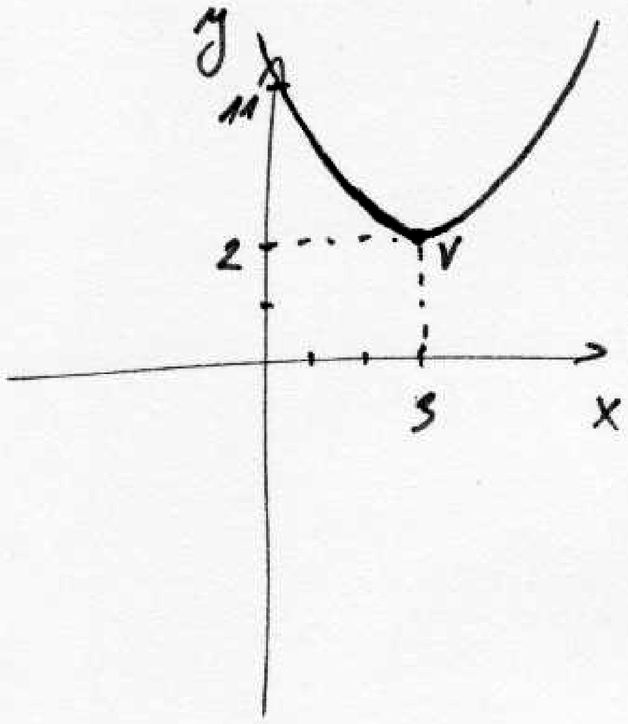
1. zápočtová písemka – A

1. a) $y = x^2 - 6x + 11$



· doplnění na čtverec:

$$y = \underbrace{x^2 - 6x + 9}_{(x-3)^2} \underbrace{-9 + 11}_{+2}$$

$$y - 2 = (x-3)^2 \Rightarrow \underline{V = [3;2]}$$

· maximum/minimum přes derivace:

$$y' = 2x - 6 = 0 \Rightarrow x = 3, \Rightarrow y = 2 \quad \Big\} \; V = [3;2]$$

· prus. s osami – s osou x: $y = 0 \Rightarrow \underline{0 = x^2 - 6x + 11}$

diskriminant: $D = 36 - 4 \cdot 11 = -8$

$\underline{D < 0}$

$\Rightarrow$ prus. s osou x neexistuje.

– s osou y: $x = 0 \Rightarrow y = 11$

· definiční ~~ob~~ obor, obor hodnot: $D(f) = \mathbb{R} = (-\infty; \infty)$; $H(f) = \langle 2; \infty)$

b) $r(t) = 2t^5 e^t + 7t^2$

$$v(t) = r'(t) = 2(5t^4 e^t + t^5 e^t) + 14t = \underline{10t^4 e^t + 2t^5 e^t + 14t}$$

$$a(t) = r''(t) = \cancel{40t^3 e^t}\, 10(4t^3 e^t + t^4 e^t) + 2(5t^4 e^t + t^5 e^t) + 14 =$$

$$= \underline{\underline{40t^3 e^t + 20t^4 e^t + 2t^5 e^t + 14}}$$

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2. $a = 0{,}3\,\mathrm{m/s^2}$

$v(t) = v_0 + at$

$s(t) = v_0 t + \frac{1}{2} a t^2$

~~a)~~

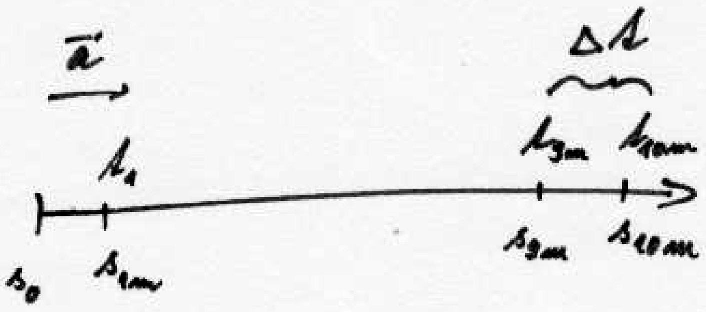
a) $s_{1m} = \underbrace{v_0 t_{1m}}_{0} + \frac{1}{2} a t_{1m}^2 \Rightarrow \underline{\underline{t_{1m} = \sqrt{\frac{2s_{1m}}{a}} = 2{,}58\,s}}$

b) $t_{9m} = \sqrt{\frac{2s_{9m}}{a}}$

$t_{10m} = \sqrt{\frac{2s_{10m}}{a}}$

$$\underline{\underline{\Delta t = t_{10m} - t_{9m} = 0{,}41\,s}}$$

c) $v_{10m} = \underbrace{v_0}_{0} + a t_{10m} = \underline{\sqrt{2 s_{10m} a} = 2{,}45\,\mathrm{m/s}}$

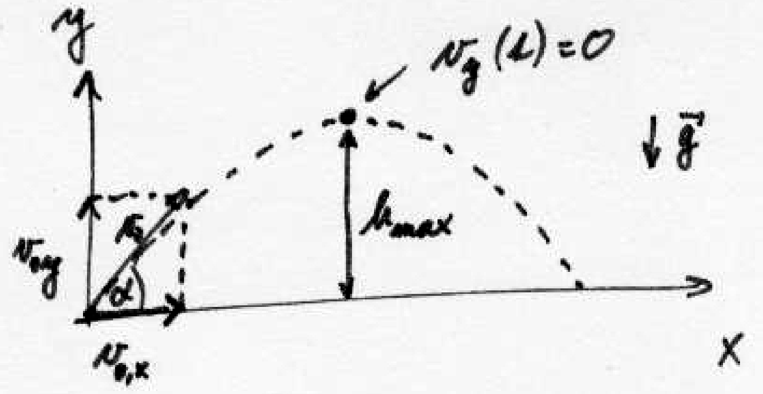


3. $t = 4\,s$

$\alpha = 60^\circ$

$g = 10\,m/s^2$

$v_0 = ?$



$x:\quad x(t) = v_{0,x} \cdot t$

$v_x(t) = v_{0,x}$

$y:\quad y(t) = v_{0,y}\, t - \frac{1}{2} g t^2$

$v_y(t) = v_{0,y} - g t$

$v_{0,x} = v_0 \cos\alpha$

$v_{0,y} = v_0 \sin\alpha$

$$\underbrace{v_y(t)}_{0} = v_{0,y} - g t$$

$$v_{0,y} = g t$$

$$v_0 = \frac{g t}{\sin\alpha} = 46{,}2\,m/s$$

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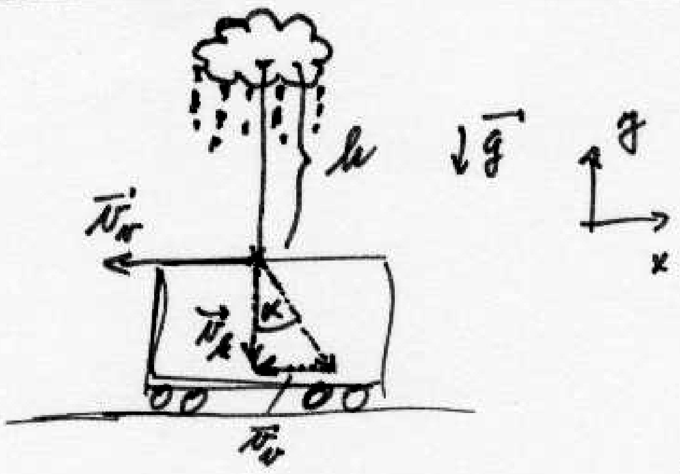
4. $v_v = 90\,km/h = 25\,m/s$

$h = 400\,m$

$g = 10\,m/s^2$

$\alpha = ?$



~~$x: x(t)$~~

$y(t) = v_{0,y}\, t - \frac{1}{2} g t^2$

$v_y(t) = v_{0,y} - g t$

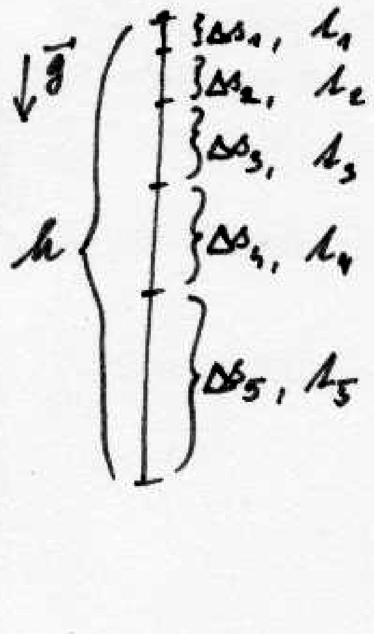
$$h = \underbrace{v_{0,y}}_{0} t_0 - \frac{1}{2} g t_0^2$$

$$\Rightarrow t_0 = \sqrt{\frac{2h}{g}}$$

$$v_k = \underbrace{v_{0,y}}_{0} - g t_0 = -g t_0 = -\sqrt{2hg}$$

$$\mathrm{tg}\,\alpha = \frac{v_v}{v_k} \Rightarrow \alpha = 15{,}7^\circ$$

BONUS



$t_1 = t_2 = \dots = t_5 = t$

$h = \Delta s_1 + \Delta s_2 + \dots + \Delta s_5$

$s_1 = \frac{1}{2} g t^2$

$s_2 = \frac{1}{2} g (2t)^2$

$\vdots$

$h = s_5 = \frac{1}{2} g (5t)^2$

$$h = \frac{1}{2} g (5t)^2$$

$$5t = \sqrt{\frac{2h}{g}} \Rightarrow t = \frac{1}{5}\sqrt{\frac{2h}{g}} = 1{,}4\,s$$

$\Delta s_1 = s_1 - 0$

$\Delta s_2 = s_2 - s_1$

$\vdots$

$\Delta s_5 = s_5 - s_4$

$\Rightarrow$

$\Delta s_1 = 9{,}8\,m$

$\Delta s_2 = 29{,}4\,m$

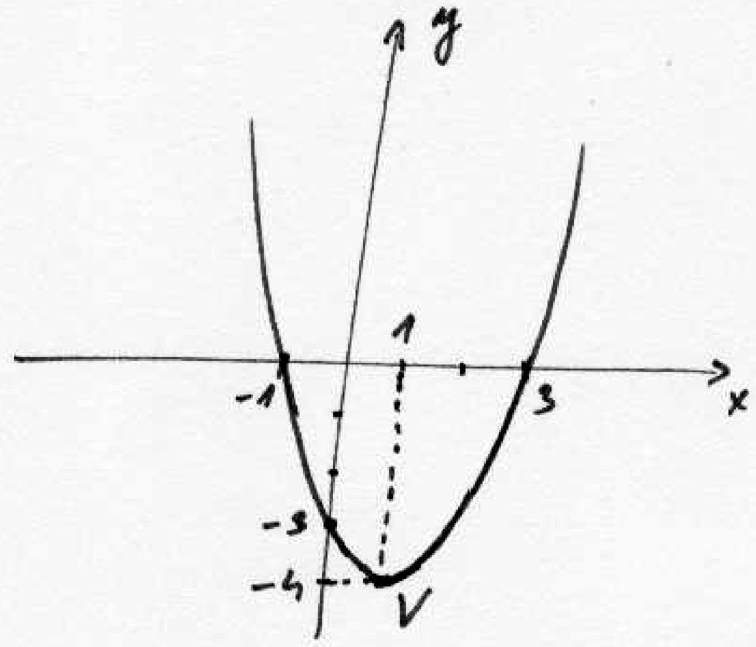
$\Delta s_3 = 49\,m$

$\Delta s_4 = 68{,}6\,m$

$\Delta s_5 = 88{,}2\,m$

1. zápočtová písemka – B

1. a) $y = x^2 - 2x - 3$



• doplnění na čtverec:

$$y = \underbrace{x^2 - 2x + 1}_{(x-1)^2} \underbrace{-1-3}_{-4}$$

$$y + 4 = (x-1)^2 \Rightarrow V = [1; -4]$$

• maximum / minimum přes derivace:

$$y' = 2x - 2 = 0 \Rightarrow \left.\begin{array}{l} x = 1 \\ y = -4 \end{array}\right\} V = [1; -4]$$

• prus. s osami – s osou x: $y = 0 \Rightarrow 0 = x^2 - 2x - 3$

diskriminant: $D = 4 + 12 = 16$

$$x_{1,2} = \frac{2 \pm \sqrt{16}}{2} = \left\langle \begin{array}{l} x_1 = 3 \\ x_2 = -1 \end{array}\right.$$

– s osou y: $x = 0 \Rightarrow y = -3$

• definiční ~~ob~~ obor, obor hodnot: $D(f) = \mathbb{R} = (-\infty; \infty)$; $H(f) = \langle -4; \infty)$

b) $r(t) = \dfrac{t^5}{e^t} - 6t = t^5 e^{-t} - 6t$

$$v(t) = r'(t) = 5t^4 e^{-t} + t^5(-e^{-t}) - 6$$

$$a(t) = r''(t) = 5\left(4t^3 e^{-t} + t^4(-e^{-t})\right) - \left(5t^4 e^{-t} + t^5(-e^{-t})\right) = 20t^3 e^{-t} - 10t^4 e^{-t} + t^5 e^{-t}$$

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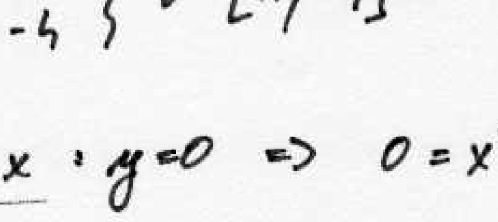
2. $v_0 = 72\,\text{km/h} = 20\,\text{m/s}$

$t_0 = 0{,}5\,\text{min} = 30\,\text{s}$

$s = ?$



$$\underbrace{v(t)}_{0} = v_0 - a t_0$$

$$v_0 = a t_0 \Rightarrow a = \frac{v_0}{t_0}$$

$$s(t) = v_0 t - \frac{1}{2} a t^2 = 300\,\text{m}$$

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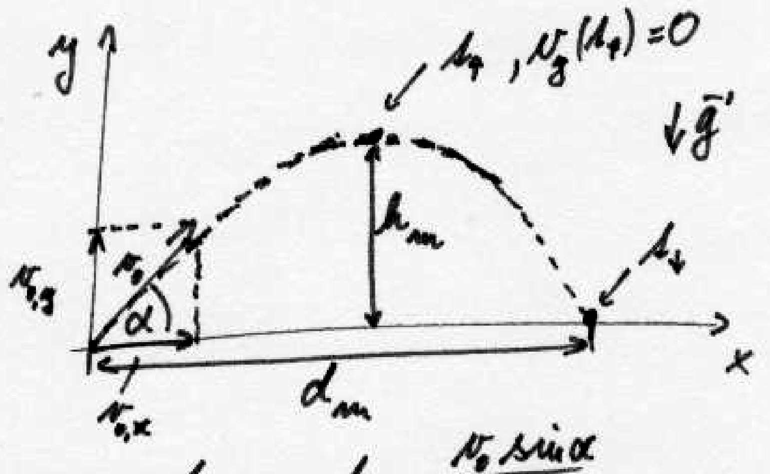
3. $v_0 = 1000\,\text{m/s}$

$\alpha = 55°$

$d_m = ?$

$h_m = ?$



x: $x(t) = v_{0,x} \cdot t$

$v_x(t) = v_{0,x}$

y: $y(t) = v_{0,y} t - \frac{1}{2} g t^2$

$v_y(t) = v_{0,y} - g t$

$v_{0,x} = v_0 \cos\alpha$

$v_{0,y} = v_0 \sin\alpha$

$$v_y(t_1) = 0: \quad 0 = v_{0,y} - g t_1 \Rightarrow t_1 = \frac{v_0 \sin\alpha}{g}$$

$$y(t_1) = h_m = v_{0,y} t_1 - \frac{1}{2} g t_1^2 = v_0 \sin\alpha \cdot \frac{v_0 \sin\alpha}{g} - \frac{1}{2} \cdot g \cdot \frac{v_0^2 \sin^2\alpha}{g^2} = \frac{1}{2} \cdot \frac{v_0^2 \sin^2\alpha}{g^2} = 33550\,\text{m}$$

$t_\downarrow = 2 \cdot t_\uparrow$

$\cancel{d_m = v_x}$ $\quad v_x(t_\downarrow) = d_m = v_{0,x} \cdot t_\downarrow = v_0 \cos\alpha \cdot \frac{2 v_0 \sin\alpha}{g} = \underbrace{\frac{2 v_0^2 \sin\alpha \cos\alpha}{g}}_{\left(= \frac{v_0^2 \sin 2\alpha}{g}\right)} = 93\,960\,m$

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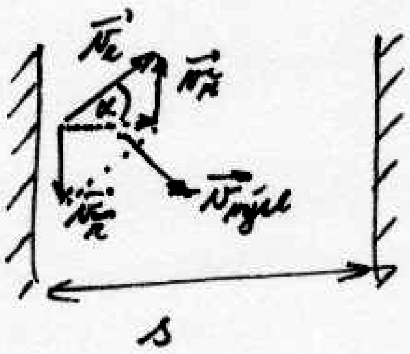
4. $s = 200\,m$
$v_ř = 3\,m/s$
$v_l = 5\,m/s$



a) $\sin\alpha = \frac{v_ř}{v_l} \Rightarrow \alpha = 36{,}9^\circ$

b) $v_{výsl} = \sqrt{v_l^2 - v_ř^2}$, $\quad s = v_{výsl} \cdot t \Rightarrow t = \frac{s}{\sqrt{v_l^2 - v_ř^2}} = 50\,s$