$u_1, \ldots, u_n$ LNZ $\Leftrightarrow$

$Ax = 0 \Rightarrow x = 0$

$\quad = h_o(A)$

$\dim[u_1, \ldots, u_n] = n \Leftrightarrow$

$\Leftrightarrow h(A) = n$

$[u_1, \ldots, u_n] \subseteq \mathbb{K}^{m \times 1} \Rightarrow \dim[u_1, \ldots, u_n] \leq m$

$\Leftrightarrow h_o(A) = m \Leftrightarrow h(A) = m$

$\varphi(x) = Ax$ $\qquad \varphi : \mathbb{K}^n \to \mathbb{K}^m$

$\dim(\operatorname{Im}\varphi) = \dim[\, C_n(A) \ldots, C_n(A)\,] = r(A)$

$r(\varphi) = r(A)$ $\qquad \operatorname{Im}\varphi = \varphi(\mathbb{K}^n)$

$\operatorname{Im}\varphi\circ\psi = \varphi(\psi(\mathbb{K}^h))$

$\psi(y) = By$ $\qquad \psi : \mathbb{K}^h \to \mathbb{K}^n$

$\varphi\circ\psi : \mathbb{K}^h \to \mathbb{K}^m$

$r(A\cdot B) \le r(A)$

$$\boxed{\begin{aligned}&\operatorname{Im}(\varphi\circ\psi) \subseteq \\ &\subseteq \operatorname{Im}(\varphi)\end{aligned}}$$

$$r(A \cdot B) \le r(A)$$

$$r(A \cdot B) = r\left((A \cdot B)^T\right) =$$

$$= r(B^T \cdot A^T) \le r(B^T) = r(B)$$

$$r(A \cdot B) \le \min\left(r(A), r(B)\right)$$

$$AB = BA = I_n$$

$$AC = CA = I_n$$

$$C = C \cdot I_n = C \cdot (A \cdot B) =$$

$$= (CA) \cdot B = I_n \cdot B = B$$

$$A^{-1}$$

$$a \neq 0 \qquad \frac{1}{a} = a^{-1}$$

$U \qquad\qquad V \qquad\qquad \varphi : V \to U$

$\alpha \qquad\qquad \beta \qquad\qquad \downarrow \quad \beta \quad\quad \alpha$

$A = (\varphi)_{\alpha,\beta}$

$\exists A^{-1} \iff \exists \varphi^{-1} : U \to V$

$$\underbrace{(\varphi)_{\alpha,\beta}}_{A} \cdot \underbrace{(\varphi^{-1})_{\beta,\alpha}}_{B} = \underbrace{(id_U)_{\alpha,\alpha}}_{I_n} \qquad \underbrace{\varphi^{-1} \circ \varphi = id_V}_{B \cdot A = I_n}$$

R A $\exists A^{-1}$ $\quad B = (x_1, \dots, x_n)$

$$A(x)_B = (\psi(x))_2$$

$$(\psi(y))_B = A^{-1}(y)_2$$

$$\psi(y) = (x_1, \dots, x_n)(\psi(y))_B$$

$$\left((\varphi \circ \psi)(y) \right)_\alpha = (\varphi)_{\alpha,\beta} (\psi)_{\beta,\alpha} (y)_\alpha$$

$$= \underbrace{A \, A^{-1}}_{I_n} (y)_\alpha = (y)_\alpha$$

$$(\varphi \circ \psi)(y) = y = \mathrm{id}(y)$$

$$(\psi \circ \varphi)(x) = x = \mathrm{id}(x)$$

$\Rightarrow \; r(I_n) = \underline{n}$

$\qquad \|$

$r(A \cdot A^{-1}) \leq r(A) \leq \underline{n}$

$\boxed{r(A) = n}$

$\Leftarrow$ Nech $r(A) = n$

$\varphi(x) = Ax$

φ je $\underline{\text{nad}}$. φ je bijek

$\varphi: \mathbb{K}^n \to \mathbb{K}^n$

$n = \dim \operatorname{Ker} \varphi \quad {}^{\text{“}}0$

$+ \dim \operatorname{Im} \varphi$

$\quad n$

$r(\varphi) = n$

$\dim \operatorname{Im}(\varphi) = n$

$\dim \operatorname{Ker} \varphi = 0$

$$A \mapsto \varphi(x) = Ax$$

$$B \mapsto \psi(y) = By$$

$$A \cdot B = I_n \qquad BA = I_n$$

$$\boxed{\varphi \circ \psi = id_{\mathbb{R}^n}}$$

$$\psi \circ \varphi = id_{\mathbb{R}^n}$$

$\varphi \circ \psi = \mathrm{id}_{\mathbb{K}^n} \Rightarrow \psi \circ \varphi = \mathrm{id}_{\mathbb{K}^n}$

$\Downarrow$ $\quad\quad$ $\Downarrow$

φ je surjekce $\quad$ ψ injekce $\quad\quad$ $\varphi, \psi : \mathbb{K}^n \to \mathbb{K}^n$

$\Downarrow$ $\quad\quad$ $\Downarrow$ surjekce

φ je injekce $\quad$ ψ bijekce $\quad\quad$ $\varphi(x) = y$

φ, ψ jsou bijekce $\quad\quad$ $\varphi \circ \psi(y) = y$

$(\psi \circ \varphi)(x) \stackrel{?}{=} x \quad\quad \cancel{\varphi} \circ \psi \circ \varphi(x) = \cancel{\varphi}(x)$

$$(A^{-1})^{-1} A^{-1} = I_n = A^{-1} \cdot ((A)^{-1})^{-1}$$

$$\|$$

$$A A^{-1} = I_n = A^{-1} A \,!$$

$$(A \cdot B)^{-1}$$

$$A \cdot B \, (B^{-1} A^{-1}) =$$

$$A \, (\underbrace{B \cdot B^{-1}}_{I_n}) A^{-1} = A A^{-1} = I_n$$

$$A \cdot A^{-1} = I_n \qquad T$$

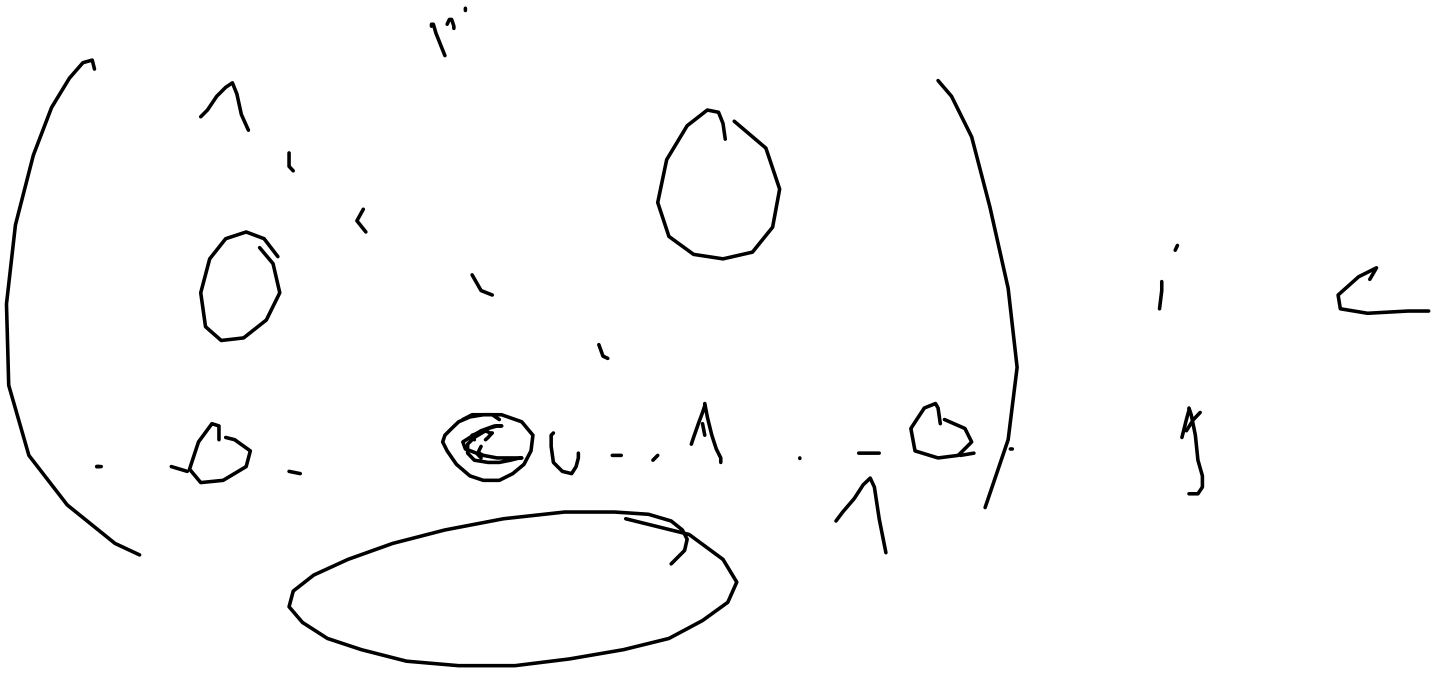
$$(A^{-1})^T A^T = I_n$$

$$\parallel$$

$$(A^T)^{-1}$$

$$A = \begin{pmatrix} \\ \hline \end{pmatrix} \quad \begin{array}{c} i \\ \\ r \end{array} \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$$I_n \to \begin{array}{c} \begin{pmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & & 1 & & \\ & & & 1 \cdots \lambda & \\ 0 & & 1 & & \end{pmatrix} \\ 1 \end{array} \begin{array}{c} \\ i \\ \\ j \end{array}$$



$$A \xrightarrow{E_P} I_n$$

$$(E_m \cdots E_1)A = I_n$$

$$A^{-1}$$

$$I_n \longrightarrow A^{-1} = (E_m \cdots E_1) I_n$$

$\Leftarrow$ JASNÉ (SOUČIN REG. MATIC JE REG.)

$\Rightarrow$ A reg. $\Leftrightarrow$ $\exists A^{-1}$

$A \cdot A^{-1} = I_n$

$\|$

$(A^{-1})^{-1} = E_k \cdots E_1$

A [illegible] B $\Leftrightarrow$

$\exists$ [illegible]

$$A \underset{E_1}{\sim} A_1 \underset{E_2}{\sim} \cdots \underset{E_l}{} A_l = B$$

$$\Updownarrow$$

$$B = \underbrace{E_l \cdot E_{l-1} \cdots E_2 \cdot E_1}_{P} A$$

$$\Updownarrow$$

$$B = PA$$

$$A \rightsquigarrow I_n$$

$$E_k \cdots E_1 = A^{-1}$$

$$B \rightsquigarrow (E_k \cdots E_1) B = \underline{\underline{A^{-1}B}}$$

$$P_{\alpha,\beta} = \left((\pi_j)_\alpha \right)_{j=1}^{n} \qquad \pi_j \in \mathcal{B}$$

$$\alpha \cdot P_{\alpha,\beta}(x)_\beta = \alpha(x)_\alpha$$

$$\shortparallel$$

$$x$$

$$P_{\alpha,\beta}(x)_\beta = (x)_\alpha$$

$$(id)_{\alpha,\beta}(x)_\beta = (id(x))_\alpha = (x)_\alpha$$

$$\psi_\delta = \partial(\psi_\delta)_\partial =$$

$$= \partial \cdot \rho_\delta(P_{\partial,\beta}) =$$

$$= \rho_\delta(\partial P_{\partial,\beta})$$

$$\partial \cdot P_{\partial,\beta} = \beta \quad \| \quad \chi_{\partial,\beta} = (id)_{\partial,\beta}$$

(I) ⇒ (II) $\quad A \cup \gamma \quad [x]_\alpha = P[x]_\beta$

(II) ⇒ (III) $\quad x \to \gamma_\gamma$

$\alpha . P = \beta \qquad$ (III) ⇐ (I)

(III) ⇒ (I) $\quad P[x]_\beta = [x]_\alpha$

$\alpha \cdot P = \beta$

$\alpha \cdot P[x]_\beta = \beta[x]_\beta = x \qquad \alpha \cdot P[x]_\beta = x$

$$\alpha = (u_1, \ldots, u_n) = \beta$$

$$(u_j)_\alpha = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \leftarrow j \qquad P_{\beta,\alpha}^{-1} (x)_\beta = (x)_\alpha$$

$$= P_{\alpha,\beta}$$

$$I_n = ((u_j)_\alpha) = P_{\alpha,\alpha}$$

$$P_{\beta,\alpha} = (P_{\alpha,\beta})^{-1}$$

$$(x)_\beta = P_{\beta,\alpha} \cdot (x)_\alpha \qquad \Big| \; P_{\beta,\alpha}^{-1}$$

$$P_{\alpha,\beta}\, P_{\beta,\gamma} = P_{\alpha,\gamma}$$

$$(id)_{\alpha,\beta} \cdot (id)_{\beta,\gamma} = (\underbrace{id \circ id}_{id})_{\alpha,\gamma}$$

$P \in K^{n \times n}$ $\underline{\text{Per}}$

$\alpha = (u_1, \ldots, u_n)$

$\exists\, \beta, \sigma$

$P = P_{\alpha, \beta} = P_{\sigma 2}$

$$P(x)_{\mathcal{B}} = (x)_{\alpha} \qquad P = P_{\alpha,\mathcal{B}}$$

$$\mathcal{B} = (v_1, \ldots, v_n)$$

$$v_j = \alpha \cdot \rho_j(P)$$

$$P(v_j)_{\mathcal{B}} = P \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - j = \rho_j(P)$$

$$(v_j)_{\alpha} = \rho_j(P_{\alpha,\mathcal{B}}) \qquad \forall j$$

$$P \qquad \exists\, P^{-1}$$

$$w_\gamma = \alpha\, D_\gamma(P^{-1})$$

$$P^{-1} = P_{\alpha,\gamma}$$

$$P = (P_{\alpha,\gamma})^{-1} = P_{\gamma,\alpha}$$

$\alpha = (u_1, \ldots, u_n)$

$\beta = (v_1, \ldots, v_n)$

$B = P_{\varepsilon\beta}$

$\alpha^{-1} = P_{\alpha\varepsilon}$

$P_{\alpha\varepsilon} \cdot P_{\varepsilon\beta} = P_{\alpha\beta}$

$\alpha^{-1} \; B = P_{\alpha,\beta}$

$$(\varphi)_{\beta_2,\beta_1} = (id \circ \varphi \circ id)_{\beta_2,\beta_1} =$$

$$= (id)_{\beta_2,\alpha_2} \cdot (\varphi)_{\alpha_2\alpha_1} \cdot (id)_{\alpha_1,\beta_1}$$

$$= P_{\beta_2,\alpha_2} (\varphi)_{\alpha_2,\alpha_1} \cdot P_{\alpha_1,\beta_1}$$

$$(\mathrm{III}) \Rightarrow (\mathrm{III})$$

$$B = PAQ \qquad r(B) = r(A)$$

$$(\mathrm{III}) \Rightarrow (\mathrm{II})$$

$m \times n$

$$\begin{pmatrix} \begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix} & 0 \\ 0 & \cdots \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & \\ 0 & \ddots & 1 \end{pmatrix}$$