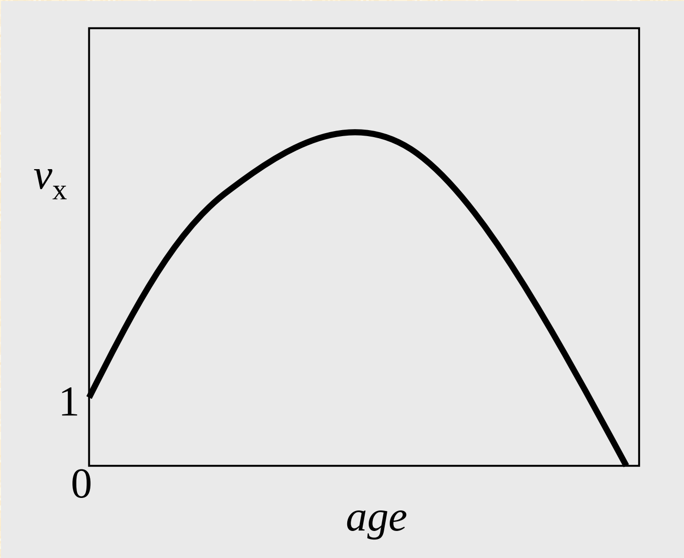
Reproductive value (RV)

- identifies age class that contributes most to the population growth
- measures relative reproductive potential of an individual of a given age
- when population increases then early offspring contribute more to v_x than older ones
- $\mathbf{v}_1$ .. left eigenvector of the dominant eigenvalue of transposed A

$$\mathbf{v}_1'\mathbf{A} = \lambda_1 \mathbf{v}_1$$

- $\mathbf{v}_1$ is proportional to the reproductive values scaled to the first category



$$v_x = \frac{\sum_x^o l_x m_x e^{-rx}}{l_x e^{-rx}}$$

$$RV = \frac{\mathbf{v}_1}{\sum_{i=1}^{S} \mathbf{v}_1}$$

Sensitivity (*s*)

▸ identifies which process (*p*, *F*, *G*) has largest effect on the population increase (λ_1)
- examines change in λ_1 given small change in processes (a_{ij})
- sensitivity is larger for survival of early, and for fertility of older classes

$$s_{ij} = \frac{v_{ij} w'_{ij}}{\mathbf{v.w}}$$

← sum of pairwise products

Elasticity (*E*)

▸ weighted measure of sensitivity
- measures relative contribution to the population increase
- impossible transitions = 0

$$E_{ij} = \frac{a_{ij}}{\lambda_1} s_{ij}$$

Conservation biology

- to adopt means for population promotion or control

Conservation/control procedure

1. Construction of a life table
2. Estimation of the intrinsic rates
3. Sensitivity analysis - helps to decide where conservation/control efforts should be focused
4. Development and application of management plan
5. Prediction of future

Excercise 6

A mouse species has spread dramatically. You perform a life-history study and find that it breeds continuously. So you distinguish age classes based upon 3-months intervals. You obtain the following data:

x	lx	mx
0	1	0
1	0.8	5
2	0.5	12
3	0.3	4

- Compare fecundity (*m*) and *RV* for each age.
- Estimate *r*.
- Predict how the population size would change in another 10 years using initial population structure (30, 10, 5).

```
(0.5+0.8)/(1+0.8)
(0.5+0.3)/(0.8+0.5)

2*(5+0.72*12)/4
2*(12+0.72*4)/4
2*(4+0.72*0)/4

A<-matrix(c(6.8,7.2,2,
0.72,0,0,
0,0.62,0),nrow=3,byrow=T);A

L<-eigen(t(A));L
v<-Re(L$vectors[,1]);v
RV<-v/v[1];RV
x<-c(1,2,3)
mx<-c(5,12,4)
plot(x,mx,type="b",ylim=c(0,12))
lines(x,RV,lty=2)

L<-eigen(A); r<-log(max(Re(L$values))); r

N0<-c(30,10,5)
N1<-A%*%N0;N1
years<-10
Nt<-matrix(0,nrow=nrow(A),ncol=years+1)
Nt[,1]<-N0
for(i in 1:years) Nt[,i+1]<-A%*%Nt[,i]
matplot(0:years,t(Nt),type="l")
legend(2,2e+10,c(1:3),lty=1:3,col=1:3)
```

Excercise 7

There is a butterfly species that appears to be rare. You perform a life-history study and gain data on survival and reproduction. You also observe which factors determine stage-specific survival.

stage	lx	mx	mortality
egg	1	0	frost in winter
larva 1	0.7	0	parasitoids
larva 2	0.3	0	bird predation
pupa	0.25	0	habitat destruction
adult	0.02	80	

- Create transition matrix, estimate λ, and find stable stage distribution.
- Perform sensitivity analysis and identify which factor has most dramatic effect on population change. Suggest a conservation plan.

```
0.7/1
0.3/0.7
0.25/0.3
0.02/0.25

A<-matrix(c(
0,0,0,0,80,
0.7,0,0,0,0,
0,0.43,0,0,0,
0,0,0.83,0,0,
0,0,0,0.08,0),nrow=5,byrow=T);A

L<-eigen(A);L
L1<-max(Re(L$values))
w<-Re(L$vectors[,3]);w
scd<-w/sum(w);scd

M<-eigen(t(A));M
M1<-max(Re(M$values))
v<-Re(M$vectors[,5]);v
s<-v%*%t(w)
ss<-s/as.numeric(v%*%w)
E<-ss*(A/L1);E
```

Temperature

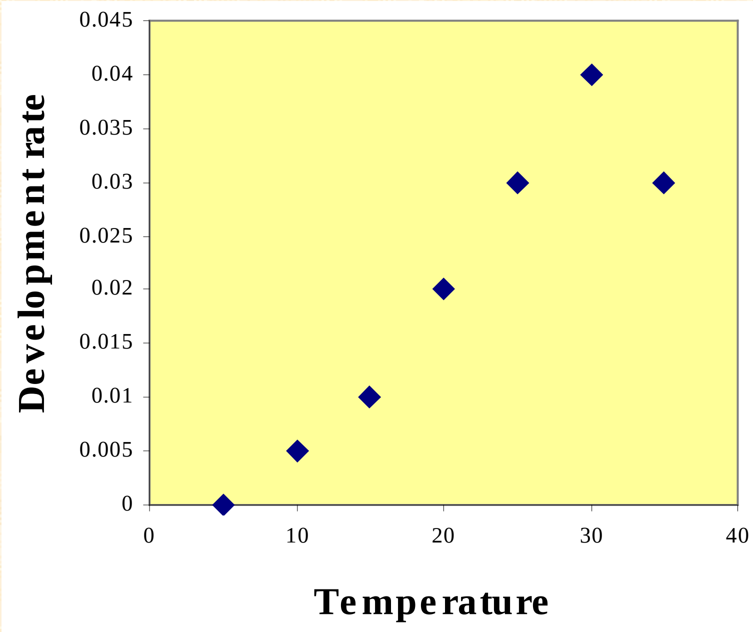
„Populační ekologie živočichů“

Stano Pekár

Linear model

- model is based on the assumption that development rate is a linear function of temperature
- valid for the region of moderate temperatures (15-25°)
- at low temperatures organisms die due to coldness, and at high temperatures organisms die due to overheating

D .. development time (days)
v .. rate of development = $1/D$
t_{min} .. lower temperature limit
.. temperature at which development rate = 0



ET.. <u>effective temperature</u> .. developmental temperature = $t - t_{min}$
S .. <u>degree-days</u> .. number of days required to complete development
.. do not depend on temperature = $D*ET$

t_{min} and *S* can be estimated from the regression line of $v = a + bt$

t_{min} :

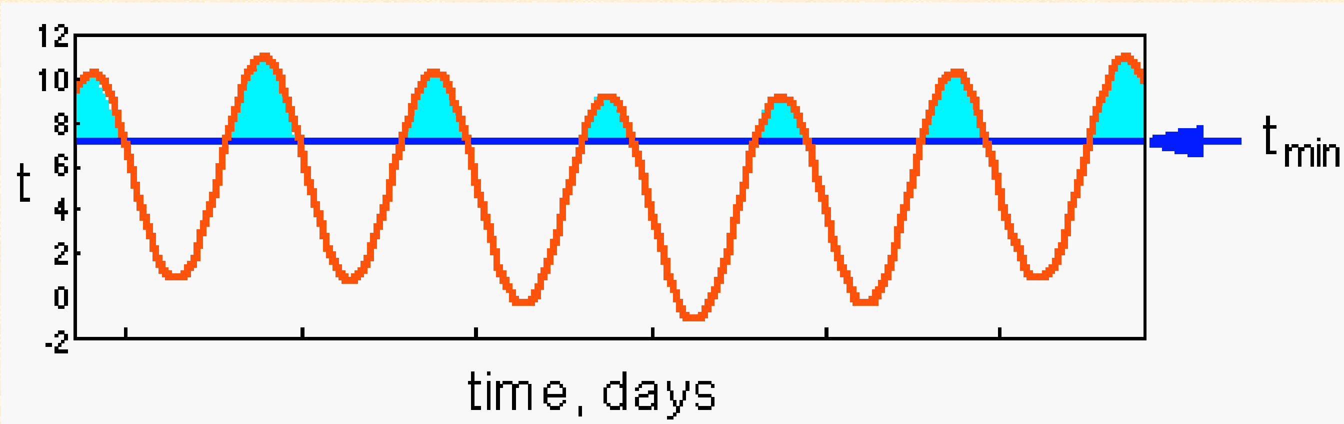
$$a + bt_{min} = 0 \Rightarrow \boxed{t_{min} = -\frac{a}{b}}$$

S :

$$S = D(t - t_{min}) = D\left(t + \frac{a}{b}\right)$$

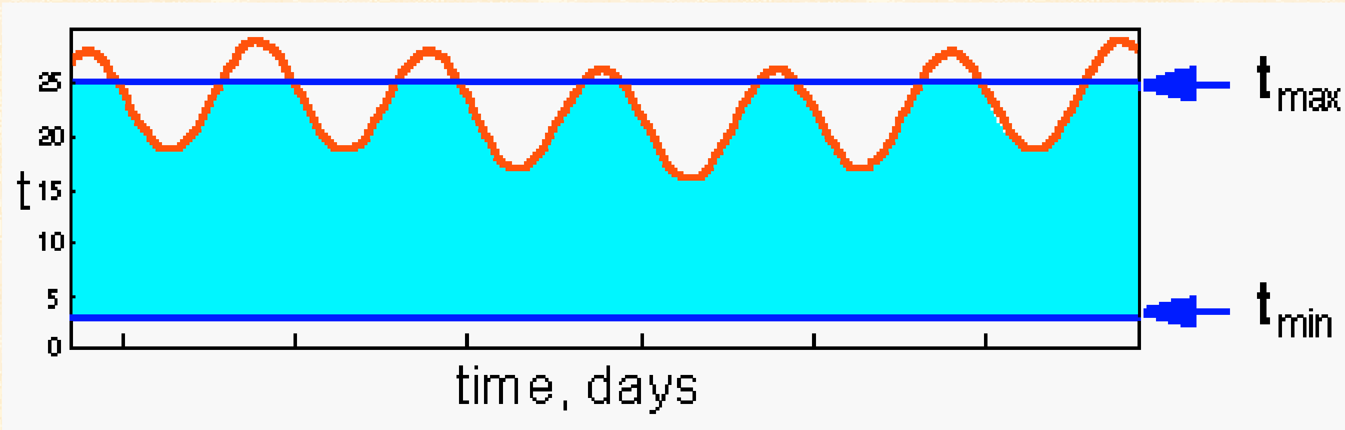
$$D = \frac{1}{v} = \frac{1}{a + bt} \Rightarrow S = \frac{t + a/b}{a + bt} \Rightarrow \boxed{S = \frac{1}{b}}$$

- accumulated degree-days (*S*) are equal to area under temperature curve restricted to the interval between current temperature and t_{min}



Non-linear models

- for temperatures between t_{min} and t_{max} (upper threshold)



- several different non-linear models (Briere, Lactin, etc.)

- allow to estimate t_{min}, t_{max} and t_{opt} (optimum temperature)

- easy to interpret for experiments with constant temperature

- instead of using average temperature, use actual temperature because below and above *ET* model is non-linear

Briere et al. (1999)

$$v = a \times t \times (t - t_{\min}) \times \sqrt{t_{\max} - t}$$

v .. rate of development (=1/D)
t .. experimental temperature
$t_{\min}$ .. low temperature threshold
$t_{\max}$ .. upper temperature threshold
a .. constant

Optimum temperature:

$$t_{opt} = \frac{4t_{\max} + 3t_{\min} + \sqrt{16t_{\max}^2 + 9t_{\min}^2 - 16t_{\min}t_{\max}}}{10}$$

- parameters are estimated using non-linear regression

Lactin et al. (1995)

$$v = e^{\rho t} - e^{(\rho t_m - \frac{t_m - t}{\Delta})} + \lambda$$

v .. rate of development
t .. experimental temperature
t_m, Δ, ρ, λ .. constants

t_{max} and t_{min} can be estimated from the formula:

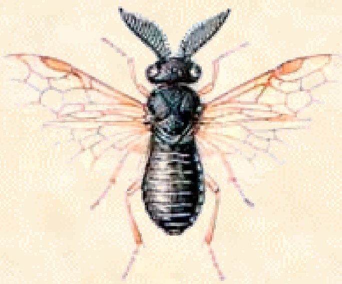
$$0 = e^{\rho t} - e^{(\rho t_m - \frac{t_m - t}{\Delta})} + \lambda$$

t_{opt} can be estimated from the first derivative:

$$0 = \rho e^{\rho \times T} - \left(\rho + \frac{1}{\Delta} \right) \times e^{(\rho T_m - \frac{T_m - T}{\Delta})}$$

Excercise 8

In the laboratory the development of *Diprion pini* was studied. Seven temperatures were used. For each temperature the development time (*D*) of the complete development were recorded:



t (°C)	***D***
5	-
10	49
15	22
20	16
25	12
30	9

- Fit linear model to the data. Estimate the minimum development temperature (t_{min}) and the degree-days (*S*).
- Estimate on which day the development is be complete if you know average day temperatures during two weeks: 17, 18, 21, 23, 24, 25, 23, 24, 21, 25, 22, 25, 26, 22 a 23 °C.

```
t<-c(5,10,15,20,25,30)
D<-c(0,49,22,16,12,9)
v<-1/D
v[1]<-0
plot(t,v)
m<-lm(v~t)
m
abline(m)

-(-0.022336/0.004351)
1/0.004351
tem<-c(17,18,21,23,24,25,23,24,21,25,22,25,26,22,23)
ET<-tem-5.13
plot(cumsum(ET),type="s")
abline(229,0)
```

Excercise 9

Effect of temperature on the development of *Nephus includens* was studied in the laboratory using a range of temperatures.

t	D
18	23.5
20	18.5
22	13
25	7.3
28	5.5
30	5
32	10.9

- Use Lactin‘s model
- Estimate minimal, maximal and optimal temperature.

```
t<-c(18,20,22,25,28,30,32)
D<-c(23.5,18.5,13,7.3,5.5,5,10.9)
v<-1/D
plot(t,v)

m1<-nls(v~exp(rho*t)-exp(rho*Tm-(Tm-t)/delta)+lambda,
start=c(rho=0,Tm=30,delta=1,lambda=0))
summary(m1)

x<-seq(15,40,0.1)
plot(t,v,xlim=c(10,35),ylim=c(0,0.25))
lines(x,predict(m1,list(t=x)))

library(rootSolve)
tminmax<-uniroot.all(function(x) exp(0.01*x)-exp(0.01*33.7-(33.7-
x)/0.7)-1.19,lower=0,upper=40); tminmax
topt<-uniroot.all(function(x) 0.01*exp(0.01*x)-
(0.01+1/0.7)*exp(0.01*33.7-(33.7-x)/0.7),lower=0,upper=40); topt
```