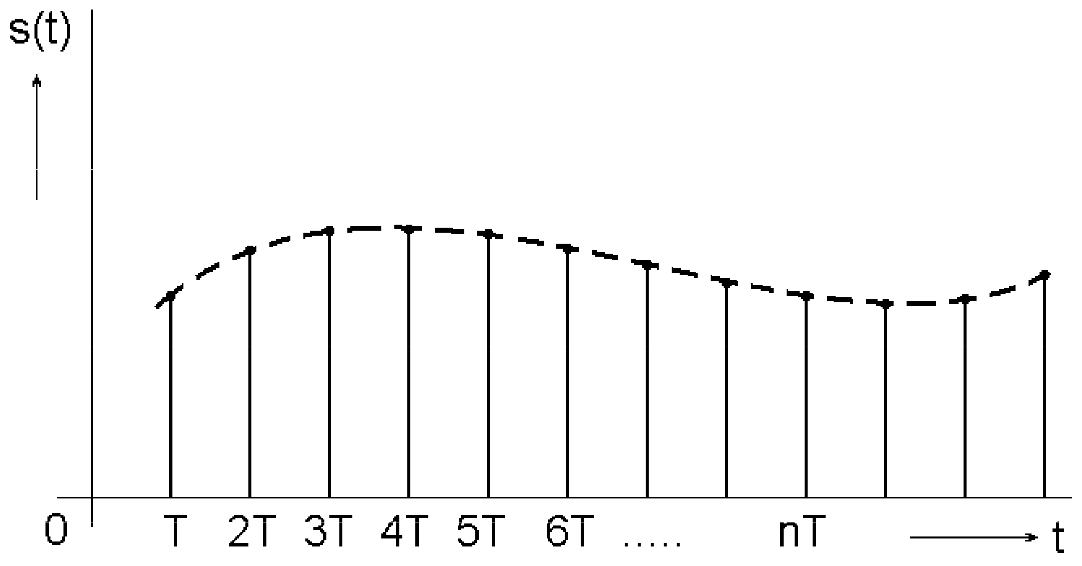


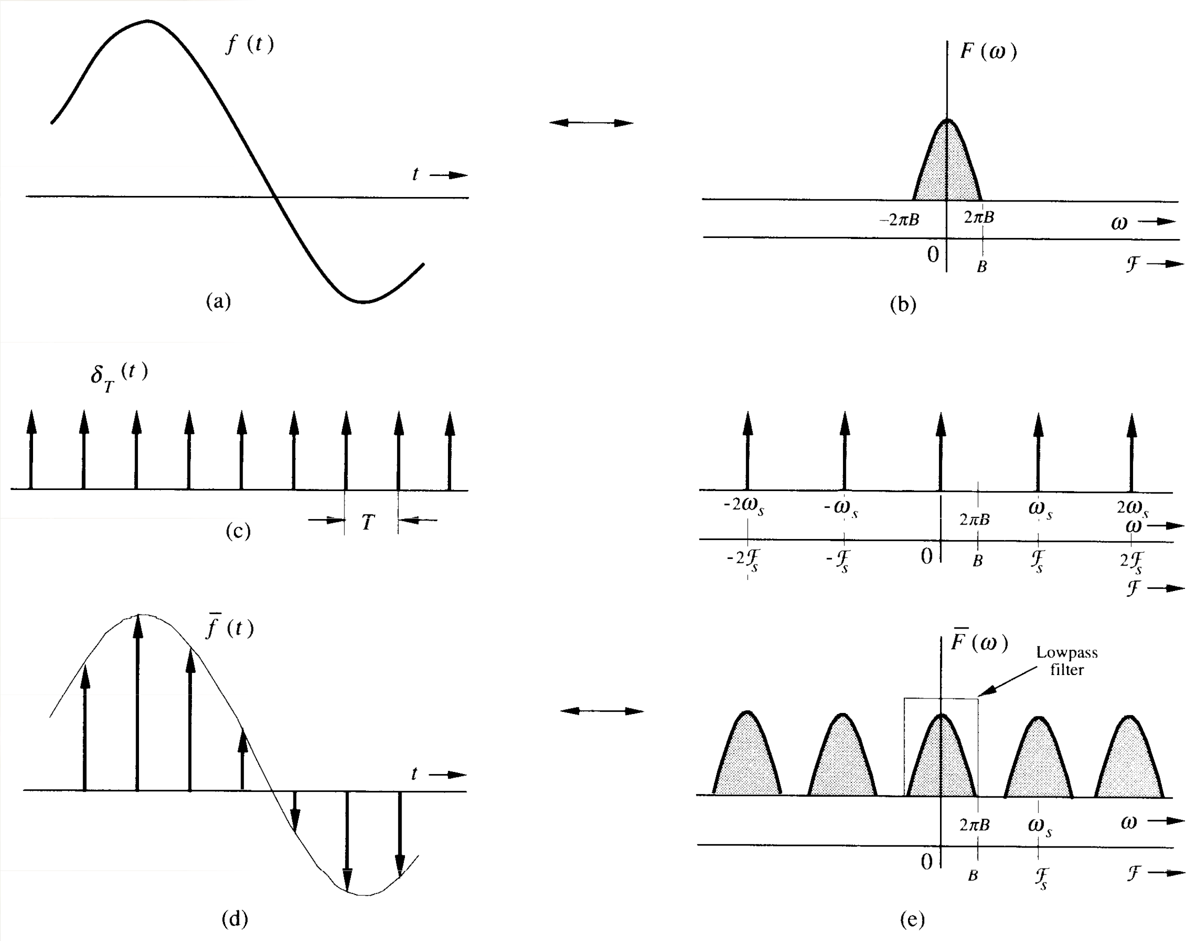
# VZORKOVACÍ TEORÉM

$$s(t) \rightarrow s(T_1), s(T_2), s(T_3), \ldots, s(T_n), \ldots$$

$$s(t) \rightarrow s(T), s(2T), s(3T), \ldots, s(nT), \ldots$$



# VZORKOVACÍ TEORÉM




**Vzorkovací frekvence:**

$f_s \geq 2B = f_N$,

kde B je maximální kmitočet ve vzorkovaném signálu

$f_N$ –

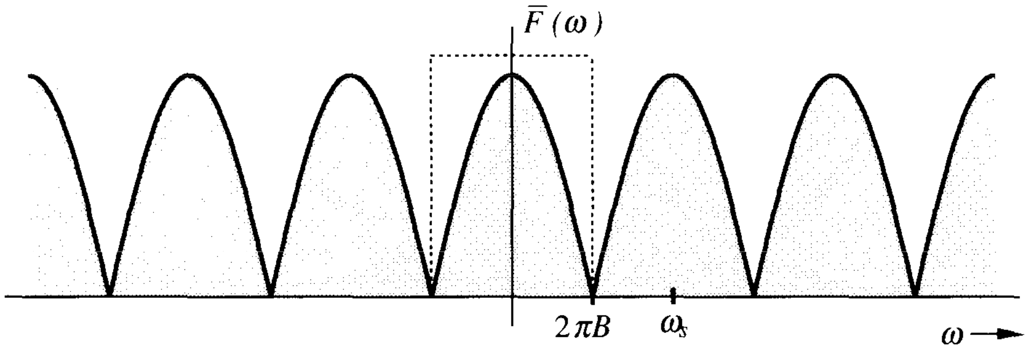
Nyquistův, (Shannonův, Kotelnikovův) kmitočet

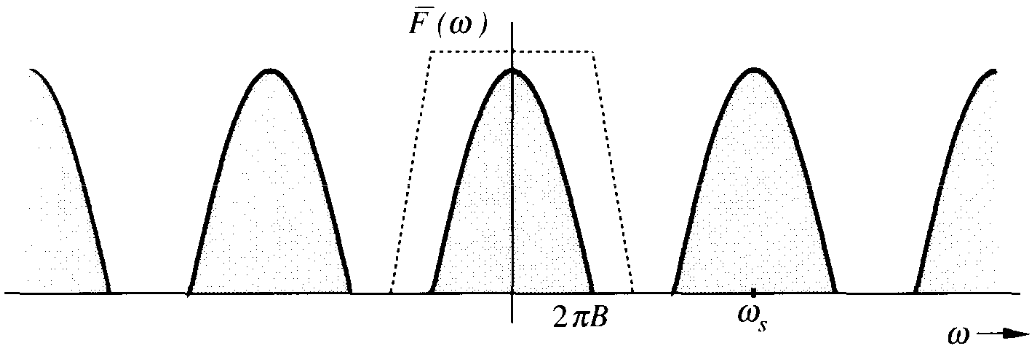
$T_N = 1/f_N = 1/2B$

Nyquistův interval (perioda), vzorkovací interval (perioda)

# VZORKOVACÍ TEORÉM

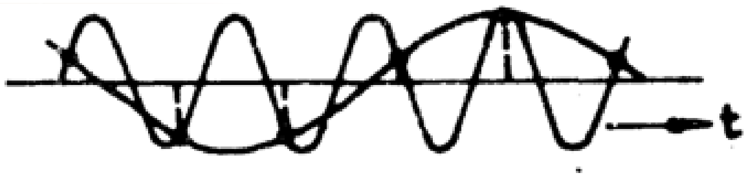
Reálné vz

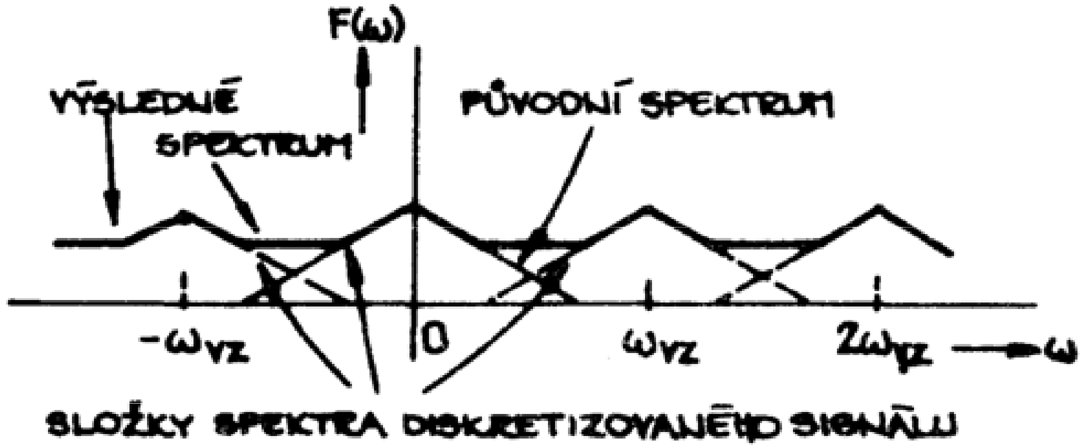


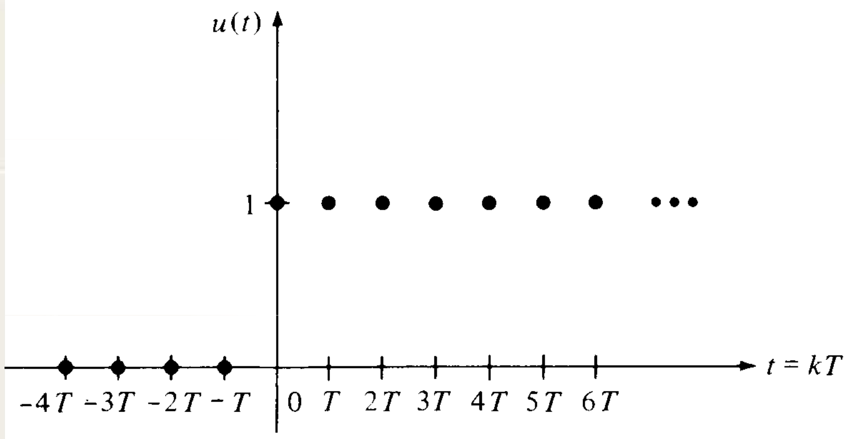

$f_{sr} = (4 \div 5).f_N$

# VZORKOVACÍ TEORÉM



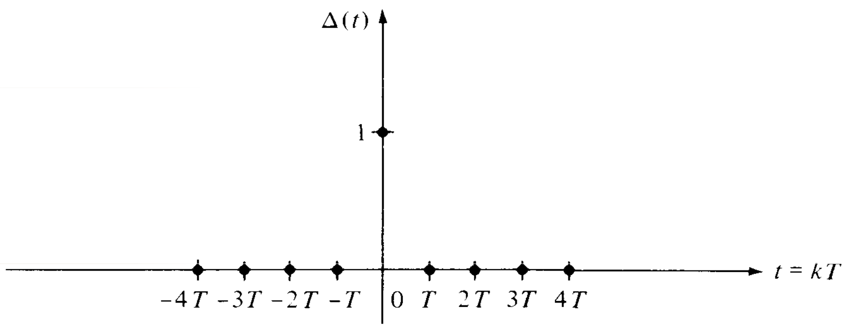


# JEDNORÁZOVÉ DISKRÉTNÍ SIGNÁLY




- ☑ jednotkový skok

$$\Sigma(t) = \begin{cases} 0, & t = kT, k = \dots, -2, -1, \\ 1, & t = kT, k = 0, 1, 2, \dots \end{cases}$$




- ☑ jednotkový impuls

$$\Delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t = kT, k \neq 0 \end{cases}$$

# PERIODICKÉ DISKRÉTNÍ SIGNÁLY

- diskrétní signál x(kT) je periodický s periodou NT, když platí

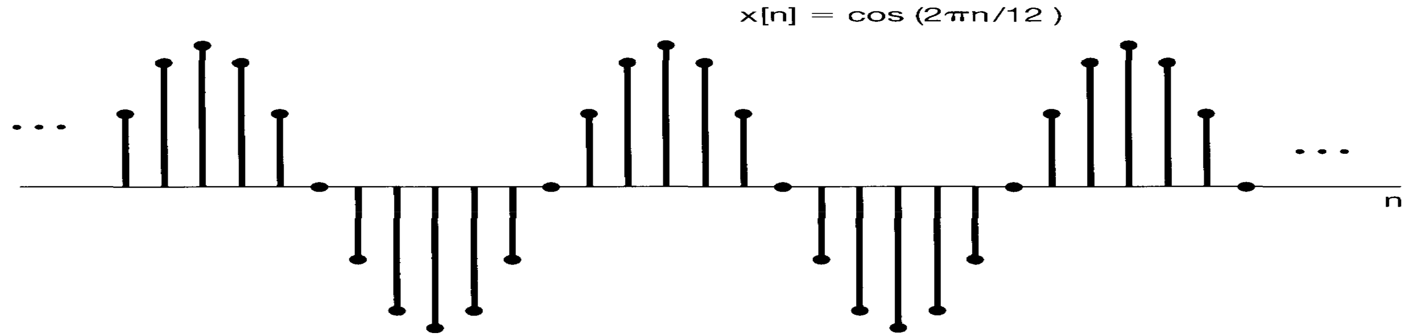
  $x[(k+N)T] = x(kT)$, pro $k = 0, \pm 1, \pm 2, \ldots$

- příklady
  - $x(kT) = A.\cos(2\pi k/N)$
  - $x(kT) = A.\sin(2\pi k/N)$
  - $x(kT) = A.\exp(j2\pi k/N)$

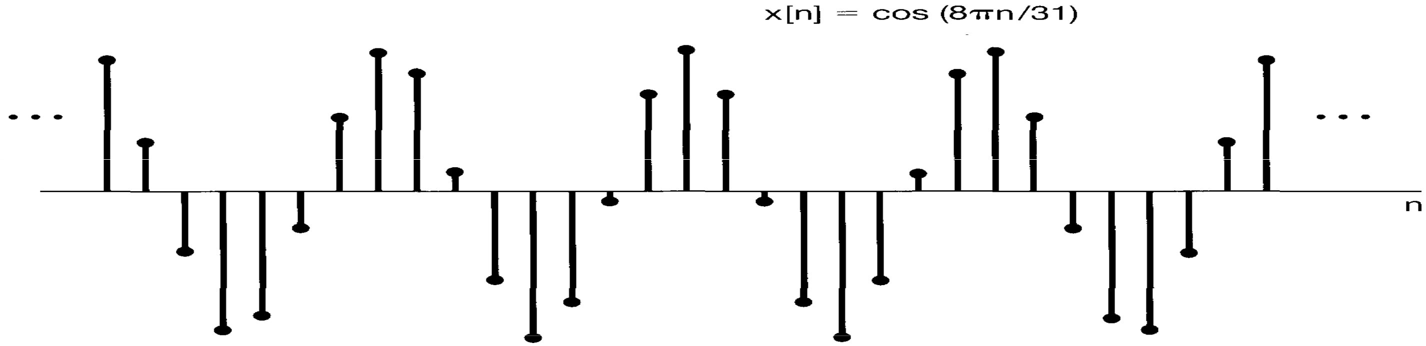
$$x[(k+N)T] = \exp\frac{j2\pi(k+N)}{N} = \exp\frac{j2\pi k}{N}.\exp(j2\pi)$$

$$\exp(j2\pi) = \cos 2\pi + j\sin 2\pi$$

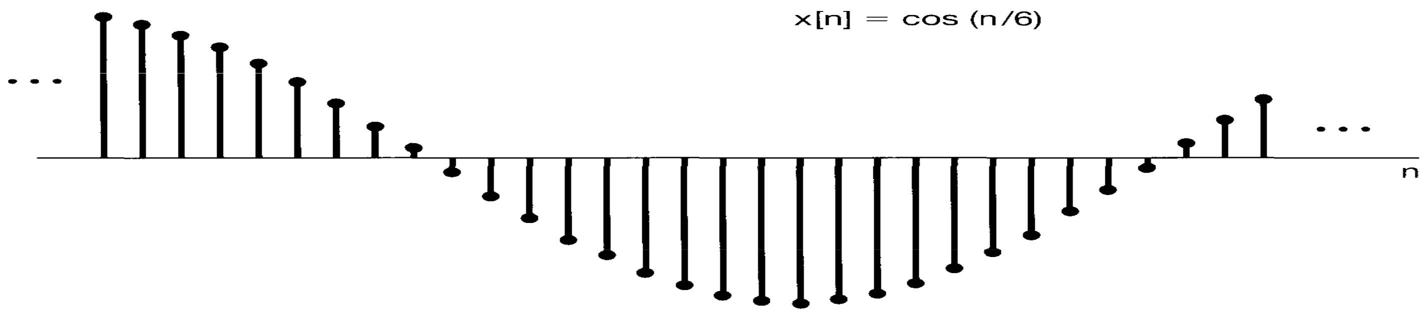
# HARMONICKÝ DISKRÉTNÍ SIGNÁL




(a)

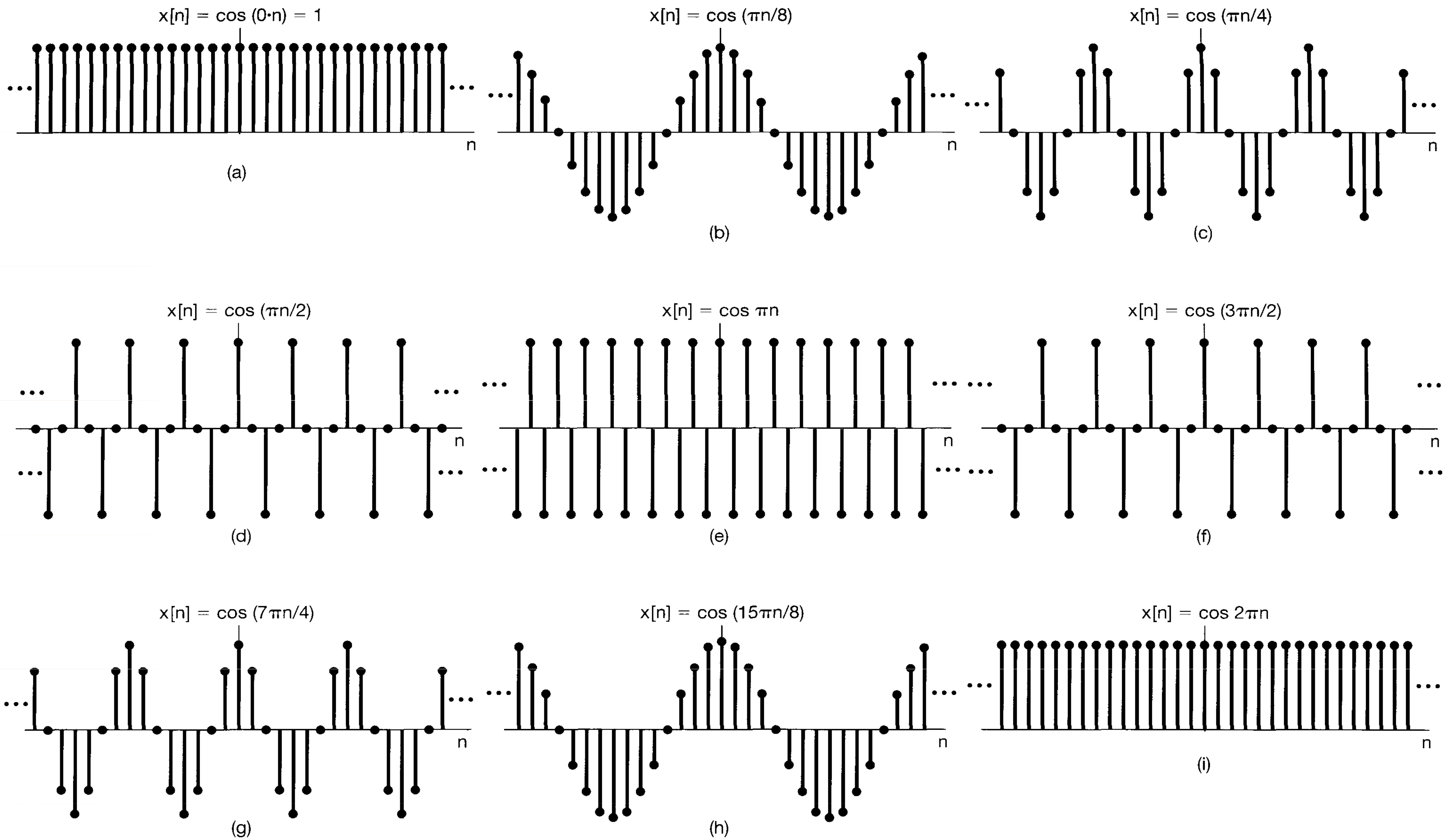



(b)




(c)

# HARMONICKÝ DISKRÉTNÍ SIGNÁL



Příprava nových učebních materiálů pro obor Matematická biologie

je podporována projektem ESF

č. CZ.1.07/2.2.00/07.0318

# „VÍCEOBOROVÁ INOVACE STUDIA MATEMATICKÉ BIOLOGIE“

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