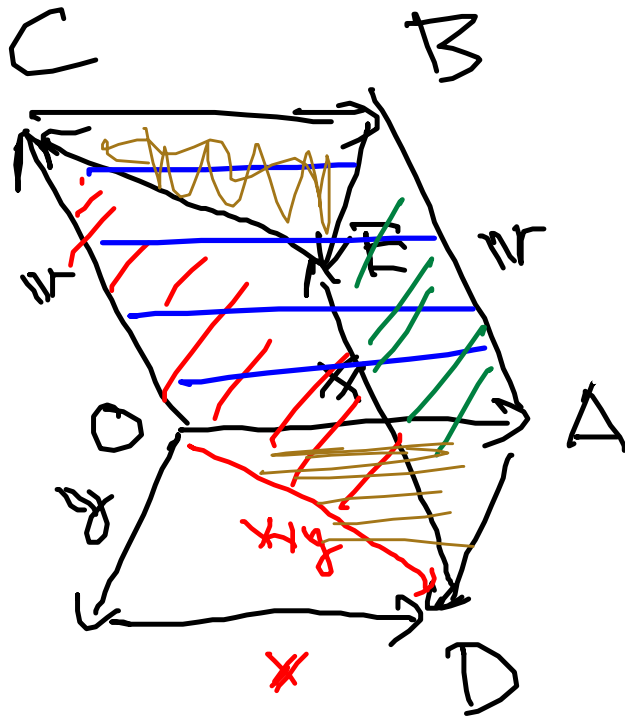


$$P(x+y, y) = P(x, y) + P(y, x)$$



$$P(x+y, A) =$$

$$= P(x, A) +$$

$$- P(y, A)$$

$$P(x, y) = -P(y, x)$$

$$P(x, cy) = cP(x, y)$$

$$P(x+y, \mathbb{N}) = P(x, \mathbb{N}) + P(y, \mathbb{N})$$

$$P(Rx, \mathbb{N}) = R P(x, \mathbb{N})$$

$$P(x, x)$$

ANTISYM \Rightarrow ALT

$$F(u_1, \dots, u_i, \dots, u_j, \dots, u_m) = 0$$

ALT + MULT. \Rightarrow ANTISYM.

$$F(u_1, \dots, \underbrace{u_i + u_j}_{\uparrow i}, \dots, \underbrace{-(u_i + u_j)}_{\downarrow j}, \dots, u_m)$$

$$\begin{aligned}
 &= F(u_1, \dots, \overset{\cdot}{u}_i, \overset{\cdot}{u}_j, \dots, u_m) + \\
 &+ F(u_1, \dots, \underset{\uparrow}{u}_i, \dots, \underset{\downarrow}{u}_j, \dots, u_m) + \\
 &+ F(u_1, \dots, \underset{\uparrow}{u}_j, \dots, \underset{\downarrow}{u}_i, \dots, u_m) + \\
 &+ F(u_1, \dots, \overset{\cdot}{u}_j, \dots, \overset{\cdot}{u}_i, \dots, u_m)
 \end{aligned}$$

(1)

① han...
han...
han...

jasni



② = ① 1^o

③

④

.

⑤

han..

$$F(x_1, \dots, x_i, \dots, x_j + c x_i, \dots, x_n) \\ = F(x_1, \dots, x_n) + c F(x_1, \dots, x_i, \dots, x_i)$$

$$N_2 = \sum_{j < R} c_j x_j$$

$$F(x_1, \dots, \sum_{j < R} c_j x_j, \dots, x_n) = \sum_{j < R} c_j F(x_1, x_j, \dots, x_n)$$

$$A = (a_{ij})$$

$$B = A^T = (b_{kl})$$

$$b_{kl} = a_{lk}$$

$$\det A^T = \det B = \sum_{\sigma \in S_n} (-1)^{|\sigma|} b_{\sigma(1)1} \cdot b_{\sigma(2)2} \cdot \dots \cdot b_{\sigma(n)n}$$

$$= \sum_{\sigma \in S_n} (-1)^{|\sigma|} a_{1\sigma(1)} \cdot \dots \cdot a_{n\sigma(n)}$$

$$\sigma \circ \rho = id$$



$$\prod_{P \in S_3} (-1)^{|P|}$$

$$a_{p(1)1} \cdot a_{1 \cdot 2} \cdot a_{p(m)m}$$

$$P = \sigma^{-1}$$

$$a_{1\sigma(1)} \cdot \dots \cdot a_{m\sigma(m)}$$

$$\sigma \circ \sigma^{-1} = \text{id}$$

$$1 = \sigma \circ \sigma^{-1} = \text{id}$$

$$\prod_{P \in S_3} (-1)^{|P|}$$

$$a_{p(1)1} \cdot \dots \cdot a_{p(m)m} = \det A$$

$$A = \begin{pmatrix} M & C \\ 0 & D \end{pmatrix}$$

$$G = \cup_{P \in \mathcal{P}_m} : \quad j \leq m \Rightarrow \rho(j) \leq m$$

$$P \in G \Rightarrow j > m \Rightarrow \rho(j) > m$$

$$Q \notin G \quad \exists j_0 \leq m, \rho(j_0) > m$$

$$Q \rho(j_0) j_0 = 0$$

$G \Rightarrow \rho$

$$|\rho| = |\rho'| + |\rho''|$$



$$\rho = (\rho', \text{id}) \circ (\text{id}, \rho'')$$

$$\begin{aligned} & \sum_{i=1}^n \frac{1}{i} = O(\ln n) \dots O(m) \quad || \\ & \sum_{i=1}^n \frac{1}{i^2} = O(1) \dots O(2) \quad || \\ & \sum_{i=1}^n \frac{1}{i^3} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^4} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^5} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^6} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^7} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^8} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^9} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{10}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{11}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{12}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{13}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{14}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{15}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{16}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{17}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{18}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{19}} = O(1) \dots O(1) \quad || \\ & \sum_{i=1}^n \frac{1}{i^{20}} = O(1) \dots O(1) \quad || \end{aligned}$$

$$\det(A \cdot B) = \det A \cdot \det B$$

$$\det: K^{m \times m} \rightarrow K$$

$$\det(A) = \det(A \cdot I)$$

$$\det(I) = \det(I \cdot B) = \det B$$

$$\det(A) = c \cdot \det A = \det B \cdot \det A$$
$$c = \det(I)$$

$$F(A) = \det(A \underline{B})$$

$$F(a_1^I, a_2, \dots, a_n) + F(a_1^{II}, a_2, \dots, a_n) = \\ = F(a_1^I + a_1^{II}, a_2, \dots, a_n)$$

$$AB =$$

$$F(\alpha a_1^I, a_2, \dots, a_n) = \alpha F(a_1^I, a_2, \dots, a_n)$$

$$\det(A \quad A^{-1}) = \det(I_n) = 1$$

$$\det A \quad \det A^{-1} \quad \det A^{-1} = \frac{1}{\det A}$$

$$? \quad \det A \neq 0 \quad \stackrel{?}{\Rightarrow} \quad A \text{ reg.}$$

$$\Rightarrow \text{invertible} \quad \Rightarrow \text{lin. indep.}$$