Enemy-Victim Models

“Populační ekologie živočichů“

Stano Pekár

Predator-prey system

Acarus

Cheyletus

Predator-prey model

- continuous model of Lotka & Volterra (1925-1928)

- continuous predation
- capture several prey items, functional response Type II

H .. density of prey
r .. intrinsic rate of prey population
a .. predation rate

P .. density of predators
m .. predator mortality rate
b .. reproduction rate of predators

- in the absence of predator, prey grows exponentially → $\frac{\mathrm{d}H}{\mathrm{d}t} = rH$
- in the absence of prey, predator dies exponentially → $\frac{\mathrm{d}P}{\mathrm{d}t} = -mP$
- predation rate is linear function of the number of prey .. *aHP*
- each prey contributes identically to the growth of predator .. *bHP*

$$\frac{\mathrm{d}H}{\mathrm{d}t} = rH - aHP$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bHP - mP$$

Analysis of the model

<u>Zero isoclines:</u>

- for prey population:

$$\frac{dH}{dt} = 0 \qquad 0 = rH - aHP \qquad \boxed{P = \frac{r}{a}}$$

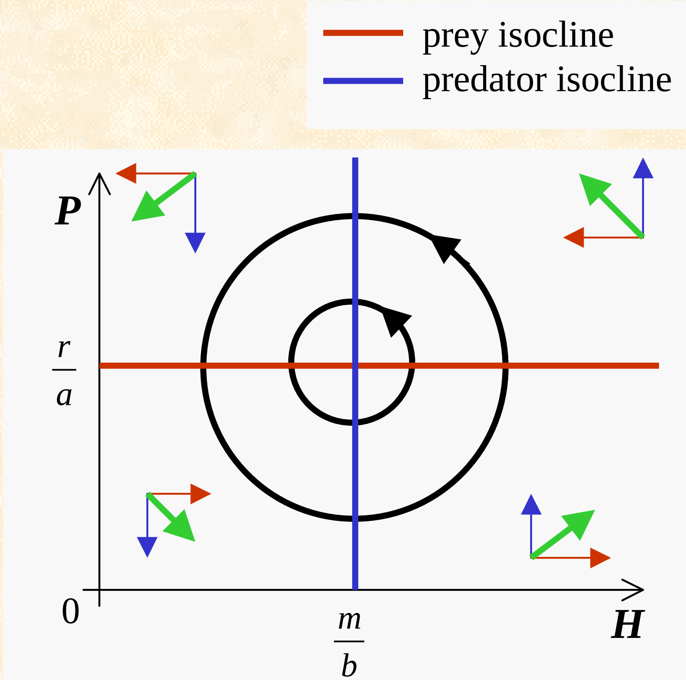
- for predator population:

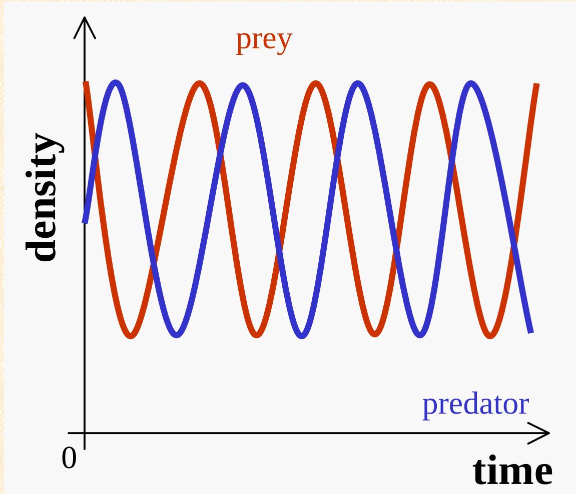
$$\frac{dP}{dt} = 0 \qquad 0 = bHP - mP \qquad \boxed{H = \frac{m}{b}}$$

- prey population would grow to infinity

$\rightarrow$ **neutral stability**

- do not converge, has no asymptotic stability (trajectories are closed lines)
- unstable system, amplitude of the cycles is determined by initial numbers
- POOR model





Incorporation of density-dependence

- in the absence of the predator prey population reaches carrying capacity *K*

$$\frac{\mathrm{d}H}{\mathrm{d}t} = rH\left(1-\frac{H}{K}\right) - aHP$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bHP - mP$$

- for given parameter values: $r = 3$, $m = 2$, $a = 0.1$, $b = 0.3$, $K = 10$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 3H\left(1-\frac{H}{10}\right) - 0.1HP \qquad \frac{\mathrm{d}P}{\mathrm{d}t} = 0.3HP - 2P$$

Zero isoclines:

- for prey population: $$\frac{\mathrm{d}H}{\mathrm{d}t}=0 \qquad 0=3H\left(1-\frac{H}{10}\right)-0.1HP$$

if H = 0 (trivial solution) or if $$0=3\left(1-\frac{H}{10}\right)-0.1P$$

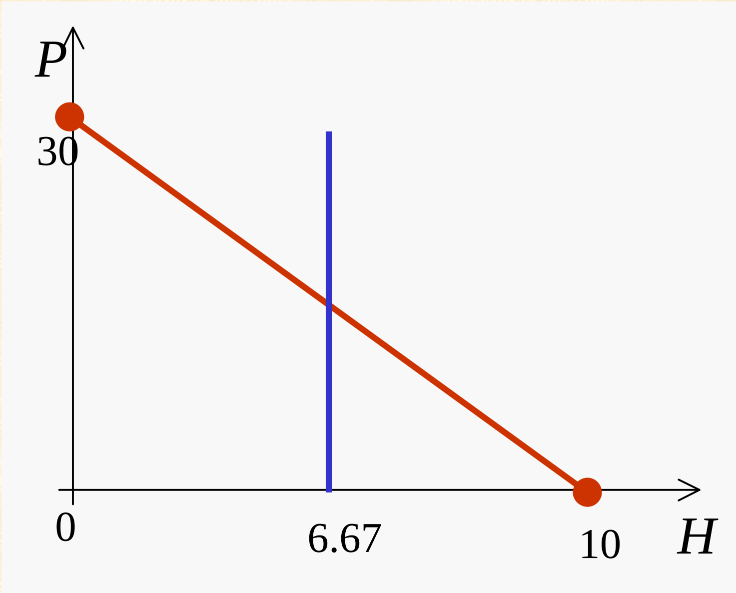
P = 30 - 3H

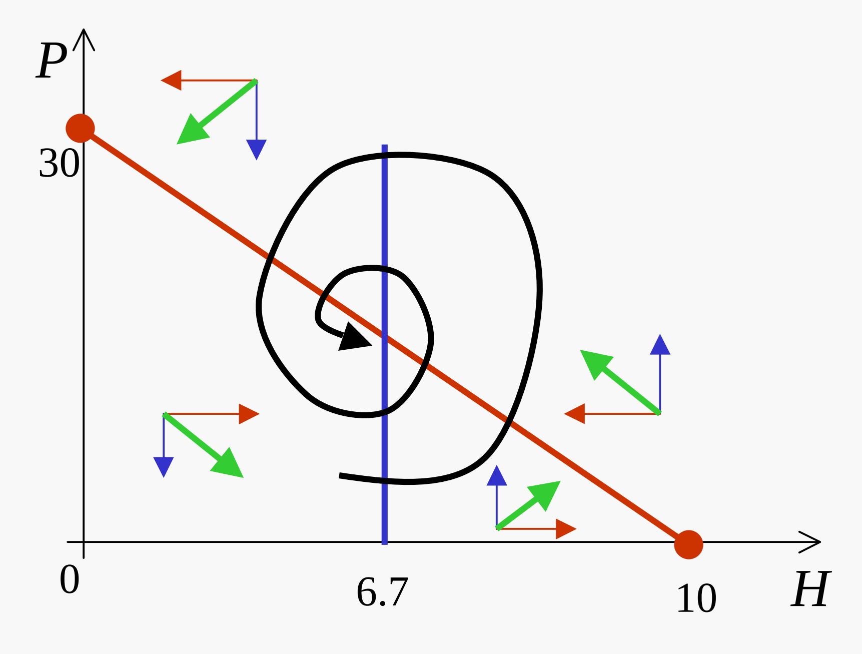
- for predator population: $$\frac{\mathrm{d}P}{\mathrm{d}t}=0 \qquad 0.3HP - 2P = 0$$

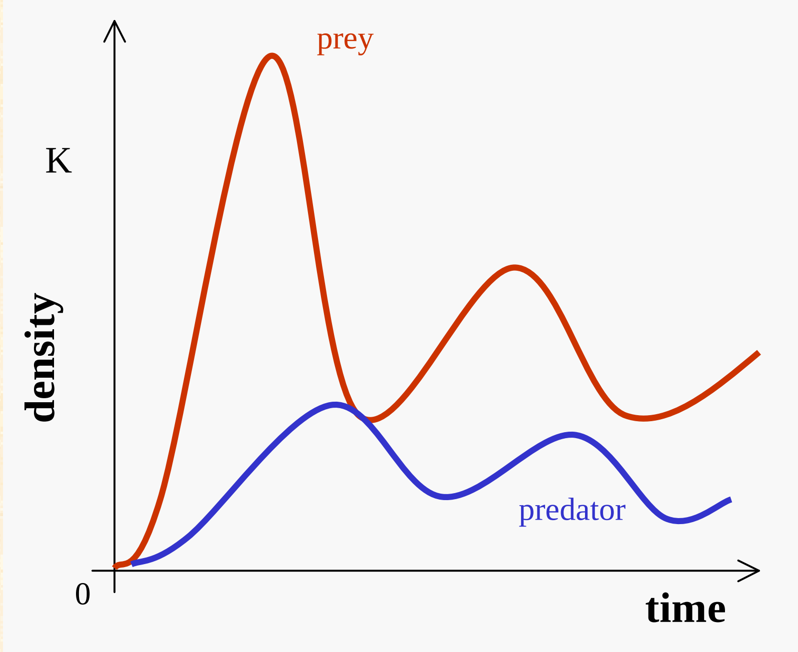
if P = 0 (trivial solution)
or if 0.3H - 2 = 0

H = 6.667

- gradient of prey isocline is negative







.. approaches stable equilibrium

Incorporation of functional response

- functional response Type II: $$H_a = \frac{aHT}{1+aHT_h}$$

- rate of consumption by all predators: $$\frac{H_a P}{T} = \frac{aHP}{1+aHT_h}$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = r_H H\left(1-\frac{H}{K}\right) - \frac{aHP}{1+aHT_h} \qquad \frac{dP}{\mathrm{d}t} = bHP - mP$$

- for parameters: $r_H = 3$, $a = 0.1$, $T_h = 2$, $K = 10$

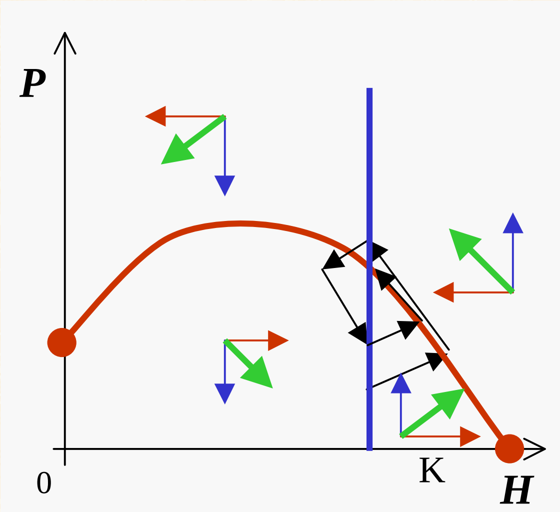
$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \qquad 0 = 3H\left(1-\frac{H}{10}\right) - \frac{0.1HP}{1+0.1H2}$$

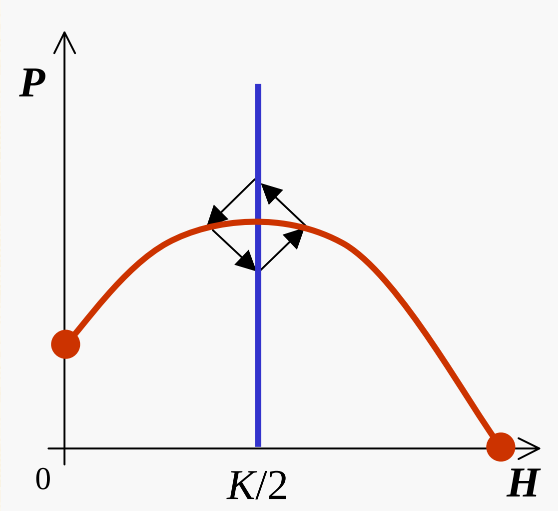
prey isocline: $P = 30 + 6H - 0.6H^2$ predator isocline: $H = constant$

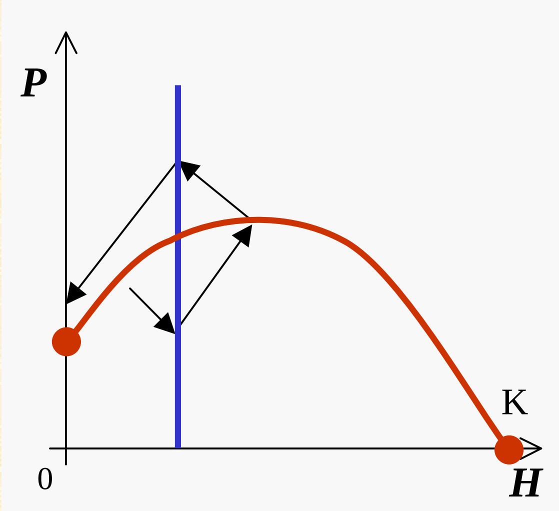
▸ predator exploits prey close to *K*
- isocline: *H* = 9

▸ predator exploits prey close to *K*/2
- isocline: *H* = 5

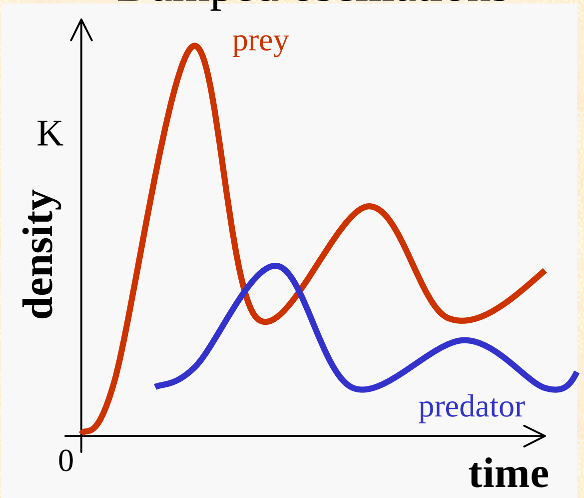
▸ predator exploits prey at low density
- isocline: *H* = 2



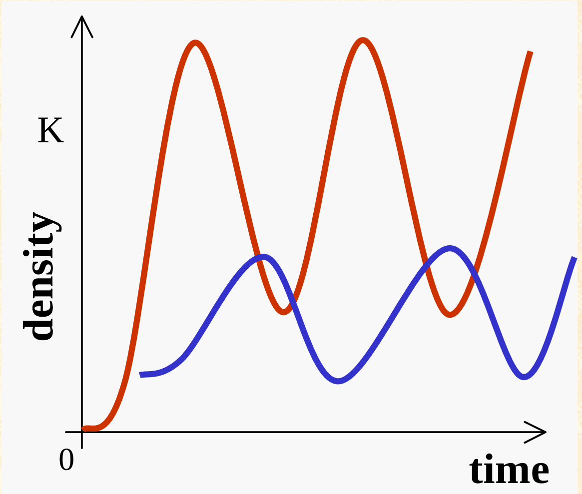




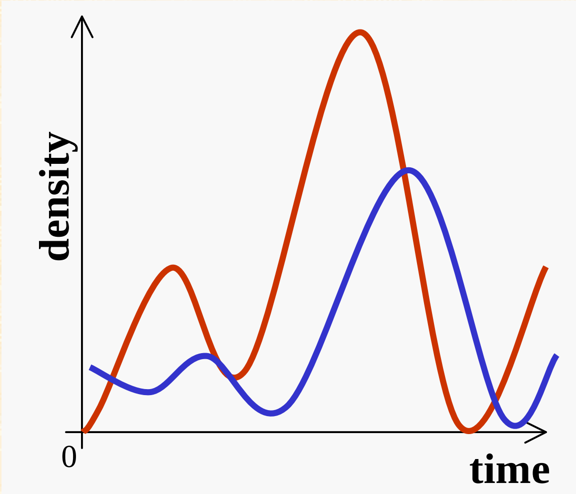
Damped oscillations



Sustained oscillations



Extinction



Rosenzweig & MacArthur (1963)

Incorporation of predator's carrying capacity

- logistic model with carrying capacity proportional to *H*
- *k* .. carrying capacity of the predator
- $r_P = bH - m$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bHP - mP$$

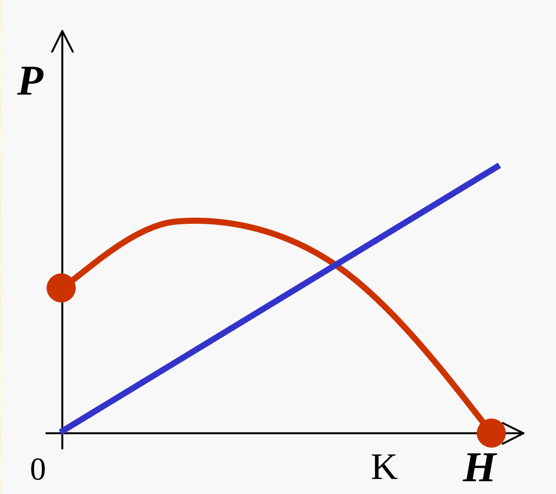
$$\frac{\mathrm{d}P}{\mathrm{d}t} = r_P P\left(1 - \frac{P}{kH}\right) \qquad \frac{\mathrm{d}H}{\mathrm{d}t} = r_H H\left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h}$$

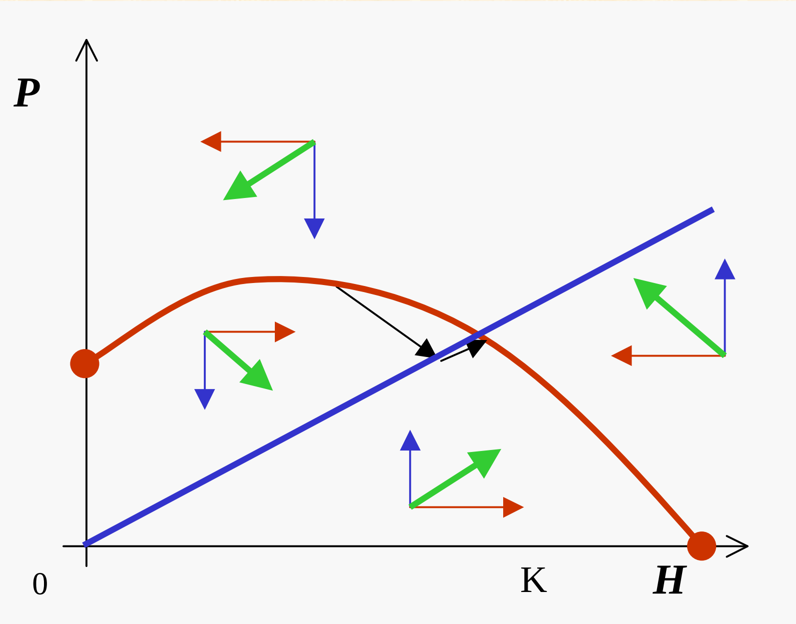
- for parameters: $r_P = 2$, $k = 0.2$

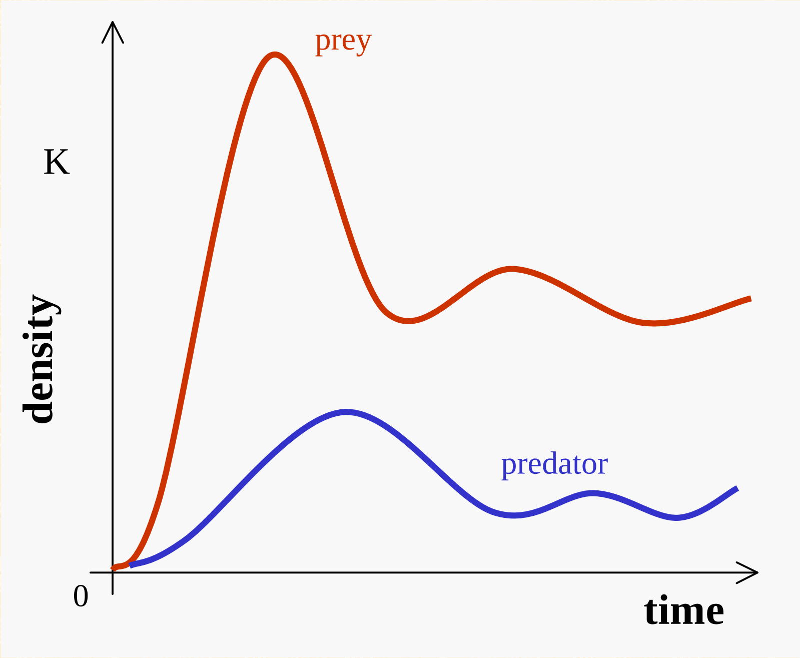
$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0 \qquad 0 = 2P\left(1 - \frac{P}{0.2H}\right)$$

predator isocline: $H = 5P$

prey isocline: $P = 30 + 6H - 0.6H^2$







.. quick approach to stable equilibrium

Host-parasitoid system

Zatypota

Theridion

Host-parasitoid model

- discrete model of Nicholson & Bailey (1935)

- discrete generations
- 1, .., several, or less than 1 host
- random host search and functional response Type III
- lay eggs in aggregation

H_t = number of hosts in time t
H_a = number of attacked hosts
λ = finite rate of increase of the host

P_t = number of parasitoids
c = conversion rate, no. of parasitoids for 1 host

$$H_{t+1} = \lambda(H_t - H_a)$$

$$P_{t+1} = cH_a = H_a$$

Incorporation of random search

- parasitoid searches randomly
- encounters (x) are random (Poisson distribution)

$$p_x = \frac{\mu^x e^{-\mu}}{x!} \qquad x = 0, 1, 2, ... \qquad p_0 = e^{-\mu}$$

p_0 = proportion of not encountered, μ .. mean number of encounters

E_t = total number of encounters
a = searching efficiency (proportion of hosts encountered)

$$E_t = a\, H_t\, P_t \longrightarrow \mu = \frac{E_t}{H_t} = aP_t \longrightarrow p_0 = e^{-aP_t}$$

- proportion of encounters (1 or more times): $p = (1 - p_0)$

$$p = (1 - e^{-aP_t})$$

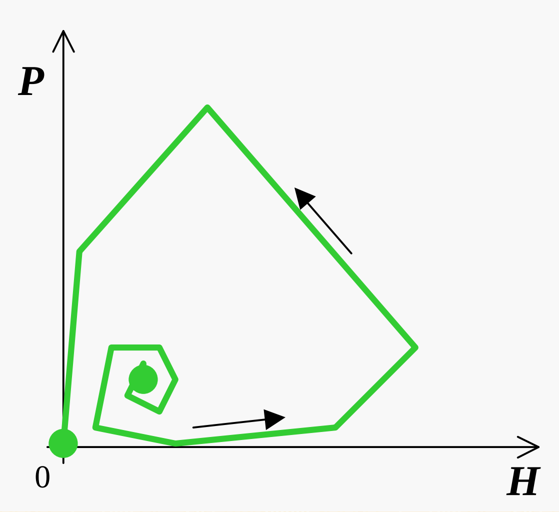
$$H_a = H_t\left(1 - e^{-aP_t}\right)$$

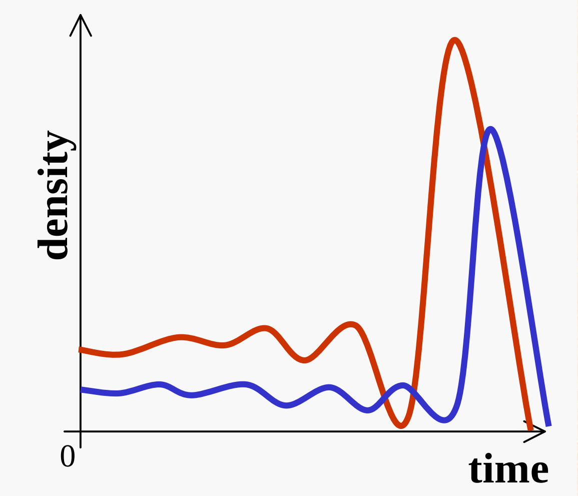
$$H_{t+1} = \lambda(H_t - H_a)$$
$$P_{t+1} = H_a$$

$$H_{t+1} = \lambda H_t e^{-aP_t}$$
$$P_{t+1} = H_t\left(1 - e^{-aP_t}\right)$$

- highly unstable model for all parameter values:

- equilibrium is possible but the slightest disturbance leads to divergent oscillations (extinction of parasitoid)





Incorporation of density-dependence

- exponential growth of hosts is replaced by logistic equation

$$H_{t+1} = \lambda H_t e^{\left(1-\frac{H_t}{K}\right)-aP_t}$$

$$P_{t+1} = H_t\left(1-e^{-aP_t}\right)$$

$$q = \frac{H^*}{K}$$

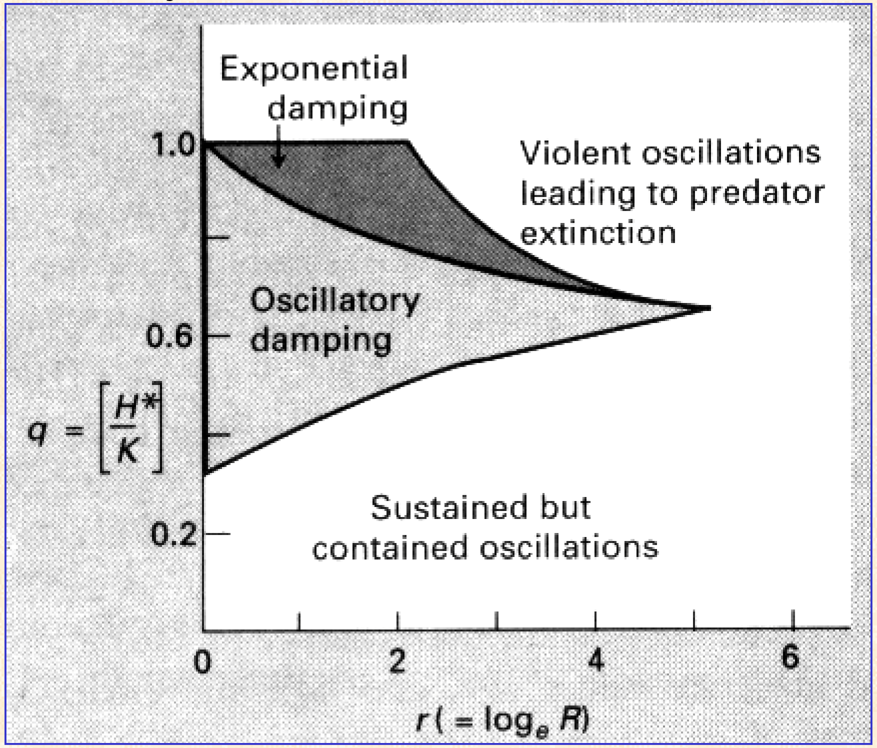
*H**.. new host carrying capacity

- depends on parasitoids' efficiency

- when a is low then $q \rightarrow 1$
- when a is high then $q \rightarrow 0$

- density-dependence have stabilising effect for moderate r and q

Stability boundaries



Beddington et al. (1975)

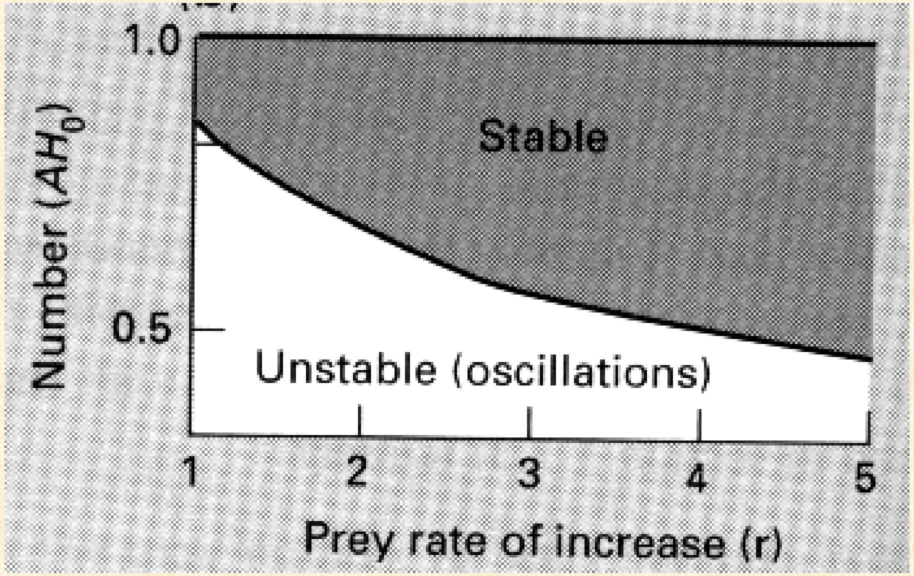
Incorporation of the refuge

- if hosts are distributed non-randomly in the space

Fixed number in refuge: H_0 hosts are always protected

$$H_{t+1} = \lambda H_0 + \lambda (H_t - H_0) e^{-aP_t}$$
$$P_{t+1} = (H_t - H_0)\left(1 - e^{-aP_t}\right)$$

- have strong stabilising effect even for large *r*



Hassell & May (1973)

Incorporation of aggregated distribution

- distribution of encounters is not random but aggregated (negative binomial distribution)

- proportion of hosts not encountered (p_0):

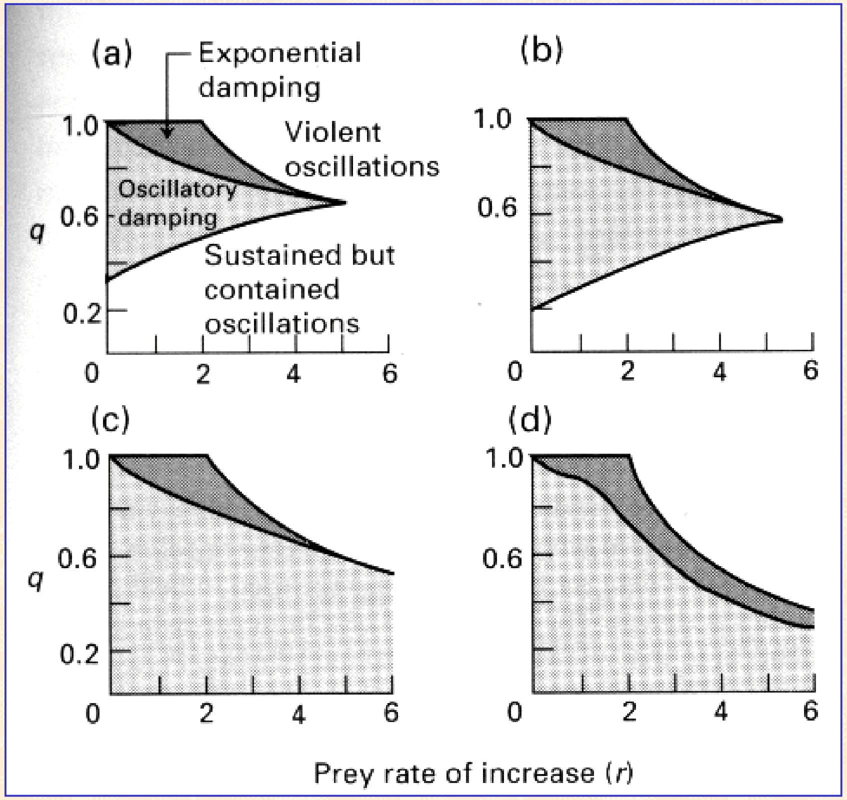
$$p_0 = \left(1 + \frac{aP_t}{k}\right)^{-k}$$

where k = degree of aggregation

$$H_{t+1} = \lambda H_t e^{\left(1-\frac{H_t}{K}\right)\left(1+\frac{aP_t}{k}\right)^{-k}}$$

$$P_{t+1} = H_t\left(1-\left(1+\frac{aP_t}{k}\right)^{-k}\right)$$

- very stable model system if $k \leq 1$



Stability boundaries:
a) $\boldsymbol{k}=\infty$, b) $\boldsymbol{k}=2$, c) $\boldsymbol{k}=1$, d) $\boldsymbol{k}=0$

Hassell (1978)

Example 21

You want to control population of mites. Before introduction of predatory mites you want to simulate the predator-prey dynamic using the following model:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = r_H H\left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h} \qquad \frac{\mathrm{d}P}{\mathrm{d}t} = \frac{acHP}{1 + aHT_h} - dP$$

Parameter estimates are obtained experimentally:
1. Rear prey population without predators. You find $r_H = 0.4$ and $K = 500$.
2. Rear predators at constant prey densities. You find predators' mortality $d = 0.08$ and conversion efficiency $c = 0.8$.
3. Perform functional response experiment. You find that $a = 0.001$ and $T_h = 0.5$.

How long it takes for the predatory mite to control mite pests. Initial densities are 200 individuals of pests and 10 individuals of predators?

```
predprey<-function(t,y,pa){
H<-y[1]
P<-y[2]
with(as.list(pa),{
dH.dt<-rH*H*(1-H/K)-a*H*P/(1+a*H*Th)
dP.dt<-a*c*H*P/(1+a*H*Th)-d*P
return(list(c(dH.dt,dP.dt)))})}

H<-200;P<-10
time<-seq(0,200,0.1)
pa<-c(rH=0.4,K=500,a=0.001,Th=0.5,c=0.8,d=0.08)
library(deSolve)
out<-data.frame(ode(c(H,P),time,predprey,pa))
matplot(time,out[,-1],type="l",lty=1:2,col=1)
legend("right",c("H","P"),lty=1:2)
```

Example 22

Caterpillars increased their population density in flour to 50 individuals/100 kg. You observed that their $\lambda = 3$ and $K = 800$. You need to control these pests using a parasitoid. You can choose from three parasitoid species (A, B, C). The three species differ in the number of eggs/host (c) and in their search efficiency (a):

	A	B	C
c	1	3	2
a	0.003	0.1	0.005

1. Use the discrete Nicholson-Bailey host-parasitoid model with density-dependence. Introduce a single parasitoid per 100 kg. Find which of the three species will achieve the quickest control.

```
time<-20
HP<-data.frame(H<-numeric(time),P<-numeric(time))
a=0.003;L=3;K=800;c=1
HP[1,]<-c(50,1)
for (t in 1:20) HP[t+1,]<-{
H<-L*HP[t,1]*exp((K-HP[t,1])/K-a*HP[t,2])
P<-c*HP[t,1]*(1-exp(-a*HP[t,2]))
c(H,P)}
matplot(HP,type="l",lty=1:2)
legend(10,2000,c("H","P"),lty=1:2)

HP<-data.frame(H<-numeric(time),P<-numeric(time))
a=0.1;L=3;K=800;c=3
HP[1,]<-c(50,1)
for (t in 1:20) HP[t+1,]<-{
H<-L*HP[t,1]*exp((K-HP[t,1])/K-a*HP[t,2])
P<-c*HP[t,1]*(1-exp(-a*HP[t,2]))
c(H,P)}
matplot(HP,type="l",lty=1:2)
legend(10,2000,c("H","P"),lty=1:2)
```

```
HP<-data.frame(H<-numeric(time),P<-numeric(time))
a=0.005;L=3;K=500;c=2
HP[1,]<-c(50,1)
for (t in 1:20) HP[t+1,]<-{
H<-L*HP[t,1]*exp((K-HP[t,1])/K-a*HP[t,2])
P<-c*HP[t,1]*(1-exp(-a*HP[t,2]))
c(H,P)}
matplot(HP,type="l",lty=1:2)
legend(10,4000,c("H","P"),lty=1:2)
```