$x \in V$ $\quad u_1, \dots, u_n$

LN $\quad [u_1, \dots, u_n] = V$

$x \in$

$x = \sum c_i u_i$

$x = \sum d_i u_i$

$0 = \sum (c_i - d_i) u_i$

$\|$

$0 \Rightarrow c_i = d_i$

$\boxed{\forall x \in V, \quad x = \sum c_i u_i}$ $\exists!\,(c_1, \ldots, c_n)$

$V = [u_1, \ldots, u_n]$

$\boxed{\sum c_i u_i = 0}$

$\overset{?}{\Longrightarrow} (c_1, \ldots, c_n) = 0$

$(0, \ldots, 0)$

✓

$$\alpha = (u_1, \ldots, u_n)$$

$$\forall x \in V \quad \exists! \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} \in K^n$$

$$x = \alpha \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = \sum_{i=1}^{n} c_i u_i$$

$$x \longmapsto (x)_\alpha$$

$$(-)_\alpha : V \longrightarrow K^n$$

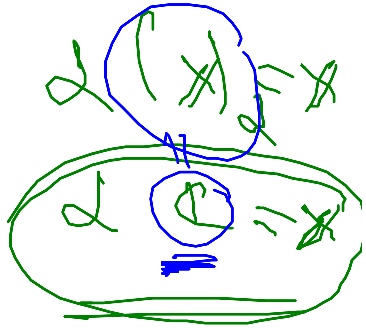
$$f(x) = f(y) \Rightarrow x = y$$

$$x \neq y \Rightarrow f(x) \neq f(y).$$

$$(x)_2 = (y)_2 \overset{?}{\Rightarrow} x = y$$

$$x = 2 \cdot (x)_2 = 2 (y)_2 = y$$

$$\forall c \in k^n \ \exists x \quad (x)_2 = c$$



$$(ax+by)_2 \overset{?}{=} a(x)_2 + b(y)_2$$

$$\partial\,(\underbrace{ax+by}_{})_2 = ax+by$$

$$\partial\cdot\left[a(x)_2 + b(y)_2\right] =$$

$$= a\,\underbrace{\partial(x)_2}_{x} + b\,\underbrace{\partial(y)_2}_{y} = ax+by$$

$$(-)_\alpha : V \to \mathbb{K}^n$$

$$(-)_\alpha^{-1} : \mathbb{K}^n \to V$$

$$\mathbb{a} \longmapsto \alpha\mathbb{a} \in V$$

$$(-)_\alpha \circ (-)_\alpha^{-1} \overset{?}{=} \mathrm{id}_{\mathbb{K}^n}$$

$$(\alpha\mathbb{a})_\alpha = \mathbb{a}$$

$$\alpha \cdot \mathbb{a} = \alpha \cdot (\alpha\mathbb{a})_\alpha \quad \checkmark$$

$$(\ -\ )^{-n}_{2} \circ (\ -\ )_{2} = \mathrm{id}_V$$

$$d(x)_2 = x$$

$S \cap T \subseteq S$ $\quad u_1, \ldots, u_k$ bāze $S \cap T$

$\dim(S \cap T) = k$

$\dim S = n$

$u_1, \ldots, u_k, u_{k+1}, \ldots, u_n$ bāze S

$\dim T = m$ bāze T

$u_1, \ldots, u_k, v_{k+1}, \ldots, v_m$

$S+T=[u_1,\ldots,u_k,u_{k+1},\ldots u_n, v_{k+1},\ldots,v_m]$

? LIN

$$\sum_{i=1}^{m} c_i u_i + \sum_{j=k+1}^{m} d_j v_j = 0$$

$$S \ni \sum_{i=1}^{m} c_i u_i = -\left(\sum_{j=k+1}^{m} d_j v_j\right) \in T$$

$$\sum_{i=1}^{m} c_i u_i \in S \cap T$$

$$\sum_{i=1}^{k} c_i u_i + \sum_{i=k+1}^{m} 0\, u_i$$

$c_1 = c_1, \ldots, c_k = c_k, \quad c_{k+1} = 0 \ldots = c_m$

$$\sum_{i=1}^{k} c_i u_i = -\sum_{j=k+1}^{m} d_j v_j$$

$$\sum_{i=1}^{k} c_i u_i + \sum_{j=k+1}^{m} d_j v_j = 0$$

$$c_1 = \dots = c_k = 0 = d_{k+1} = \dots = d_m$$

$$V \cong V \times \{0\}$$

$$(v, w) = (v, 0) + (0, w)$$

$$W \cong \{0\} \times W = \{(0, w) : w \in W\}$$

$$\underbrace{V \times W}_{n+m} = \underbrace{V \times \{0\}}_{n} \oplus \underbrace{\{0\} \times W}_{m}$$

$\varphi(0)$ $\qquad$ $\varphi : V \to U$ $\quad$ L2

$\varphi(cx) = c\,\varphi(x)$ $\qquad \ominus \quad x \in V$

$0_V = 0_K \cdot 0_V$

$\varphi(0_V) = 0_K \cdot \varphi(0) = 0_U$

$\varphi(0_V) = \varphi(0_K \cdot 0_V) =$

$= 0_K \cdot \underbrace{\varphi(0_V)}_{\in U} = 0_U$

$$\Theta_u = \Theta_v + \Theta_x$$

$$\Theta_{u+v}\,\varphi(\cancel{\Theta_x}) = \varphi(\cancel{\Theta_x}) + \varphi(\Theta_x)$$

$$\Theta_u = \varphi(\Theta_x)$$

$$\varphi(-x) = -\varphi(x)$$

$$\varphi((-1)x) = (-1)\varphi(x) = -\varphi(x)$$

$$\varphi_a(x) = ax \qquad \varphi_a : V \to V$$

$$\varphi_a(x+y) = \varphi_a(x) + \varphi_a(y)$$

$$a(x+y) = ax + ay$$

$$\varphi_a(cx) \stackrel{?}{=} c\,\varphi_a(x)$$

$$= a\cdot(cx) = (ac)x = (ca)x =$$

$$= c(ax) = c\,\varphi_a(x)$$

(I) $\Rightarrow$ (II) (III) $\rightarrow$ (I) 2 : ...

$$\varphi(\underline{ax} + \underline{by}) = \varphi(ax) + \varphi(by) =$$

$$= a\varphi(x) + b\varphi(y)$$

(II) $\Rightarrow$ (III) $\varphi\left(\sum_{i=1}^{n+1} c_i x_i\right) = \varphi\left(1 \cdot \sum_{i=1}^{n} c_i x_i\right) + c_{n+1} x_{n+1}$

$n = 0$, $n = 1$ $= 1 \cdot \varphi\left(\sum_{i=1}^{n} c_i x_i\right) + c_{n+1} \varphi(x_{n+1})$

$$= \sum_{i=1}^{n+1} c_i \varphi(x_i)$$

$$(\varphi \circ \psi)(ax+by) =$$

$$= \varphi(\underbrace{\psi(ax+by)}) =$$

$$= \varphi(a\psi(x) + b\psi(y)) =$$

$$= a\varphi(\psi(x)) + b\varphi(\psi(x)) =$$

$$= a \cdot (\varphi \circ \psi)(x) + b \cdot (\varphi \circ \psi)(x)$$

$\varphi : V \to U$

$S \subseteq V$ hu.

$\underset{\varphi}{0}$ $\quad 0 = \varphi(0) \in \varphi(S) \neq \emptyset$

$x \in \varphi(S)$ $\quad c, d \implies cx + dy \overset{?}{\in} \varphi(S)$

y

$x = \varphi(u) \quad u \in S \qquad \varphi(cu + dv) =$

$y = \varphi(v) \quad v \in S \qquad = c\varphi(u) + d \cdot \varphi(v) = cx + dy$

$$\varphi^{-1}(T) = \{ x \in V : \varphi(x) \in T \}$$

$$\varphi(0) = 0 \in T \Rightarrow 0 \in \varphi^{-1}(T)$$

$x, y \in \varphi^{-1}(T)$, $c, d \in K$

$$\Rightarrow cx + dy \in \varphi^{-1}(T)$$

$\varphi(x) \in T$, $\varphi(y) \in T$ $\qquad cx+dy \Leftarrow \in \varphi^{-1}(T)$

$$c\varphi(x) + d\varphi(y) = \varphi(cx + dy)$$

V v.s. over $\mathbb{R}$

$x \in X$

$g: V \times X \longrightarrow V$

$f: X \to V$

$\downarrow$

$f(x) \in V$

$g(f) = f(x)$

$g(cf + dg) =$

$= c\, g(f) + d\, g(g)$

$(cf + dg)(x) =$

$= c f(x) + d g(x)$

$= c\, g(f) + d\, g(g)$

(x_n) $\quad x_n \to x$

(y_n) $\quad y_n \to y$

$$c(x_n) + d(y_n) \xrightarrow{?} cx + dy$$

$$\lim\ c\,x_n + d\,y_n =$$

$$= c \lim x_n + d \lim y_n = \underline{cx + dy}$$

φ is $\Rightarrow$ $\operatorname{Ker}\varphi = \{0\}$
$\Uparrow$

$x \in \operatorname{Ker}\varphi \stackrel{?}{\Rightarrow} x = 0$

$\varphi(x) = 0 = \varphi(0) \Rightarrow \underline{\underline{x = 0}}$

$\operatorname{Ker}\varphi = \{0\} \Rightarrow \varphi$ inj.

$\stackrel{?}{\ }$

$\varphi(x) = \varphi(y) \Rightarrow x = y \Leftarrow \underline{\underline{x - y = 0}}$

$\underline{\underline{\varphi(x-y) = 0}}$

$\varphi : V \to W$

$\mathrm{Ker}(\varphi) \subseteq V$ $\qquad$ $\mathrm{Im}\,\varphi \subseteq W$

$\dim \mathrm{Im}\,\varphi = n - k$

$v_1, \ldots, v_k$

bázis $\mathrm{Ker}(\varphi)$

$v_1, \ldots, v_k, v_{k+1}, \ldots, v_n$

bázis V

$n = k + (n-k)$

$x \in V \qquad \varphi(x) \qquad \sum_{i=k+1}^{m} c_i v_i = \sum_{i=1}^{k} c_i v_i$

$x = \sum_{i=1}^{m} c_i v_i \qquad \sum_{i=1}^{m} c_i v_i = 0$

$\varphi(x) = \cancel{\sum_{i=1}^{k} c_i \varphi(v_i)} + \sum_{i=k+1}^{m} c_i \varphi(v_i)$

$\sum_{i=k+1}^{m} c_i \varphi(v_i) = 0 \Rightarrow \varphi\left(\sum_{i=k+1}^{m} c_i v_i\right)$