[X] $\quad c_1 u_1 + \dots + c_k u_k = 0$

(1) $\Rightarrow$ (11) $\quad u_1, \dots, u_n$ LZ

(1) $\Rightarrow$ (11')

$$c_1 u_1 + c_2 u_2 + \dots + c_n u_n = 0$$

$$\exists\, (c_1, \dots, c_n) \neq 0$$

$$k = \max\{ l : c_l \neq 0 \}$$

$$j > k \Rightarrow c_j = 0$$

$$u_k = -\sum_{i=1}^{k-1} \frac{c_i}{c_k} u_i$$

(II) $\Rightarrow$ (III) JASNÉ

(III) $\Rightarrow$ (II)

$$u_k = \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i u_i$$

$$0 = c_1 u_1 + \quad + c_k u_k + \quad + c_n u_n$$

$$c_k = -1 \neq 0$$

$$\Rightarrow \quad x \in [u_1, \ldots, u_m]$$

$$x = \sum_{i=1}^{m} c_i u_i = \sum_{i=1}^{m} d_i u_i$$

$$0 = \sum_{i=1}^{m} \underbrace{(c_i - d_i)}_{=0} u_i$$

$$c_i = d_i \quad \forall i$$

$\Leftarrow$ $\quad c_1 u_1 + \dots + c_n u_n = 0$

$\left(\begin{matrix} z \\ \Rightarrow \end{matrix}\right) \quad (c_1, \dots, c_n) = (0, \dots, 0)$

$0 u_1 + \dots + 0 u_n = 0$

$c_1 = c_2 = \dots = c_n = 0$

$$\underbrace{u_1, \ldots, u_n}_{LN}, v$$

$[u_1, \ldots, u_n] \ni v$.

(i) $\Rightarrow$ (ii)

$v \in [u_1, \ldots, u_n]$

$$v = \sum_{i=1}^{n} c_i u_i$$

$u_1, \ldots, u_n, v$

$(11) \Rightarrow (11')$

$$[u_1, \dots, u_m] \subseteq [u_1, \dots, u_m, v]$$

$$x = \sum_{i=1}^{m} c_i u_i + c v$$

$$v = \sum_{i=1}^{m} d_i u_i$$

$$x = \sum_{i=1}^{m} (c_i + c d_i) u_i \Rightarrow x \in [u_1, \dots, u_m]$$

$$\Big[\underbrace{u_1, \ldots, u_m}_{LN}, \; v^T_1, \ldots, v^T_m\Big]$$

$$\underbrace{\Big[u_1, \ldots, u_m, \; v^T_1, \ldots, v^T_{13}\Big]}_{LN} =$$

$u_1, \ldots, u_n$ LNZ

$u_1, \ldots, u_n, v_1$ LZ

LNZ

$i_1 = 1$ $\quad V_0 = [u_1, \ldots, u_n]$

$k = 1$ $\quad V_1 = [u_1, \ldots, u_n, v_1]$

$V_0 = V_1$ $\left\{ \begin{array}{l} \text{NIC NEPROVEDU} \\ \text{NE ZA RADÍM} \\ \quad \text{index 1 do J} \end{array} \right.$

$k = j$ $\qquad V_j = [\, [V_{j-1}] \cup \{u_j\} \,]$

$V_j = V_{j-1}$ $\qquad$ Nic

$V_j \neq V_{j-1}$ $\qquad$ dominanta j do J

$J = \{ i_1, \ldots, i_k \}$

$u_1, \ldots, u_m, u_{m+1}, \ldots, u_{i_k}$ LN

$$\left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 1 & 1 & 1 & 0 & 5 & 1 \\ -1 & 0 & -3 & 1 & -2 & 1 \\ -1 & 1 & -5 & 2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & -2 & 2 & 4 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cc} 1 & 0 & 3 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & \cancel{2}\,1 & 1\cancel{2} & 1\cancel{2} \end{array}\right) \sim \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \left(\begin{array}{cccc|c|c} 1 & 0 & 3 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \end{array}$$

$\mathbb{R}^2 \notin [x_1, \dots, x_4]$

$v \in [x_1, \dots, x_4]$

$c_4 = 1$

$c_2 = 2 + 2c_3$

$c_1 = 3 - 3c_3$

$X, Y \in K^{m\times n}$ $\quad X' = ?$

$\boxed{X \sim Y}$ $\quad [D_1(Y) \ldots D_{s_k}(Y)] \sim Y'$

$X_j = D_j(Y)$ $\quad \underline{\text{LNZ}}$

$j_1, \ldots, j_k$

$X_{j_1}, \ldots, X_{j_k}$ $\quad$ LNZ

$Y' \cdot c = 0 \Leftrightarrow Y' \cdot R = 0$

$j \notin \{ j_1, \ldots, j_k \}$ $\quad X \sim Y$

$$X_j = \sum_{i=1}^{k} c_{j_i} X_{j_i}$$

$\underline{d} = \max \{ j_1, \ldots, j_k \leq d \}$ $\quad$ RST

$$\left(p_{a_1}(X), \ldots, p_{a_k}(X) \mid X_j \right) \sim \left(Y \mid y' \right)$$

$Y' e = X_j \iff Y' e = y \quad \left(\frac{}{0} \quad \frac{}{0} \right)$

$$\begin{bmatrix} \underline{x_{j_1} \quad \cdots \quad x_{j_k}} \end{bmatrix} = \begin{bmatrix} \underline{x_1 \quad \cdots \quad x_n} \end{bmatrix}$$

$$\begin{pmatrix} 1 & 1 & 3 & -1 & 1 \\ 2 & 0 & 2 & 1 & 3 \\ 1 & 1 & 3 & 2 & 4 \\ 2 & 0 & 2 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 & -1 & 1 \\ 2 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 3 & -1 & 1 \\ 0 & -2 & -4 & 3 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

x_1, x_2, x_4,

(i) $\Rightarrow$ (ii)

$v \in [X]$ $\qquad \exists\, x_1, \ldots, x_k \in X$

$\qquad c_1, \ldots, c_k \in K$

$$v = \sum_{i=1}^{k} c_i x_i$$

$c_1 x_1 + \ldots + c_n x_n - 1v = 0$

$\{x_1, \ldots, x_n, v\}$

$u_1, \ldots, u_n$ LNZ

$C = (c_{ij})$

$u_j \in [v_1, \ldots, v_m]$

$$u_j = \sum_{i=1}^{m} c_{ij} v_i$$

$$(v_1, \ldots, v_m)\begin{pmatrix} c_{1j} \\ \\ c_{mj} \end{pmatrix} = u_j$$

$$\underline{(v_1, \ldots, v_m)\, C = (u_1, \ldots, u_n)}$$

$$(v_1, \ldots, v_m) \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = 0$$

?

$m \geq n$

$m < n$

C $m \times n$ СЛР

$\exists x \quad Cx = 0, \quad x \neq 0$

$$(v_1 \ldots v_m) \cdot C x = 0 = (u_1 \ldots u_n) \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$\underline{\underline{\quad}} \; 0 \Rightarrow x = 0$

(I) ⇒ (II) $\exists\, X \subseteq Y$ $[X] = Y$

$x_n \ldots$

$\exists\, x$. nah. lin. nez. m. (Y)

$y_1 \ldots \ldots y_k \ldots \ldots$

$\mathrm{card}(X) = n$ $y_1 \ldots y_{n+1}$ LNZ

$\in [X]$ $\in [X] = V$

$n+1 \leq n$ SPOR

(ii) $\Rightarrow$ (i)

$y_1 \in V, \quad y_1 \neq 0$

$[y_1] = V$

$\neq$

$\exists\, y_2 \in V - [y_1]$

$y_1, y_2 \qquad V = [y_1, y_2]$

y_1, y_2, y_3

LNZ

$[y_1, \ldots, y_n] = V$

$$Y = [X] \quad X ? \ldots$$

$$X = ? \; x_1 \ldots \ldots x_n \; \}$$

$$[x_1 \ldots \ldots x_n] = [X] = Y$$

LNL

$$\underline{u_1, \ldots, u_k}, v_1, \ldots, v_m$$

$$v_{i_1}, \ldots, v_{i_l}$$

$$[u_1, \ldots, u_k, v_{i_1}, \ldots, v_{i_l}] = L\,U_2$$

$$[u_1, \ldots, u_k, v_1, \ldots, v_m] = V$$

$u_1, \dots, u_n$ baze V

$v_1, \dots, v_m$

LNZ

$[v_1, \dots, v_m] = V$

$\underline{m \le n \le m} \Rightarrow \underline{n = m}$

$\mathbb{C}$ над $\mathbb{R}$

$$z = x + iy = \underset{\in \mathbb{R}}{x} \cdot 1 + \underset{\in \mathbb{R}}{y} \cdot (i)$$

$1, i$ $\quad$ Lin $\quad$ $\dim_{\mathbb{R}} \mathbb{C} = 2$

бz.

$\mathbb{C}$ над $\mathbb{C}$ $\qquad z = z \cdot 1.$

$\dim_{\mathbb{C}} \mathbb{C} = 1.$