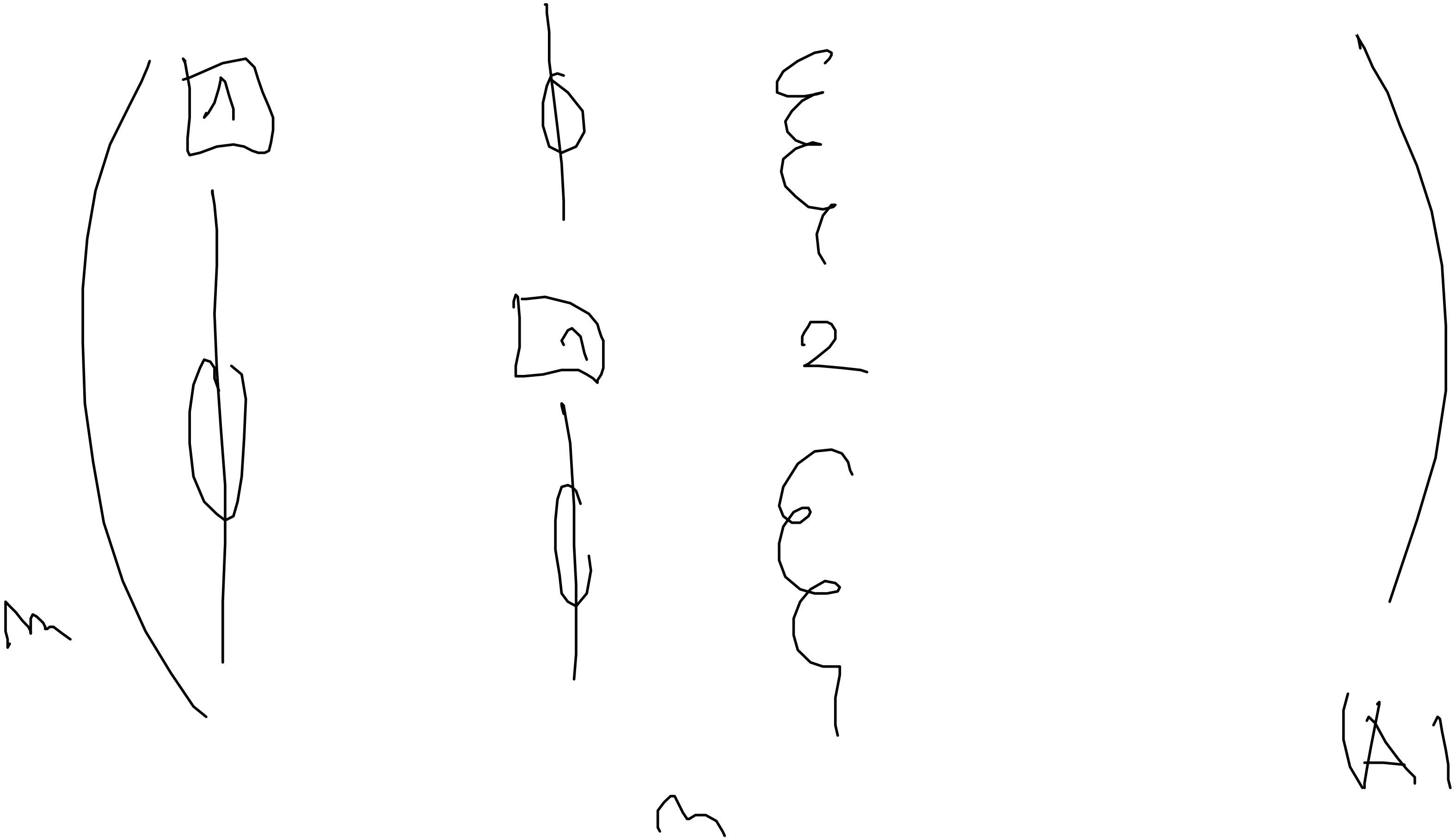


$Ax=b$

$Bx=c$

$(A|b) \to (B|c)$

$Bx = c$ $\qquad k \le m$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ j_1 & j_2 & j_k \end{array}$$

$$\left(\qquad\qquad\qquad \right) x = c$$

$J = \{j_1, \ldots, j_k\}$ $\qquad J' = \{1, \ldots, m\} \setminus J$

$$B'\begin{pmatrix} x_{j_1} \\ \\ x_{j_k} \end{pmatrix} = c - B''\begin{pmatrix} x_{q_1} \\ \\ x_{q_{n-k}} \end{pmatrix}$$

$$\{q_1, \dots, q_{n-k}\} = S'$$

$$\begin{array}{l} k\} \\ \\ n-k \end{array}\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ \hline & 0 & \end{pmatrix}\begin{pmatrix} x_{j_1} \\ \\ x_{j_k} \end{pmatrix} = c - B''\begin{pmatrix} x_{q_1} \\ \\ q_{n-k} \end{pmatrix}$$

$$n-k \quad \begin{pmatrix} x_{j_1} \\ x_{j_2} \\ 0 \\ 0 \end{pmatrix} = e' = e - B'' \begin{pmatrix} x_{q_1} \\ \vdots \\ x_{q_{n-k}} \end{pmatrix} \begin{matrix} x^0 \\ = 0 \\ \\ = 0 \end{matrix}$$

$n-k$

$k \leq m < n$

$q_1 \in J' = \{1, \ldots, n\} - J$

$Ax = b$,

$\exists\ \bar{x} \quad A\bar{x} = b$

$\exists\ x_0 \quad Ax_0 = 0$

$x_0 \neq 0$

$A(\bar{x} + x_0) = b + 0 = b$

$\neq$

$\bar{x}$

$$\begin{array}{cccccc} & & v & & & u \\ \end{array}$$

$$\left(\begin{array}{cccccc|c} 0 & 2 & 3 & 0 & -1 & 4 & 1 \\ 0 & 0 & 0 & -2 & 5 & 4 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 4 \end{array}\right)$$

$$x_2 = -\frac{3}{2}v + \frac{4}{6} - \frac{u}{6}$$

$$-2u + \frac{1}{2}$$

$$= \frac{7}{6} - \frac{3}{2}v - \frac{13u}{6}$$

$$2x_2 + 3x_3 - x_5 + 4u = 1$$

$$3x_5 = 4 - u \Rightarrow x_5 = \frac{4}{3} - \frac{u}{3}$$

$$-2x_4 + 5x_5 + 4u = 0$$

$$-2x_4 + \frac{20}{3} - \frac{5u}{3} + 4u = 0$$

$$\frac{10}{3} + \frac{7u}{6}$$ PODEZŘELÉ! $= x_4$

?

$\mathbb{R}^3$

$$(x_1, x_2, x_3) = x_1(1,0,0) + x_2(0,1,0)$$
$$+ x_3(0,0,1)$$

(A1) $x + y = y + x$

$x + y \in S$

$\boxed{S} \subseteq V$

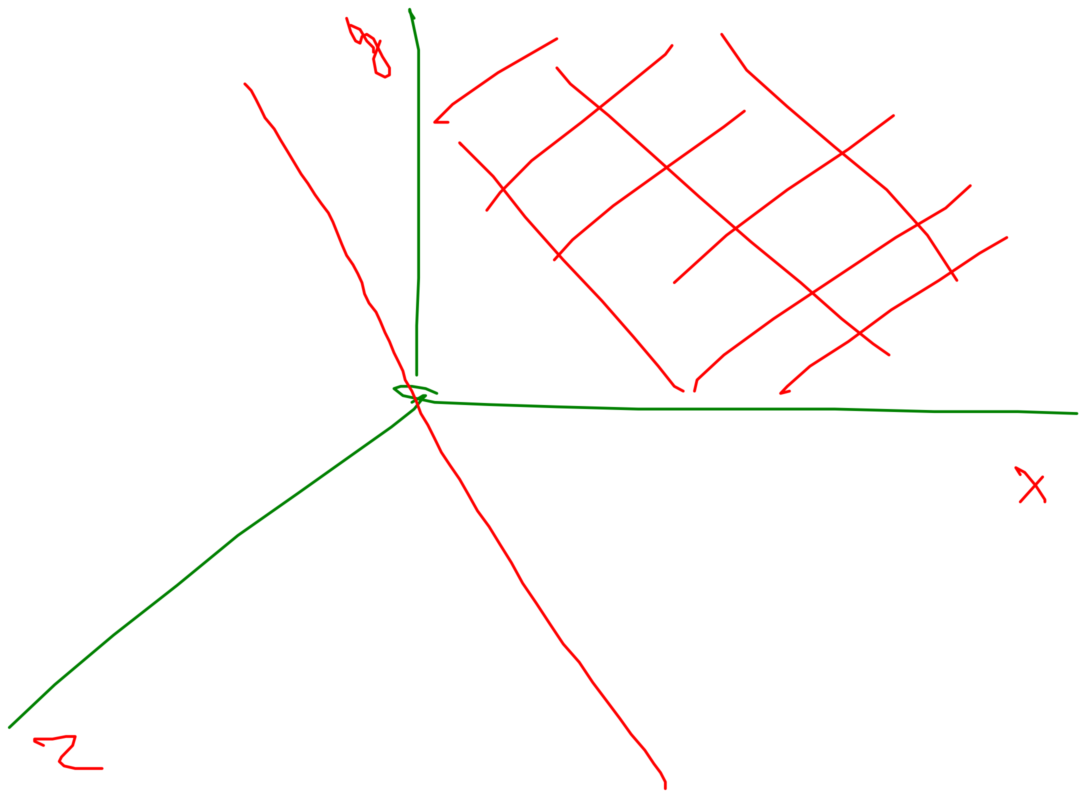
$\underline{x, y \in S}$

$1 x = x$

$\in S$

$(a \cdot b) x = a \cdot (b \cdot x)$

(A8)



y
x
z

(i) $\Rightarrow$ (ii) $\quad S \neq \emptyset$

$x, y \in S, \; a, b \in K \overset{?}{\Longrightarrow} ax + by \in S$

$\Rightarrow \begin{matrix} ax \in S \\ by \in S \end{matrix} \Rightarrow ax + by \in S$

(ii) $\Rightarrow$ (iii) induktiv!

$\underline{n = 0}$ $\quad \sum_{i=1}^{0} a_i x_i = \underline{0 = 0x + 0x} \in S$

$n \Rightarrow n+1$

$a_1, \dots, a_{n+1} \in K$

$x_1, \dots, x_{n+1} \in S \Rightarrow \sum_{i=1}^{n+1} a_i \cdot x_i \in S$

$$\sum_{i=1}^{n+1} a_i \cdot x_i = \left(\underbrace{\sum_{i=1}^{n} a_i \cdot x_i}_{\in S} \right) + \underbrace{a_{n+1} x_{n+1}}_{\in S}$$

$\in S$

(iii) $\Rightarrow$ (i) $\qquad S \neq \emptyset$

$$0 \in S$$

$$a \in K,\ x \in S \Rightarrow ax \in S$$

$$x, y \in S \Rightarrow x + y \in S$$

$$\begin{array}{c} x + y \\ \| \\ 1 \cdot x + 1 \cdot y \end{array}$$

K^X

$K^{(X)} = \{ f \in K^X : \{x \in X : f(x) \neq 0\} \}$

è finito

$f, g \in K^{(X)}$

$(a f + b g)(x) \neq 0$

$f(x) \neq 0$ o $g(x) \neq 0$

$$\mathbb{R}^X \supseteq C(X)$$

$$x_n \to x \implies f(x_n) \to f(x)$$

$$(f+g)(x_n) = f(x_n) + g(x_n)$$

$$(a\,f)(x_n) = a \cdot f(x_n) \to \underline{\underline{af(x)}} \qquad f(x) + g(x) = (f+g)(x)$$

$x \in X$ $\qquad y_1 = \sum_{i=1}^{m} a_i \overset{\in X}{x_i}$ $\qquad 0 \in [X]$

$x = 1x \in [X]$ $\qquad X \subseteq [X]$

$[X]$ is VP

$y_2 = \sum_{j=1}^{n} b_j \overset{\in X}{z_j}$

$y_1 \in [X]$, $y_2 \in [X]$, $a, b \in K$

$\Rightarrow a y_1 + b y_2 \in [X]$

$= \sum_{i=1}^{m} (a a_i) \overset{\in X}{x_i} + \sum_{j=1}^{n} (b \cdot b_j) \overset{\in X}{z_j}$

$X \subseteq S$, S [illegible].

$x_1, \ldots, x_m \in X \subseteq S$

$a_1, \ldots, a_m \in \mathbb{K}$ $\Rightarrow$ $\cancel{\sum_{i=1}^{m} a_i x_i \in S}$

$[X] \subseteq S$

(c) $X \subseteq Y \Rightarrow [X] \leq [Y]$

$\Downarrow$

π

$$\pi = \sum_{i=1}^{n} a_i \, x_i \, , \quad a_i \in \mathbb{K}, \; x_i \in X \leq Y$$

$x_i \in Y$

$$\overline{\overline{\hphantom{xxxxxxxxxxxx}}}$$

$\in [Y]$

(a) X is VP $\overset{?}{\iff} X = [X]$

$X \subseteq X \implies [X] \subseteq X \subseteq [X]$

$\Longleftarrow \quad [X] = X$ $\qquad \overline{}$ $\quad X = [X]$

(e) $[[X]] \overset{?}{=} [X]$

VP $\qquad \Leftarrow$ (a)

(a) $W \in [X] \Longrightarrow W = \sum_{i=1}^{k} b_i y_i, \quad y_i \in X$

$[X \cup \{W\}] = [X]$

$\supseteq$

$T \in [X \cup \{W\}]$ $\qquad [X]$

$T = \sum_{i=1}^{n} a_i x_i + b \cdot W = \qquad x$

$= \sum a_i x_i + \sum b_i y_i$

$\qquad \in X$

$a_i \in K, \quad x_i \in X$

(ii) $\Leftarrow$ $[x) = [x_0, x_0+1] \Rightarrow r$

S_i VP $\quad i \in I \Rightarrow \bigcap_{i \in I} S_i$ is VP

$0 \in S_i \Rightarrow 0 \in \bigcap_{i \in I} S_i$

$x, y \in \bigcap S_i \subseteq S_i \Rightarrow ax + by \in \bigcap_{i \in I} S_i$

$a, b \in K$

$\Downarrow$

$ax + by \in S_i \quad \forall i \in I$

$S + T \leq VP$

$\overset{\in S}{0} + \overset{\in T}{0} = 0 \in S+T$

$x = s_1 + t_1$

$y = s_2 + t_2$

$s_1, s_2 \in S, \quad t_1, t_2 \in T$

$x \in S+T, \quad a, b \in K$?

$y \in S+T \quad \Longrightarrow \quad \underline{ax + by \in S+T}$

$ax + by = a(s_1 + t_1) + b(s_2 + t_2)$

$= as_1 + at_1 + bs_2 + bt_2 \in S+T$

$as_1 + bs_2 \in S, \quad at_1 + bt_2 \in T$

$$S+T = [S \cup T]$$

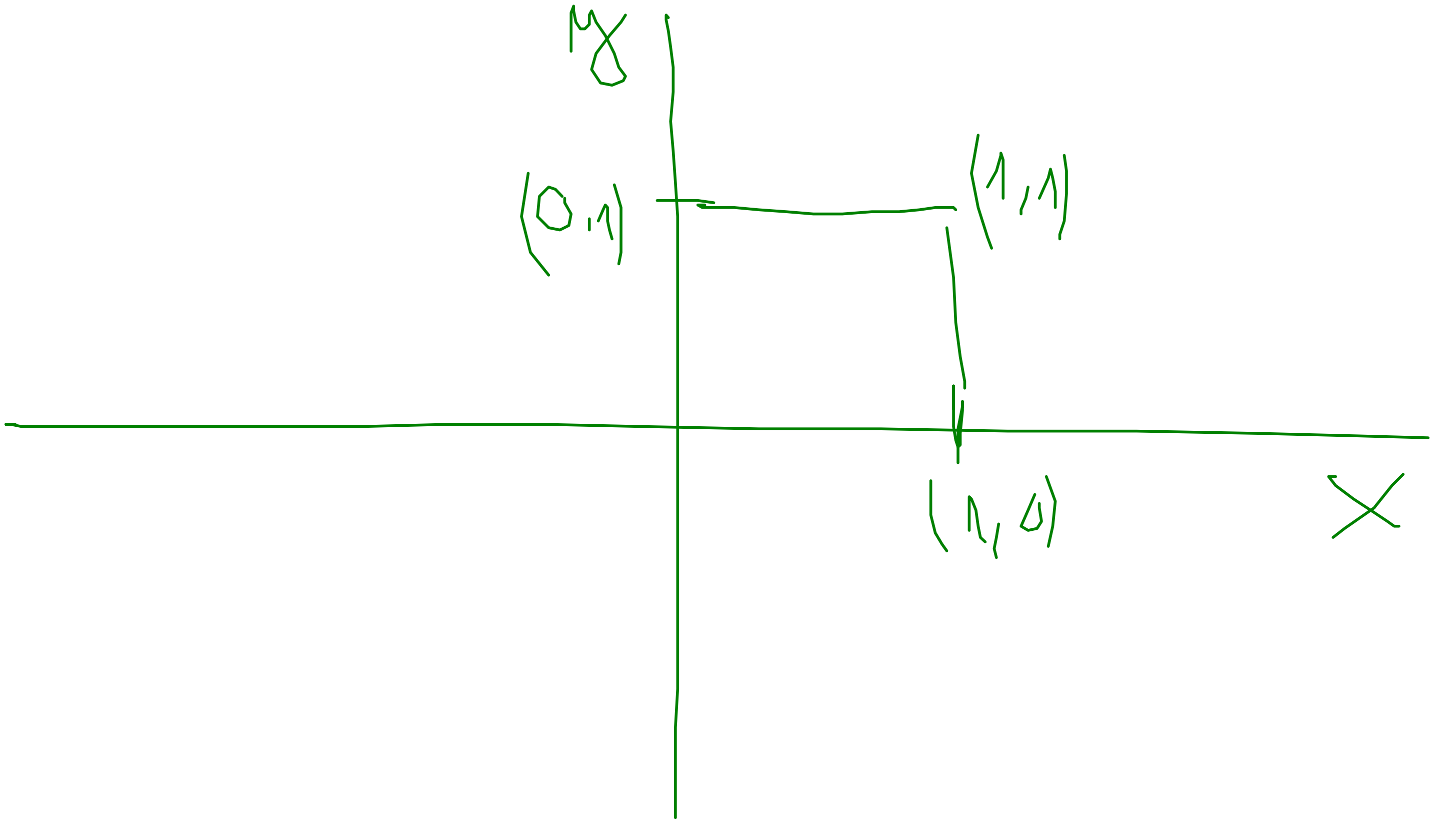
$$S+T \supseteq S = S+0$$

$$S+T \supseteq T \Rightarrow S+T \supseteq S \cup T$$

$$S+T \supseteq [S \cup T]$$

$$\cap$$

$$[S \cup T]$$


y
(0,1)
(1,1)
(1,0)
x

(i) $\Rightarrow$ (ii)   Ans. $\Rightarrow$ yes.

$$\pi_2 = x + y = x' + y' \quad , \quad x, x' \in S$$

$$\pi_2 = x' + y' \qquad\qquad y, y' \in T$$

$$x - x' \in S$$

$$0 = (x - x') + (y - y')$$

$$x = x'$$

$$y = y'$$

$$T \ni y' - y = x - x' \in S \cap T = \{0\}$$

(ii) $\Rightarrow$ (i)

$$x \in S \cap T \Rightarrow \underline{\underline{x = 0}}$$

$$\Downarrow$$

$$x \in S, \quad x \in T$$

$$x = \underset{\in S}{x} + \underset{\in T}{0} \Rightarrow \underline{\underline{x = 0}}$$

$$x = \underset{\in S}{0} + \underset{\in T}{x}$$

$$u_1, \quad \ldots, \quad u_i \qquad u_n$$

$$0 \quad \ldots \quad 1 \qquad 0$$

$$0u_1 + \cdots + 1 \cdot 0 + \cdots + 0u_n = 0$$

$$u_1, \quad \ldots, \quad u_i, \quad u_i \quad u_n$$

$$0 \quad 1 \quad 0 \quad -1 \quad 0$$

$$0 + 1u_i - 1u_i + 0$$

$S_1 \subseteq S$, Say.

$$\overset{=0}{c_1} u_1 + \cdots + \overset{=0}{c_n} u_n + 0 u_{n+1} + \cdots + 0 u_m$$

$\{u_1, \ldots, u_n\} = S_1$

$\{u_1, \ldots, u_m\} = S$

(1, 0, 0), (0, 1, 0) LU

(1, 0, 0), (0, 1, 0), (2, 0, 0) LZ

(1, 0, 0), (0, 1, 0), (0, 0, 1) LU