



MODULARIZACE VÝUKY EVOLUČNÍ A EKOLOGICKÉ BIOLOGIE  
CZ.1.07/2.2.00/15.0204



# MORFOMETRIE NA PRŮSEČÍKU BIOLOGIE A GEOMETRIE

**Miloš Macholán**



evropský  
sociální  
fond v ČR



EVROPSKÁ UNIE



MINISTERSTVO ŠKOLSTVÍ,  
MLÁDEŽE A TĚLOVÝCHOVY

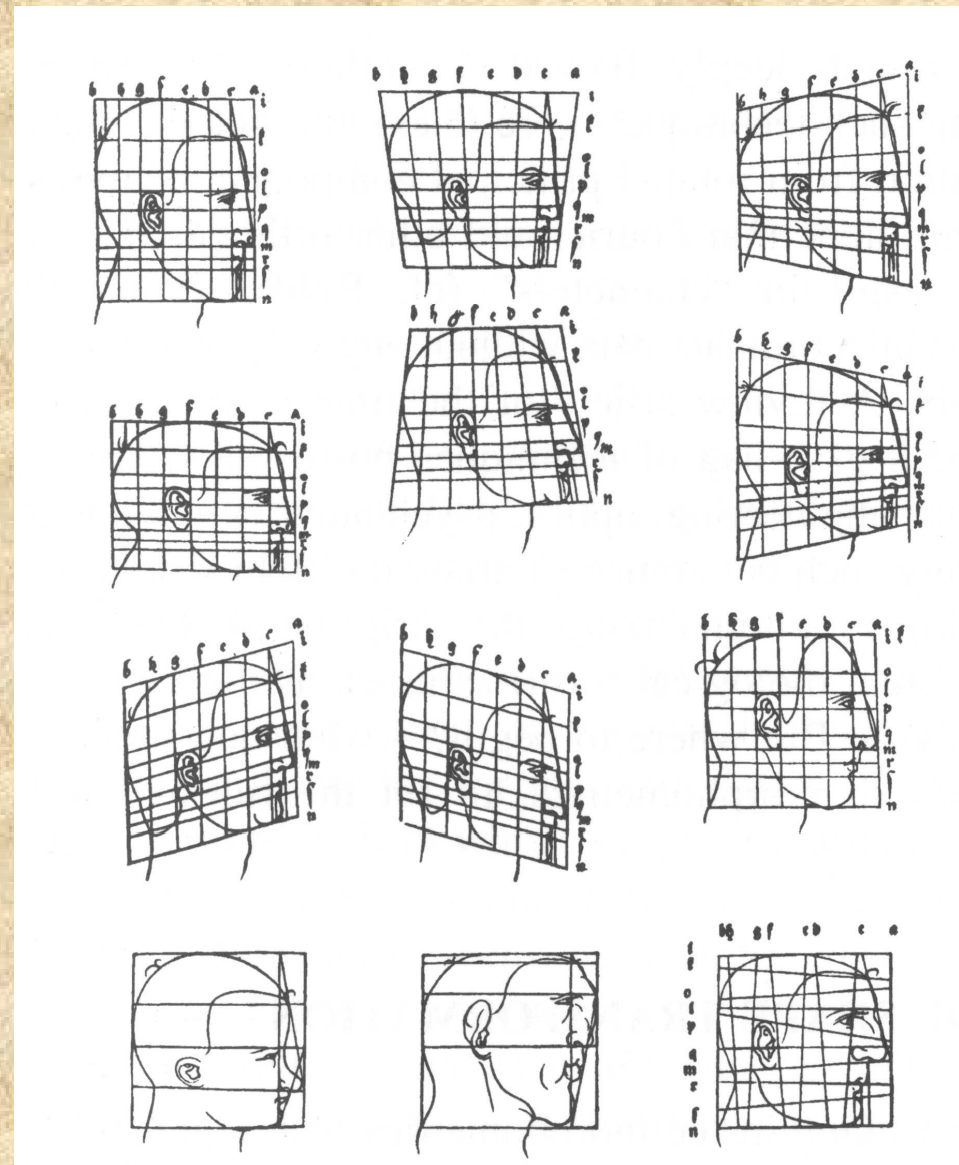


OP Vzdělávání  
pro konkurenceschopnost



INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

# Snaha zachytit tvarové změny a rozdíly mezi objekty



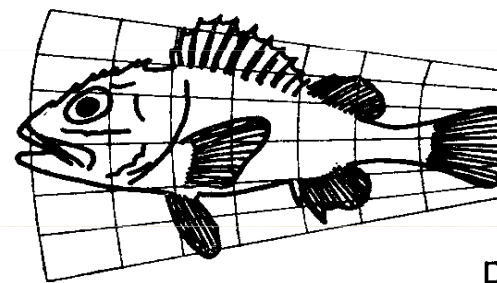
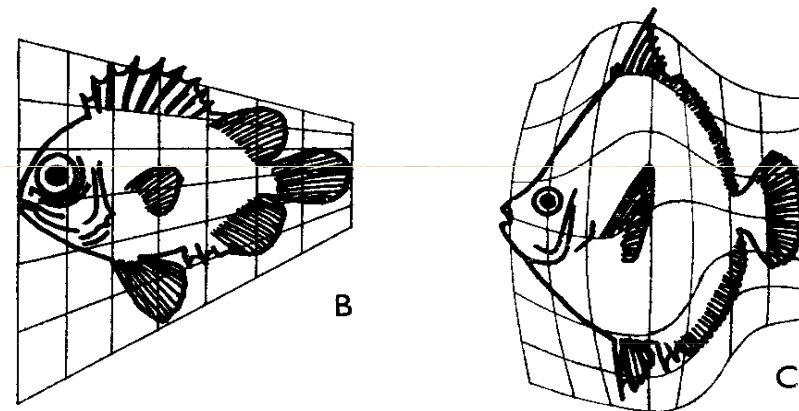
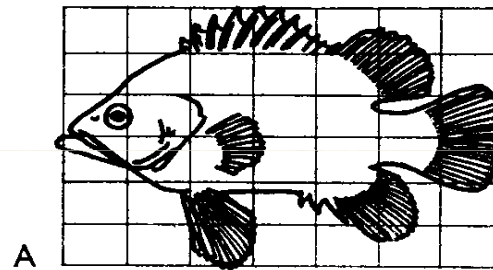
**A. Dürer (1524): Vier Bücher von Menlicher Proportion.**

# V historii zkoumání tvaru biologických objektů 2 odlišné strategie:

## 1) Sir W. D'Arcy Thompson

Absence kvantifikace  
tvarových změn!

W. A. Thompson (1917):  
On Growth and Form.



# **V historii zkoumání tvaru biologických objektů 2 odlišné strategie:**

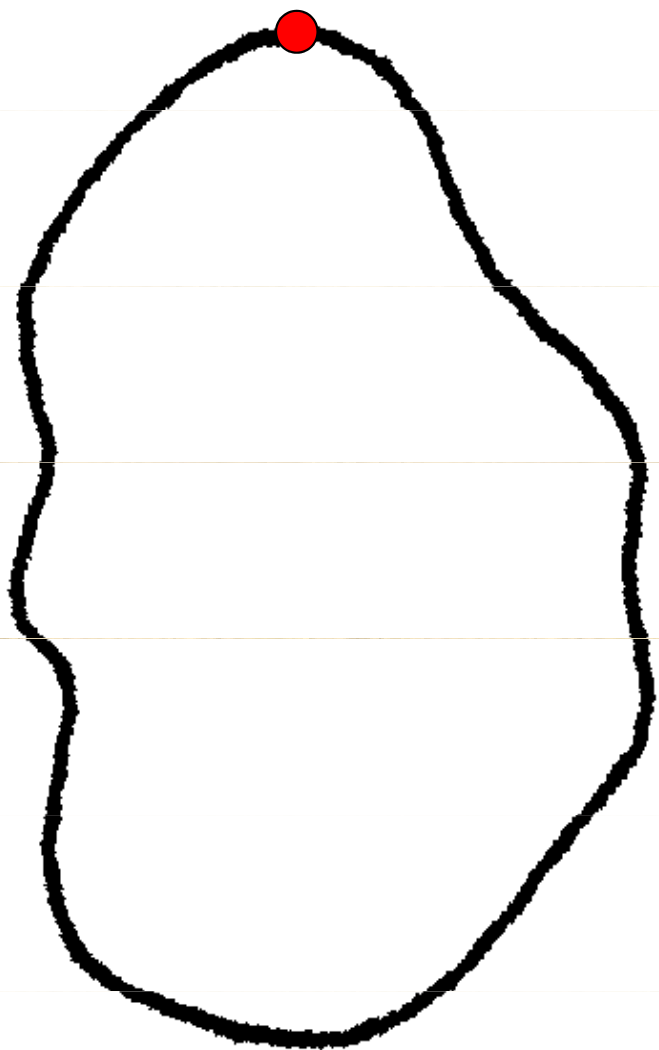
## **2) Tradiční morfometrie:**

- F. Galton, K. Pearson, R. A. Fisher, S. Wright, H. Hotelling, ...**
- rozměry, váhy, úhly, plochy, ...**
- PCA, FA, DFA, CVA, PCOA, shluková a., ...**

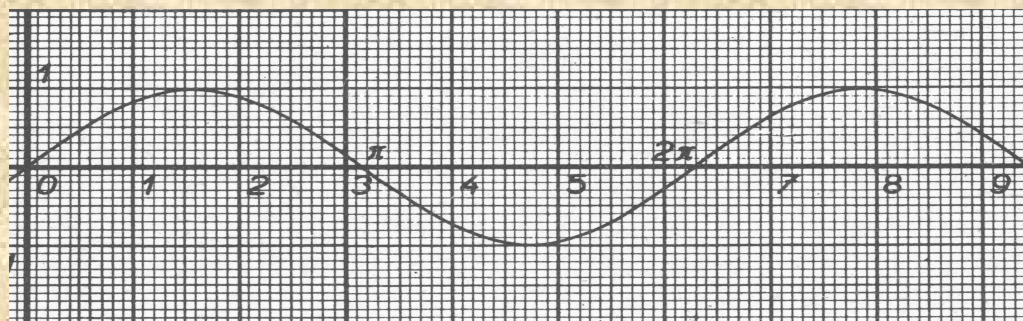
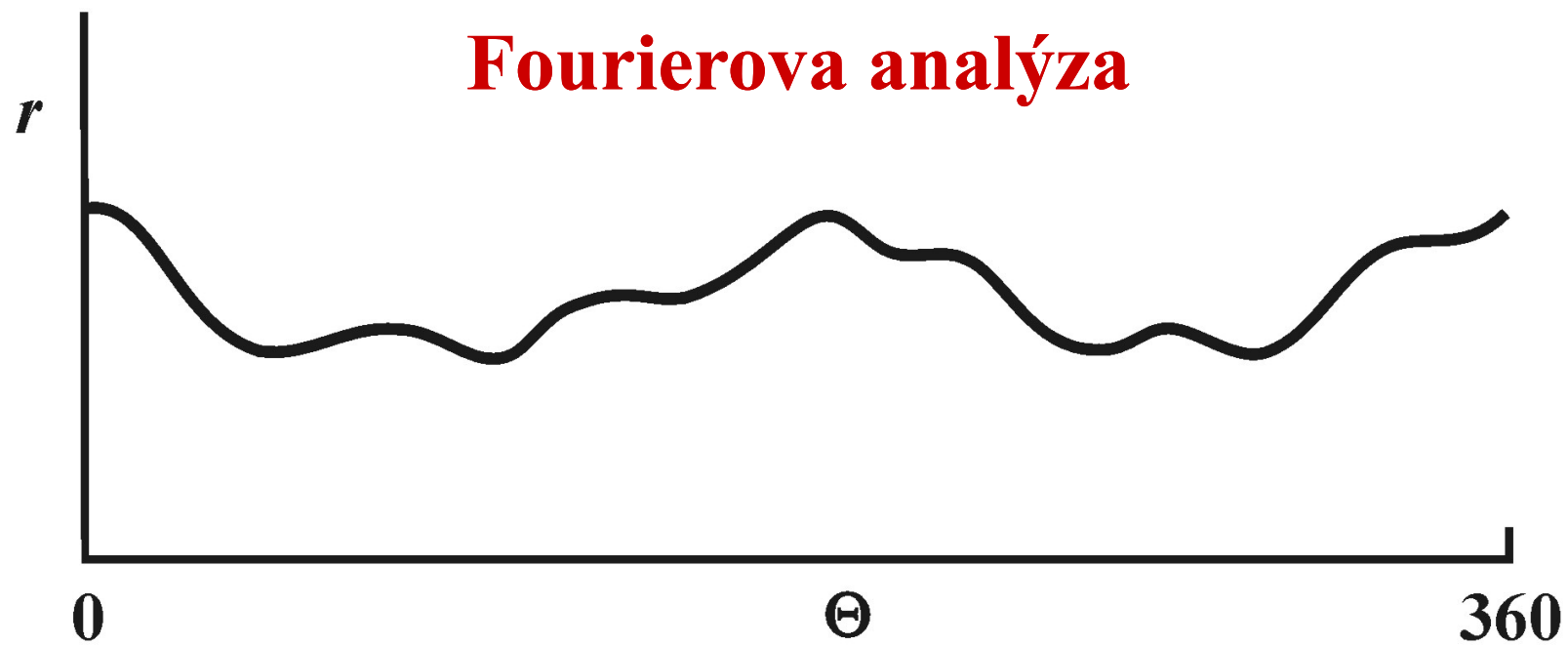
**1983-1989**

**C. R. Goodall, K. V. Mardia, I. Dryden, D. G. Kendall,  
F. L. Bookstein:  
„morfometrická syntéza“, „nová morfometrie“**

- **význačné body (landmarks)**
- **kvantifikace tvaru pomocí tvarových koordinát**
- **odlišení různých tvarových složek**
- **informace o tvaru zachována po celou dobu matematické analýzy**
- **velikostní standardizace a možnost samostatné práce s vektorem velikosti**
- **možnost zpracování dat pomocí tradičního morfometrického aparátu**

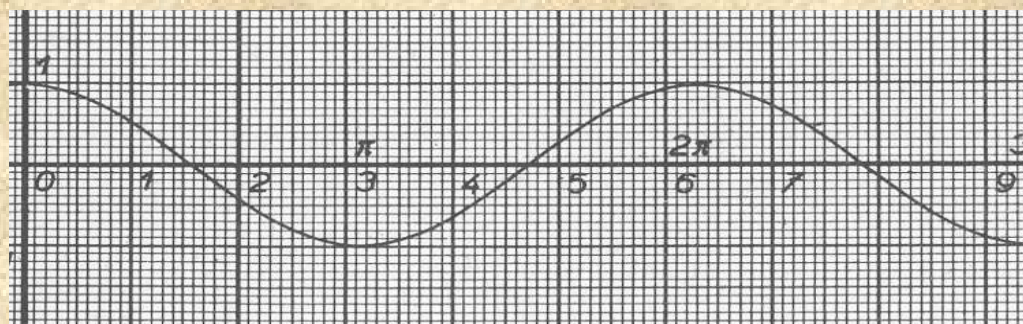


# Fourierova analýza

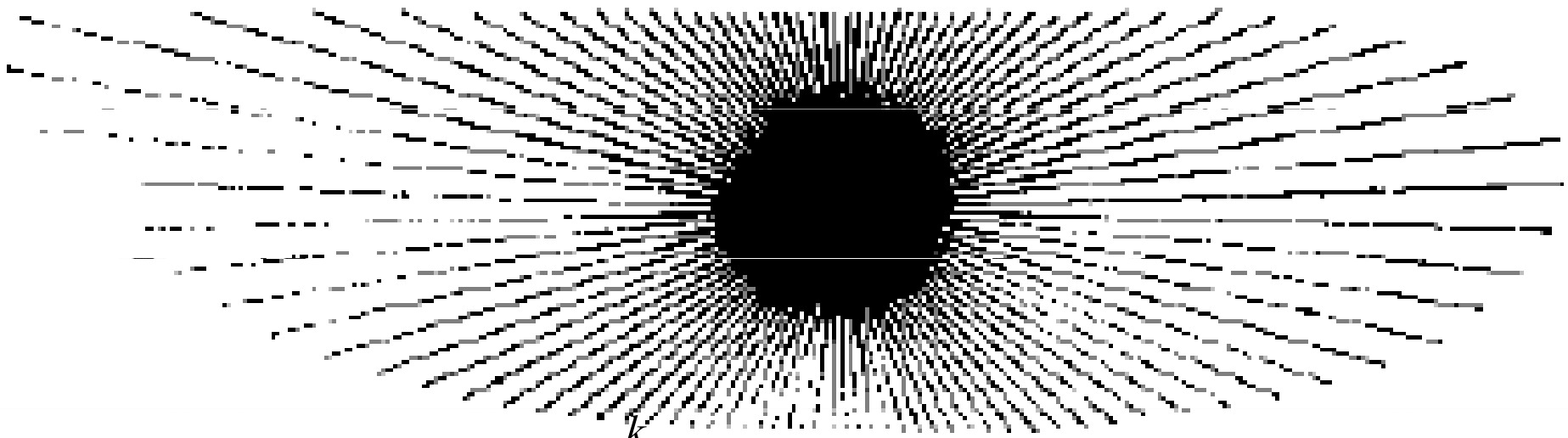
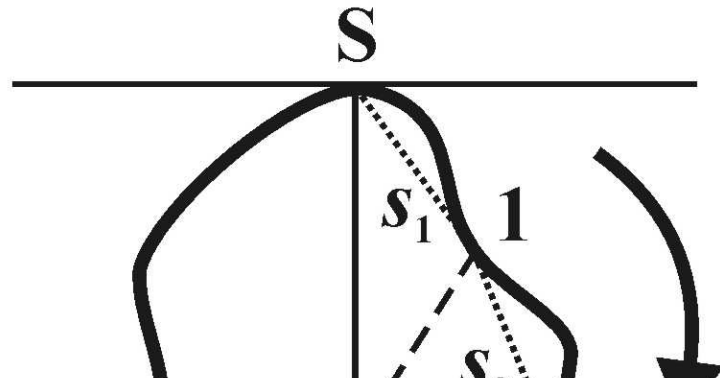
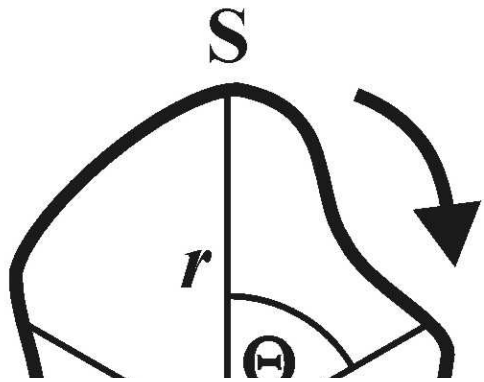


**sin**

**harmonické složky  
(harmonics), koeficienty**



**cos**

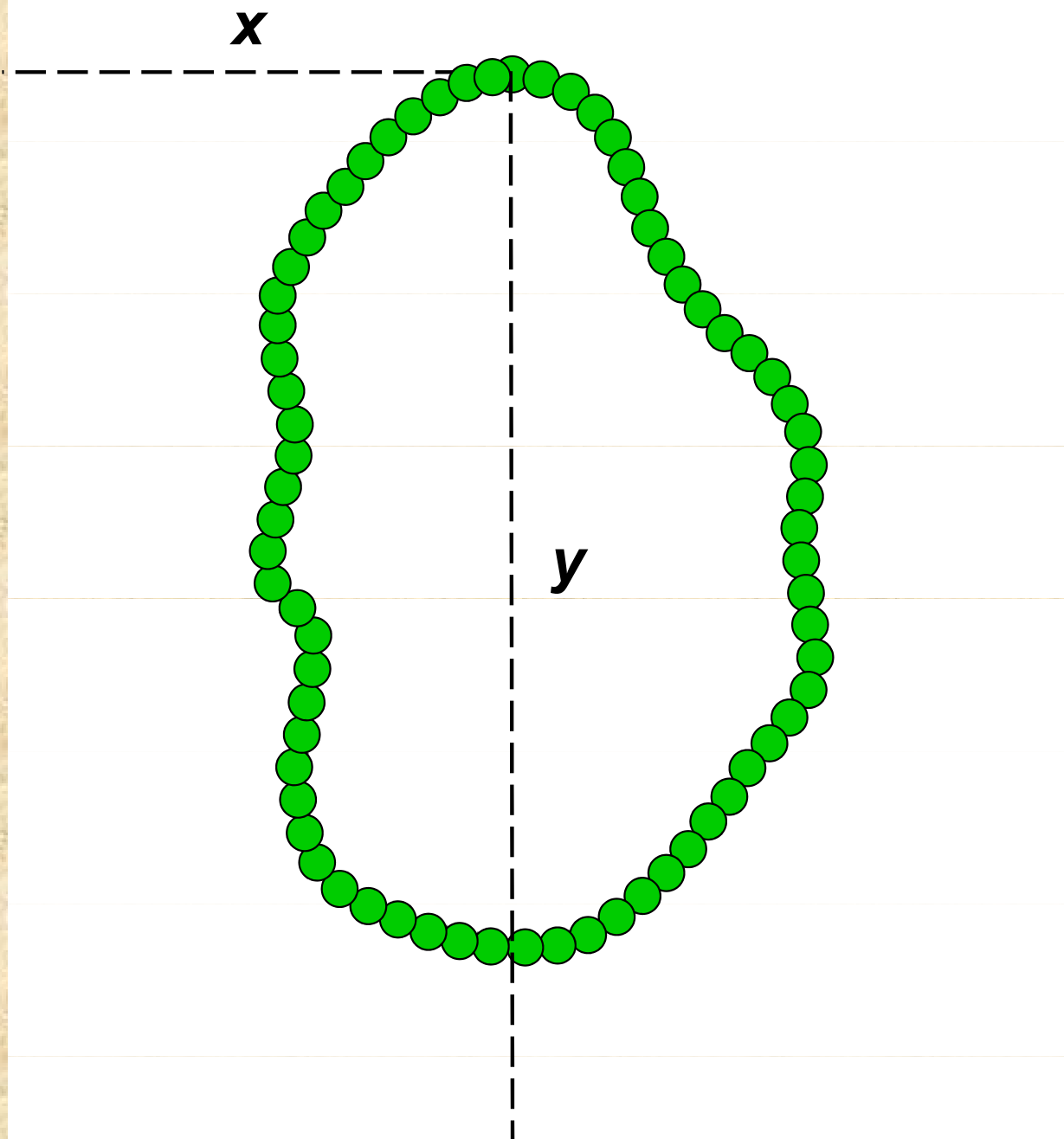


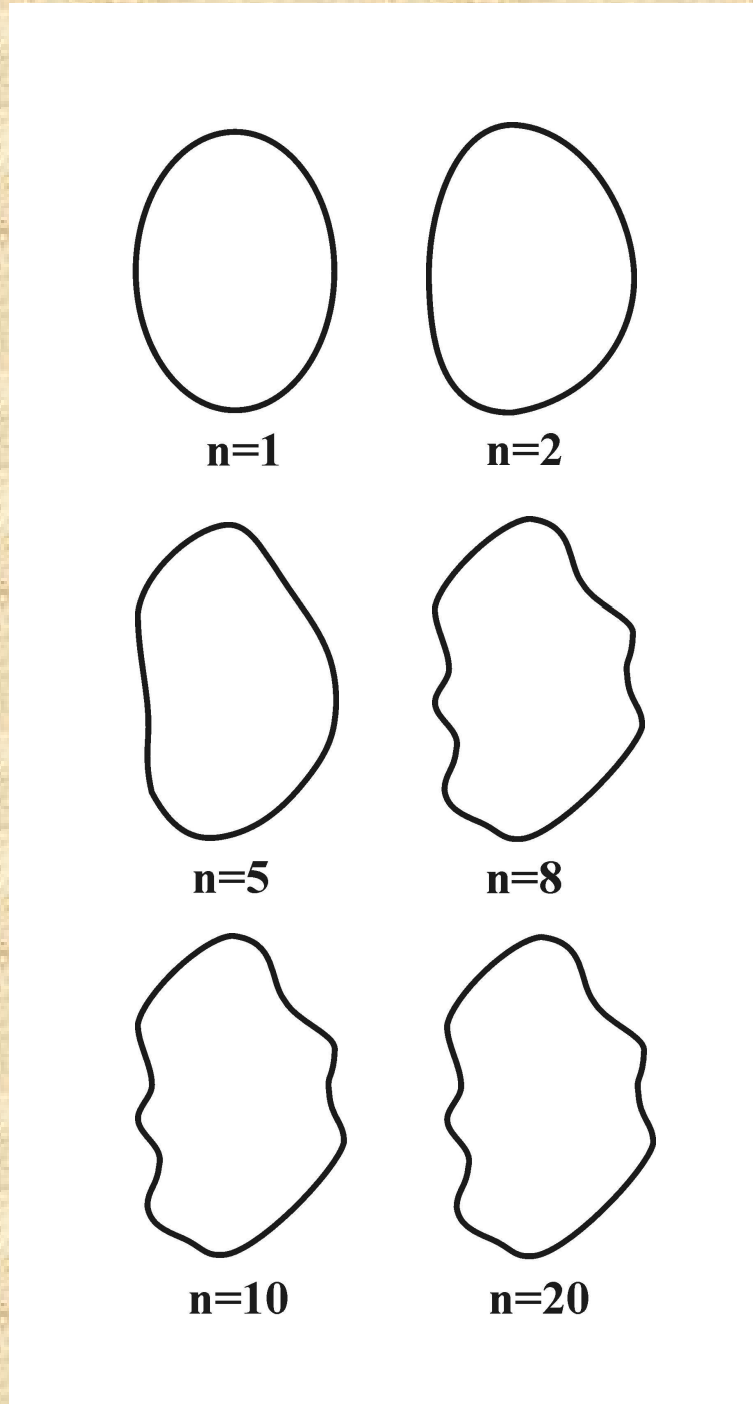
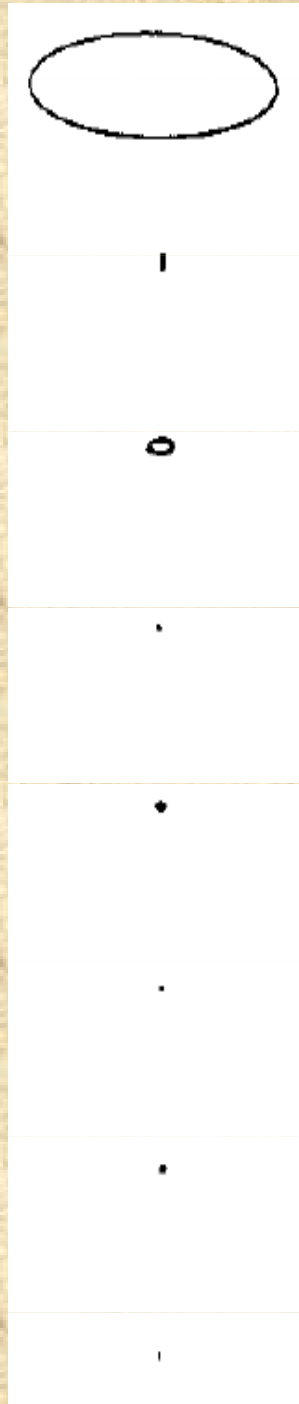
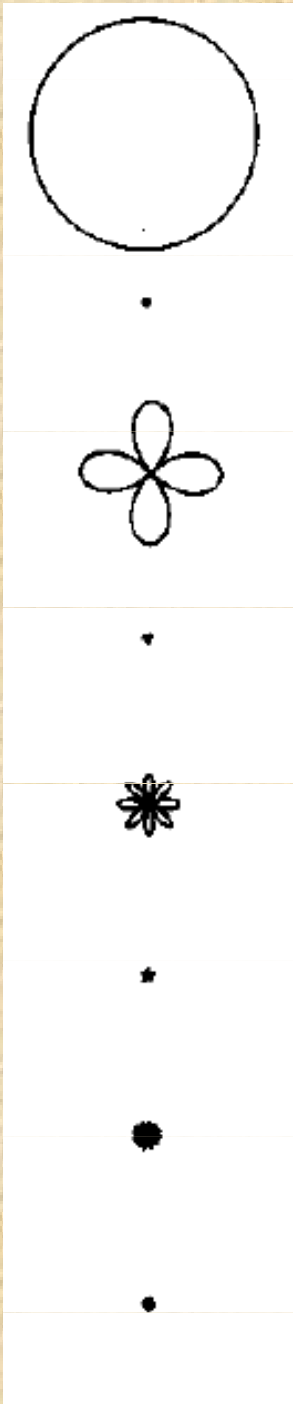
$$r_{\theta} = a_0 + \sum_{i=1}^k (a_i \cos_i \theta + b_i \sin_i \theta)$$

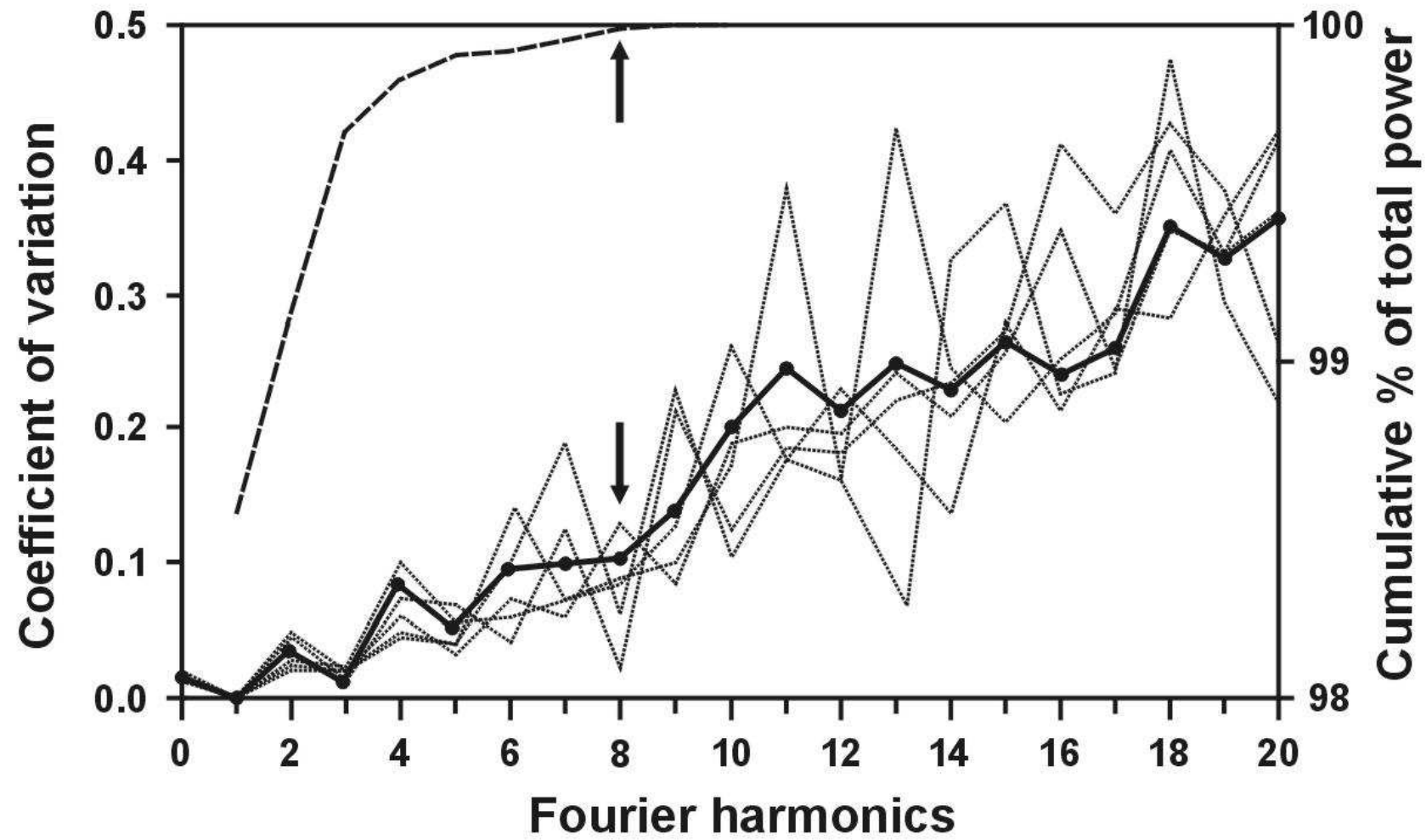
$$a_0 = \sqrt{\frac{1}{p} \sum_{i=1}^p \theta_i} \quad a_i = \sqrt{\frac{2}{p} \sum_{i=1}^p \theta_i \cos_i \theta_i} \quad b_i = \sqrt{\frac{2}{p} \sum_{i=1}^p \theta_i \sin_i \theta_i}$$



# Eliptická Fourierova analýza



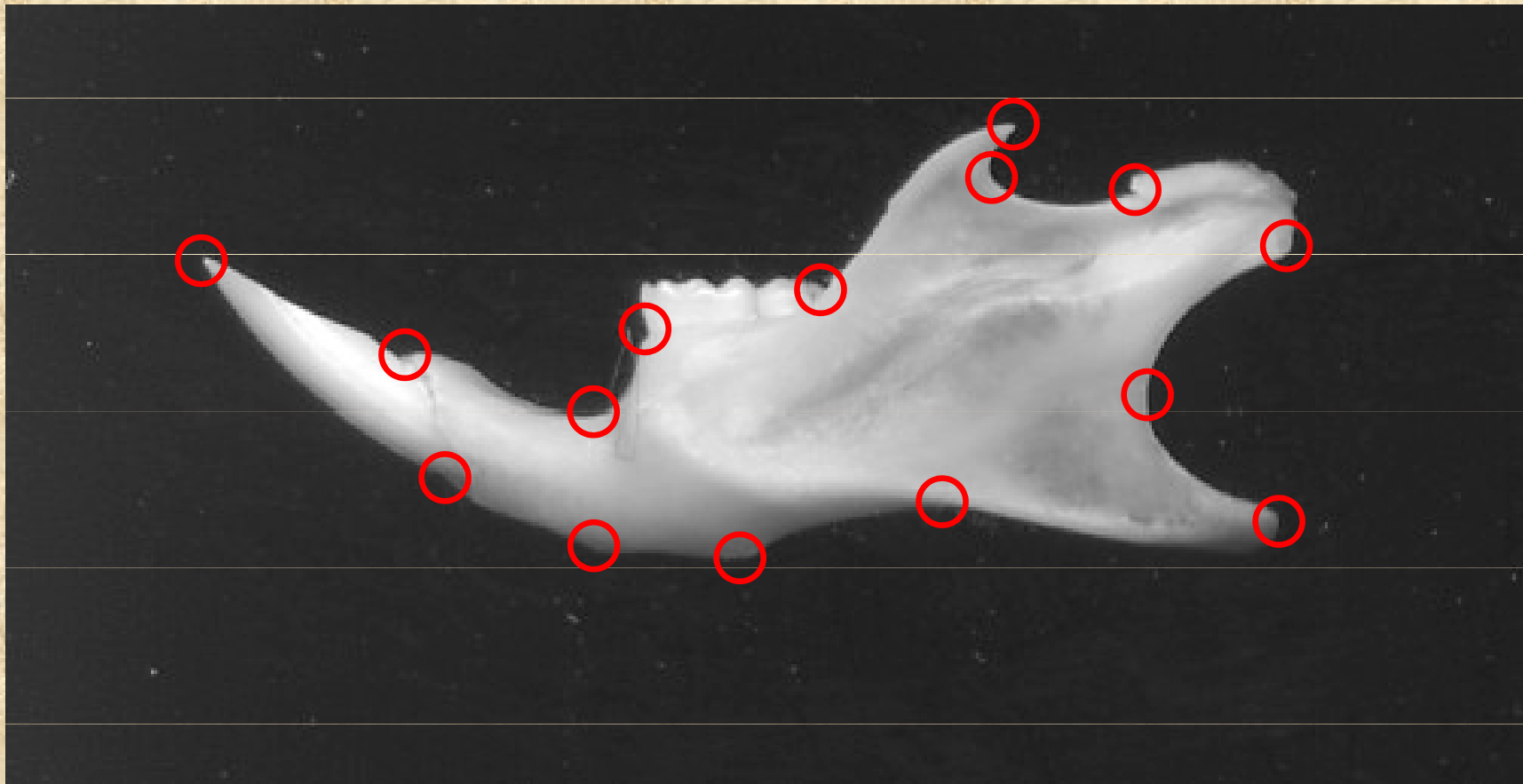




# Geometrická morfometrie II.

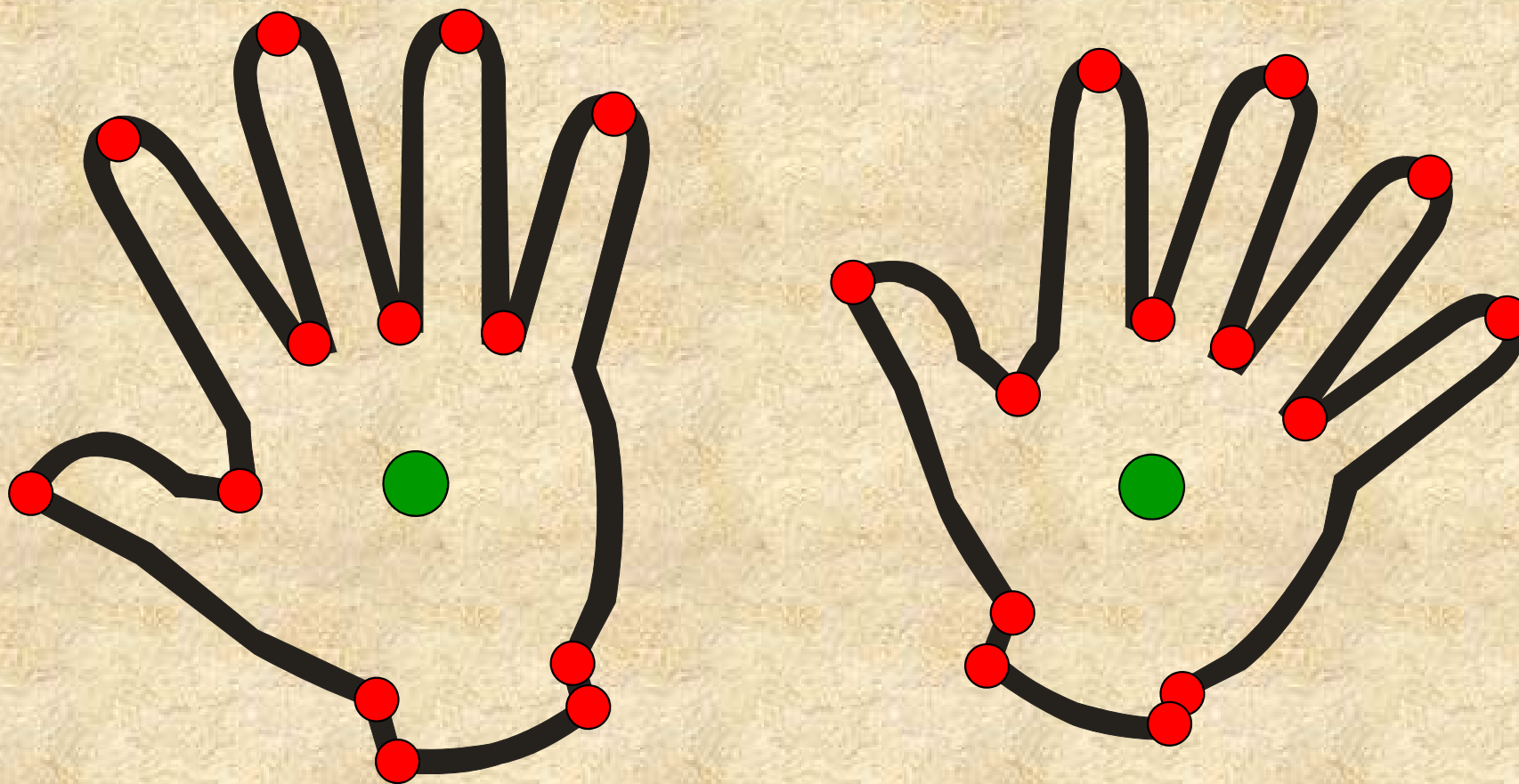
## Význačné body, tvarové koordináty

**význačné body (landmarks) = body, které lze přesně lokalizovat a které jsou alespoň v geometrickém smyslu mezi objekty homologické**



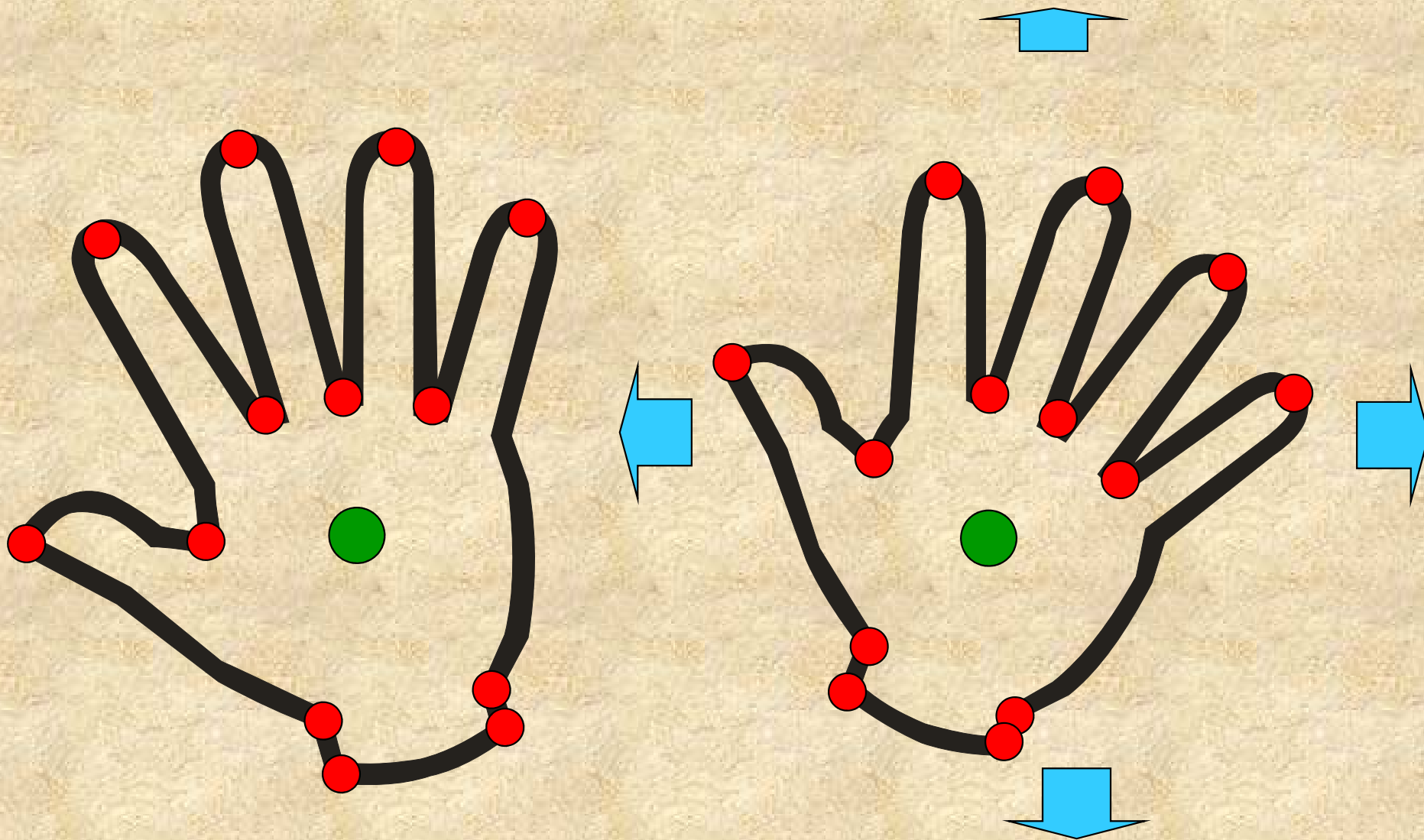
# Prokrustovská superpozice = GLS (Generalized Least Squares)

tvar = vše kromě informace o velikosti, pozici a orientaci objektu



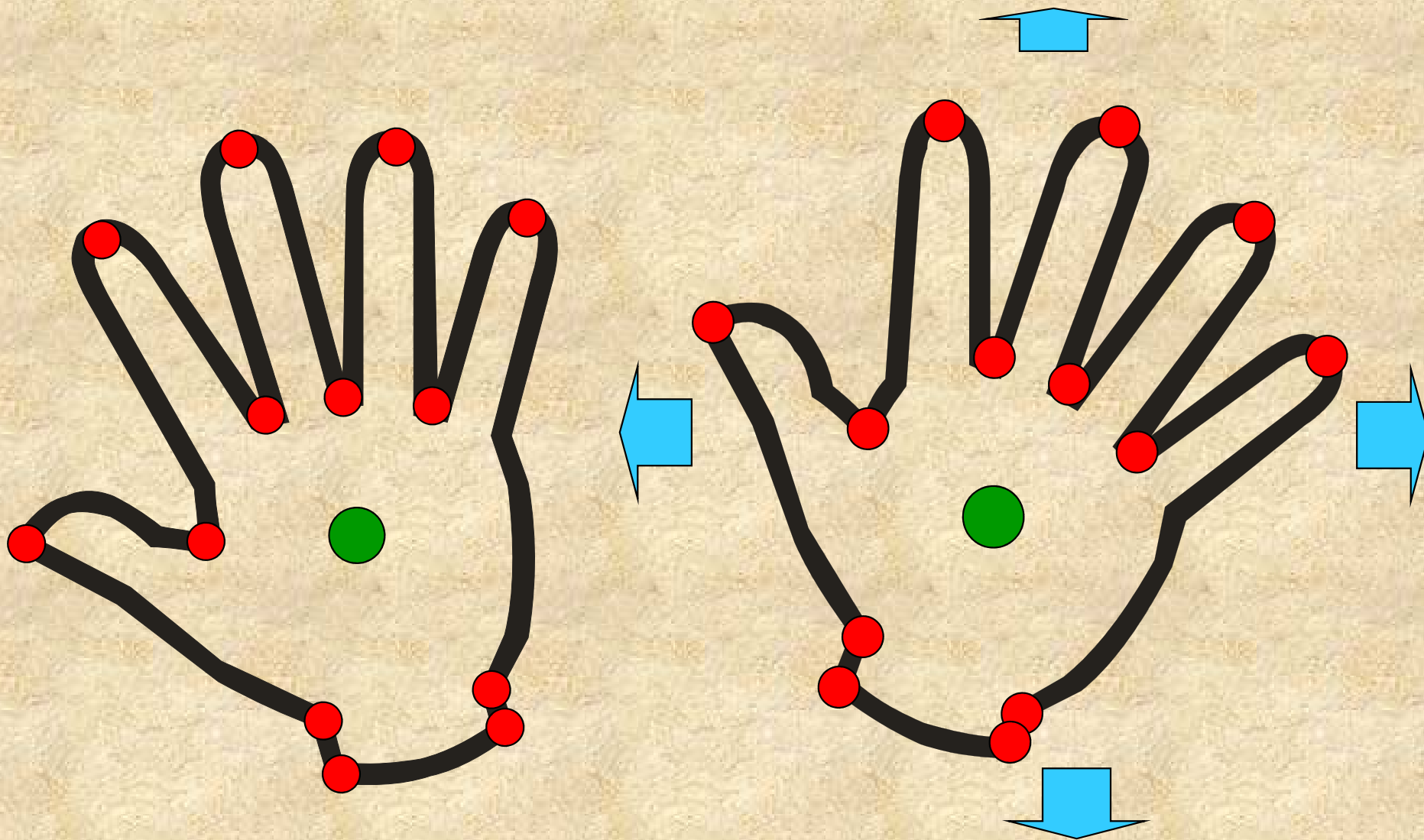
OBRAZOVÝ PROSTOR:  $p$  – rozměrů,  $k$  – bodů;  $n = pk$

# Prokrustes – krok 1



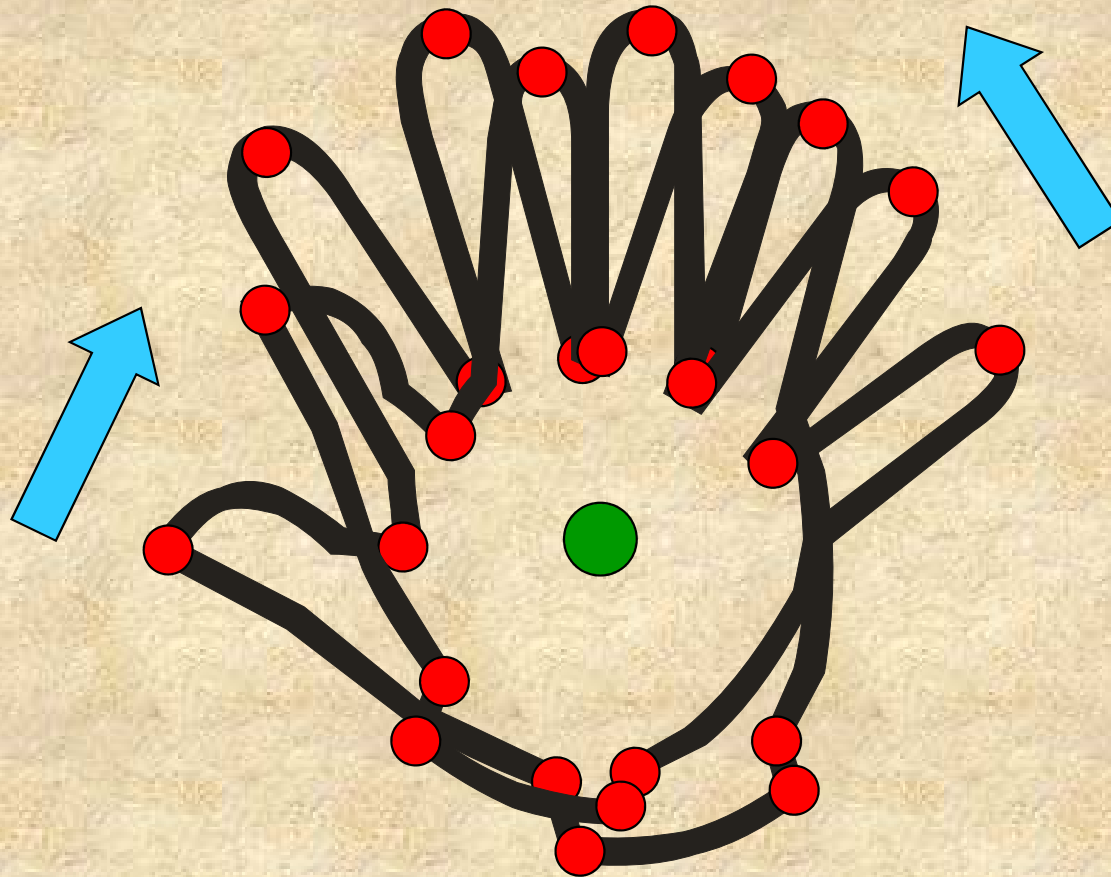
**OBRAZOVÝ PROSTOR:  $p$  – rozměrů,  $k$  – bodů;  $n = pk$**

# Prokrustes – krok 1



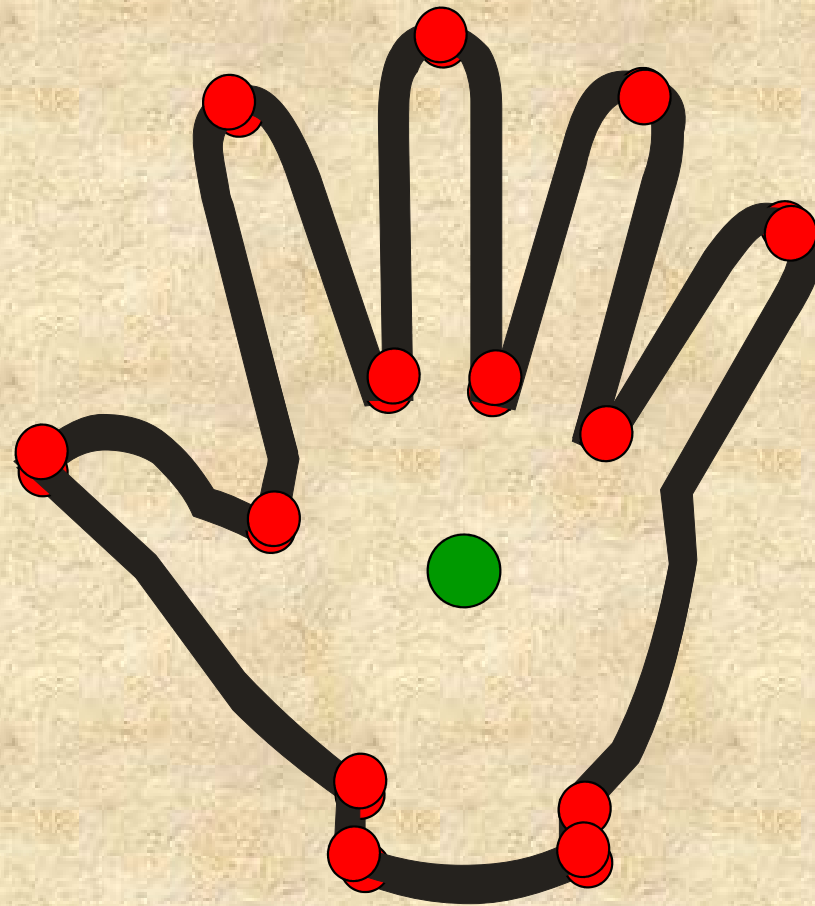
**OBRAZOVÝ PROSTOR:  $p$  – rozměrů,  $k$  – bodů;  $n = pk$**

# Prokrustes – krok 2





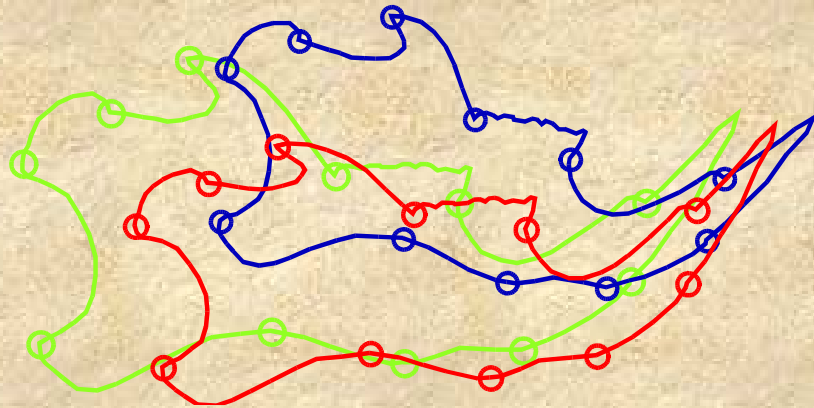
## Prokrustes – krok 3



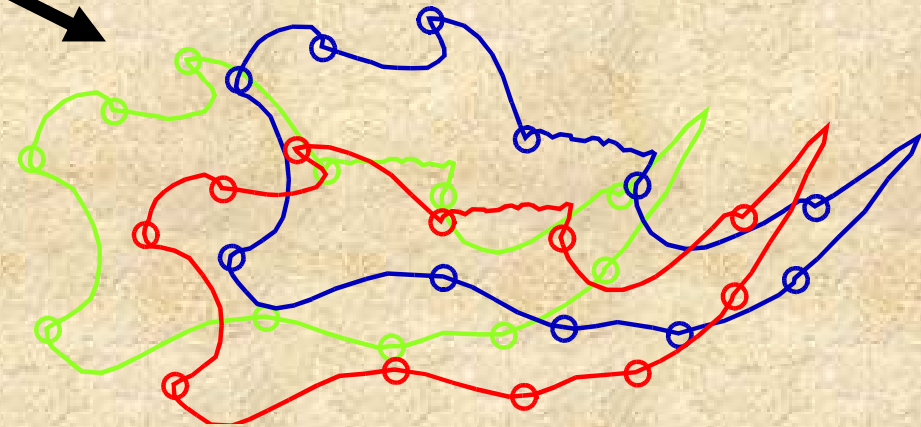
**TVAROVÝ PROSTOR:  $n = pk - k - k(k-1)/2 - 1$**

# Extracting shape information: Procrustes superposition

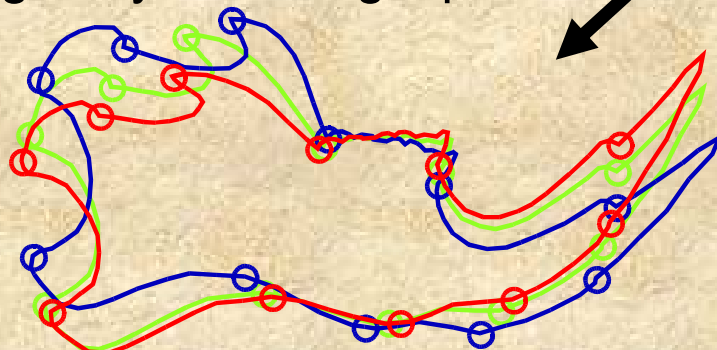
Original landmark configurations



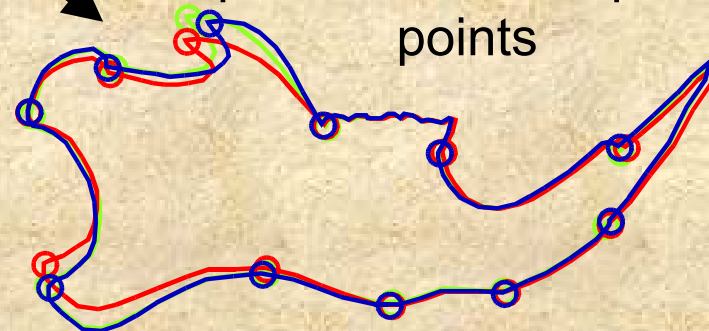
1. Change scale so that all configurations have the same size

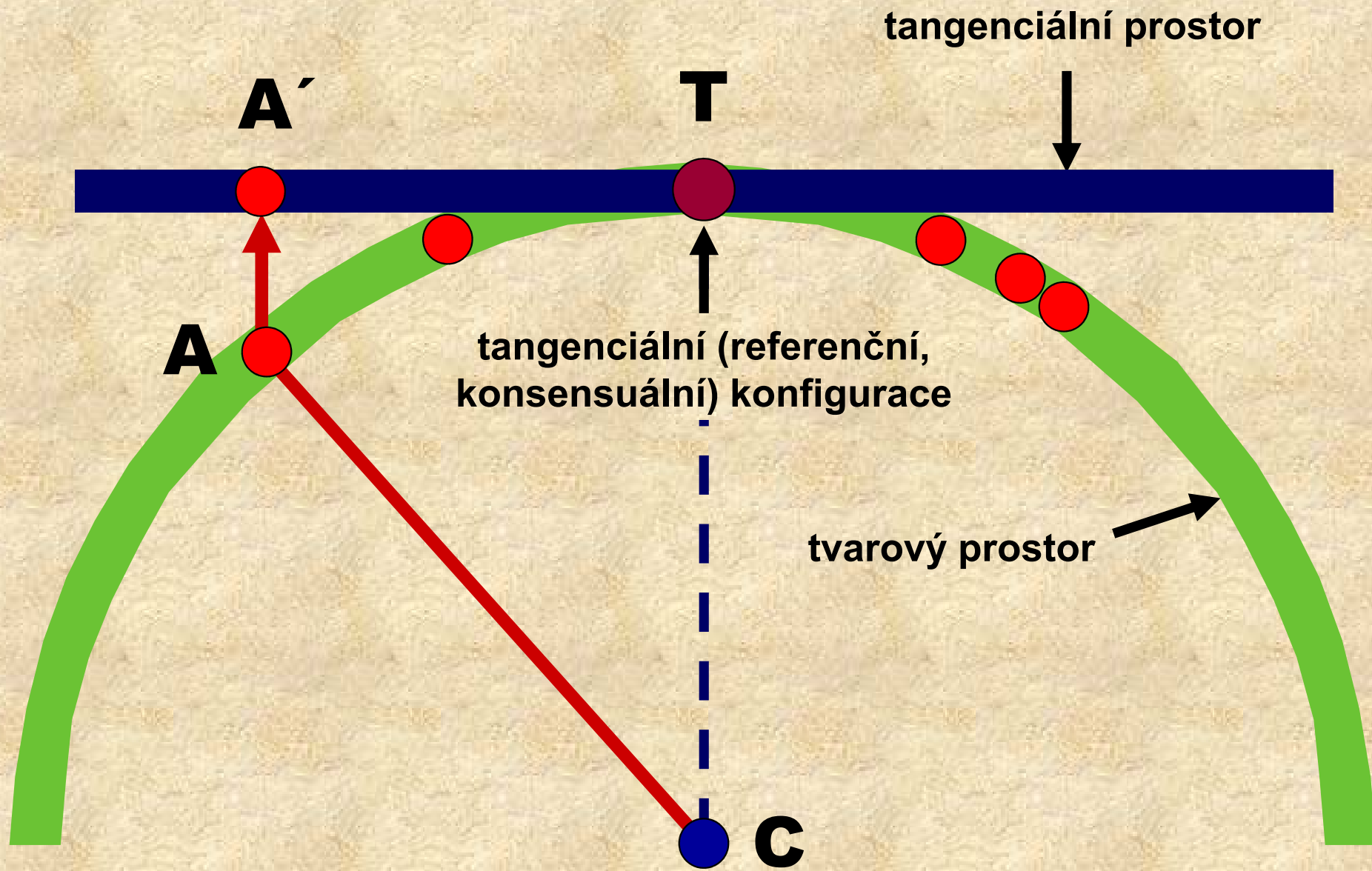


2. Superposition of the centers of gravity on a single point

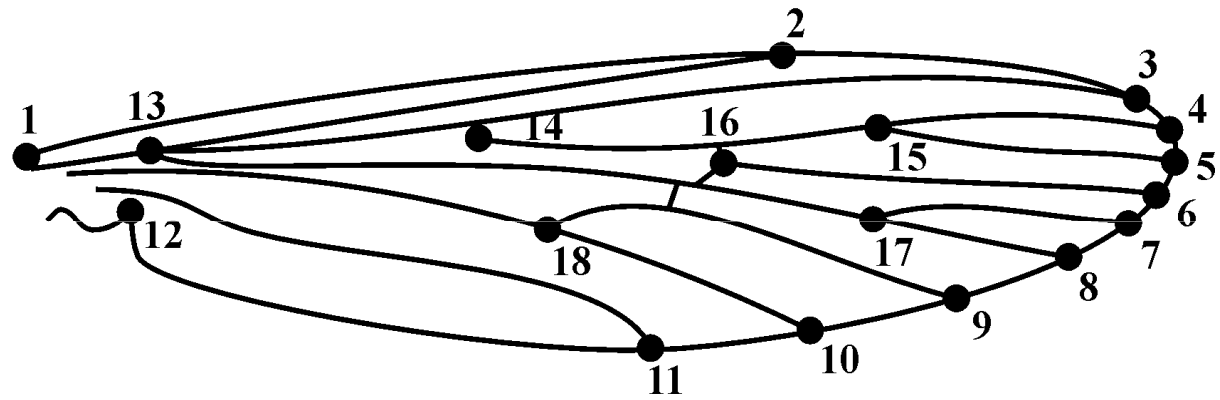


3. Rotation to minimize the dispersion of corresponding points

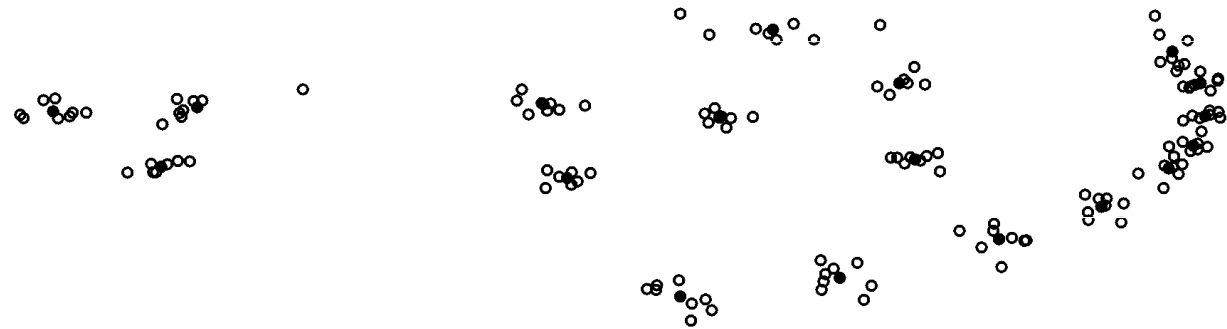




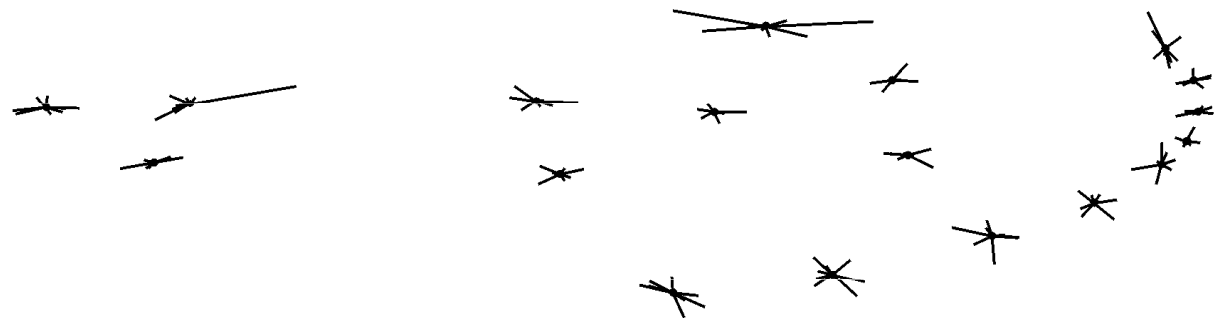
**a)**



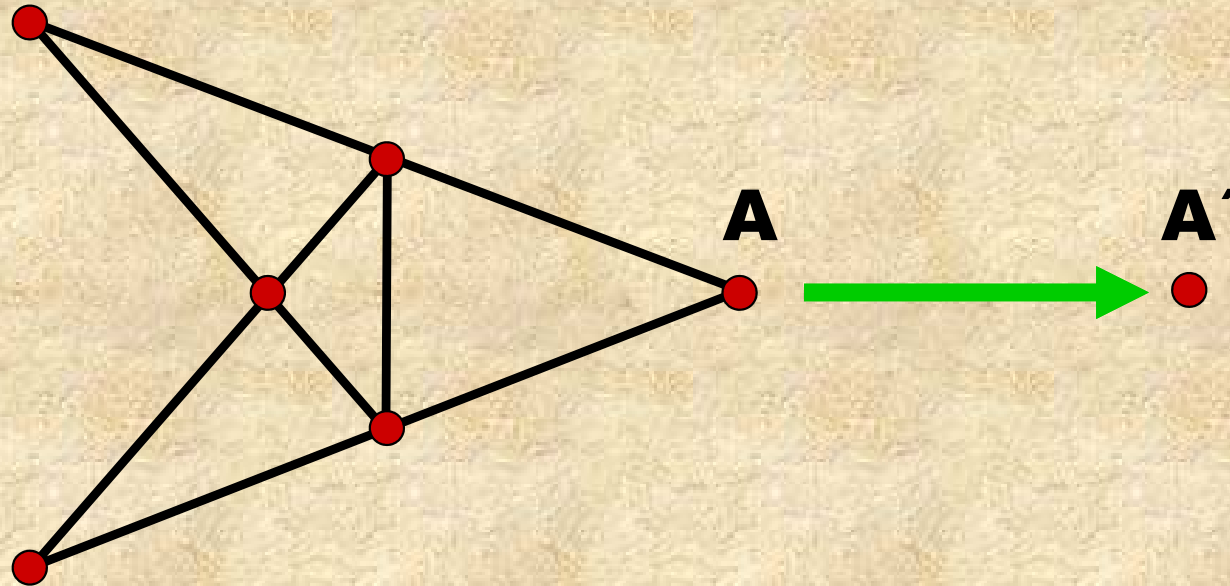
**b)**



**c)**



## Problém nejmenších čtverců:



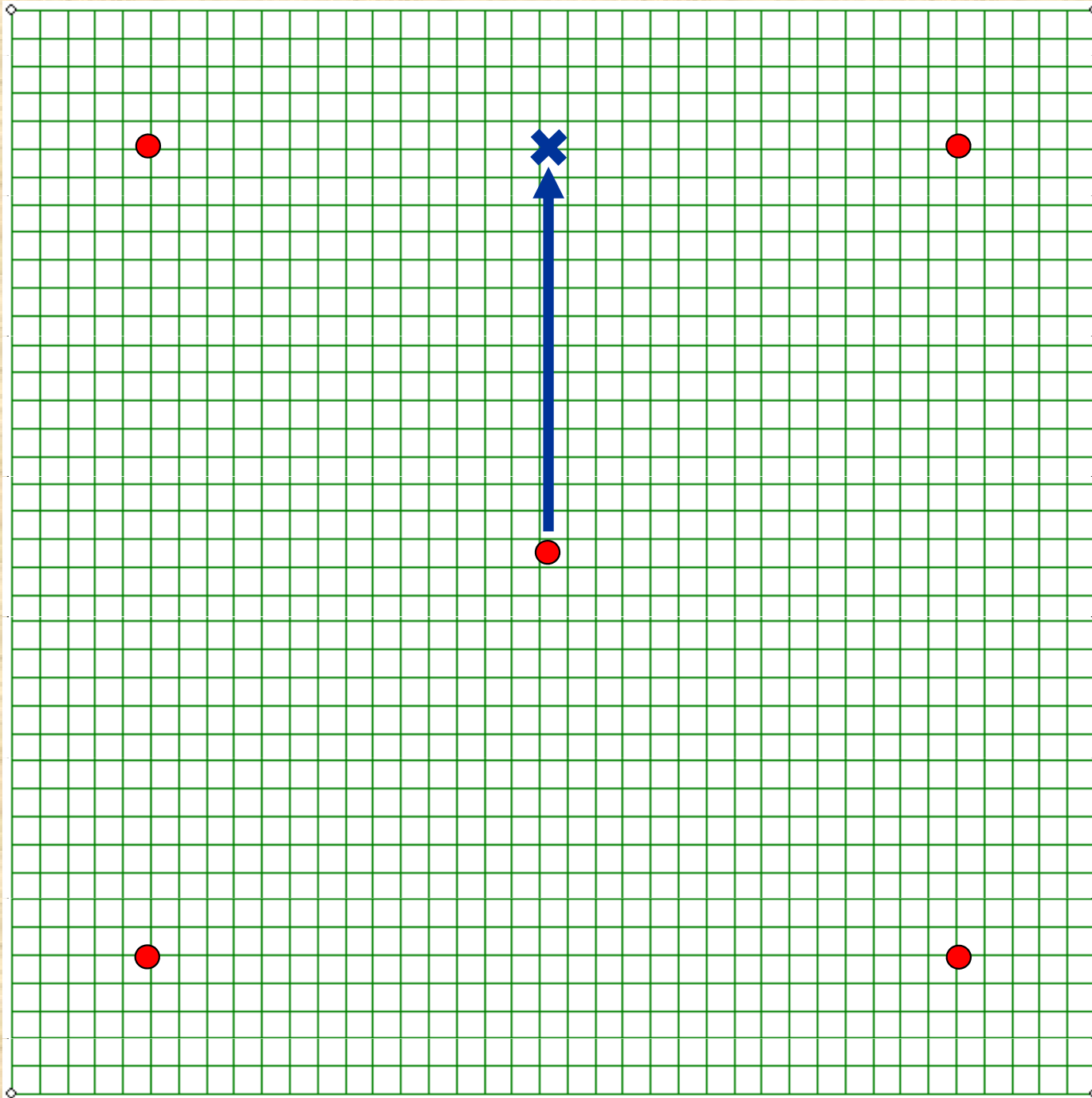
→ Rezistentní přizpůsobení  
GRF (Generalized Resistant Fit)

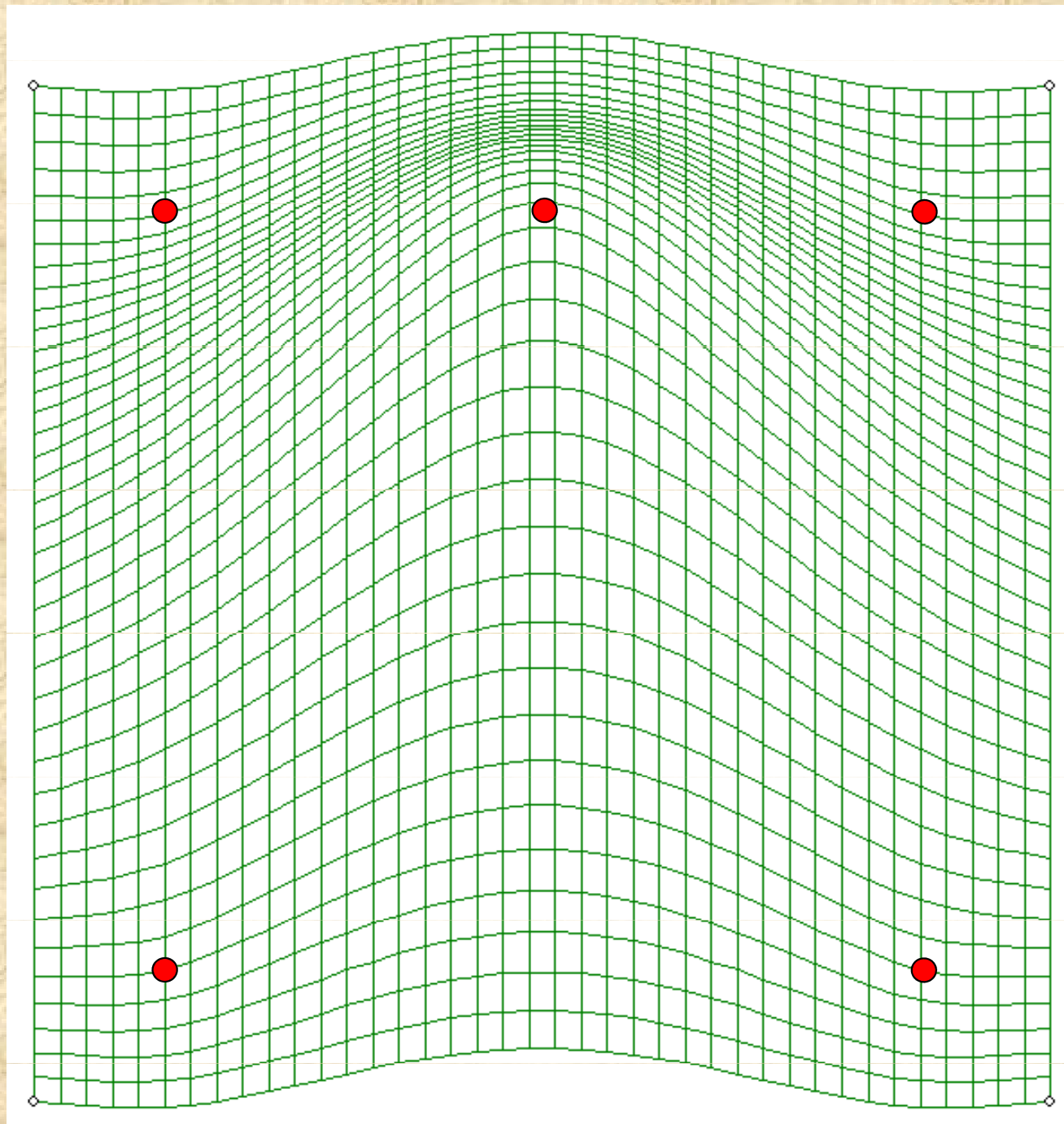
# **Geometrická morfometrie III.**

## **Deformace souřadnicových sítí**

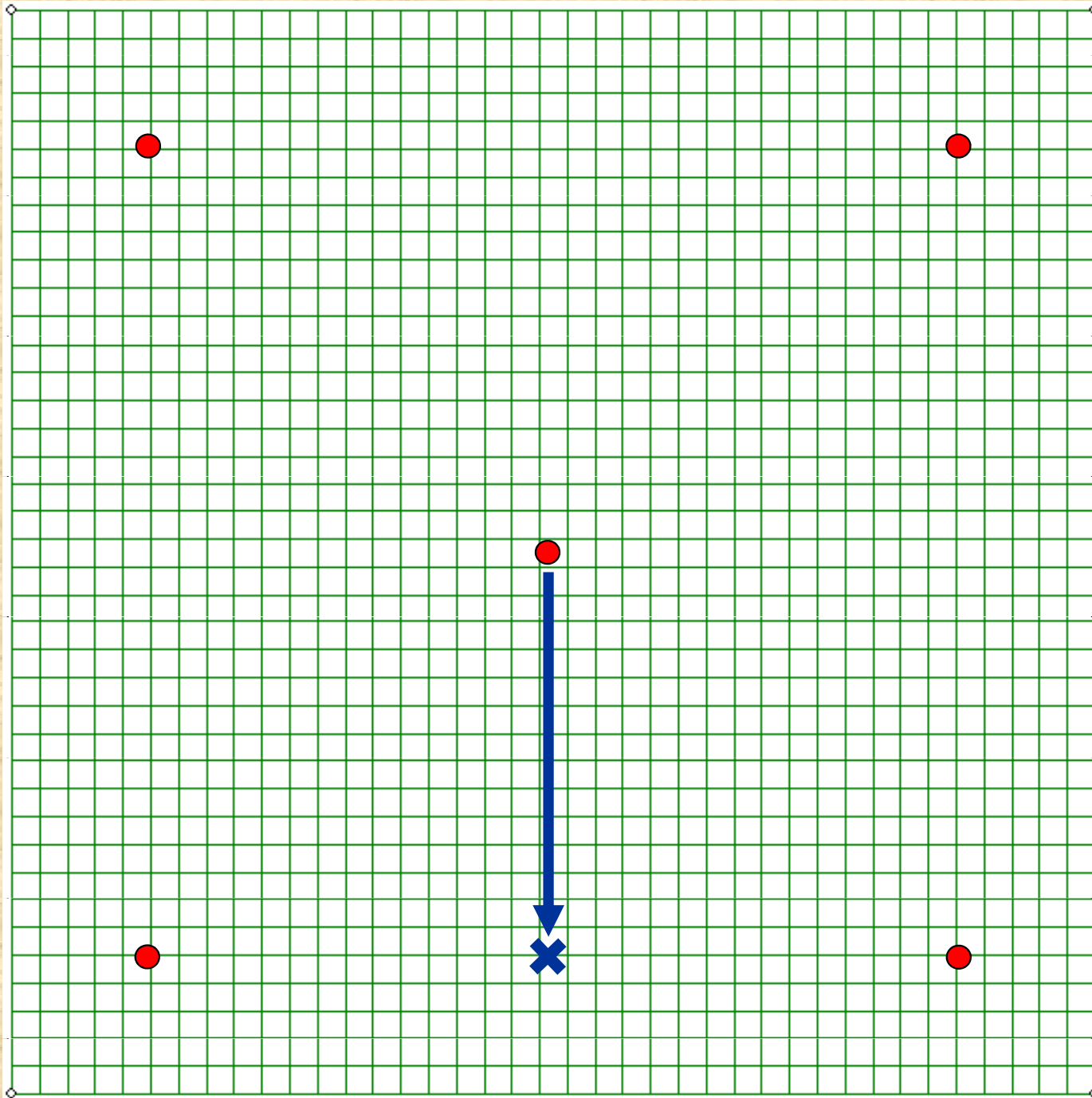
### **Metoda ohebných pásků, TPS (Thin-Plate Spline)**

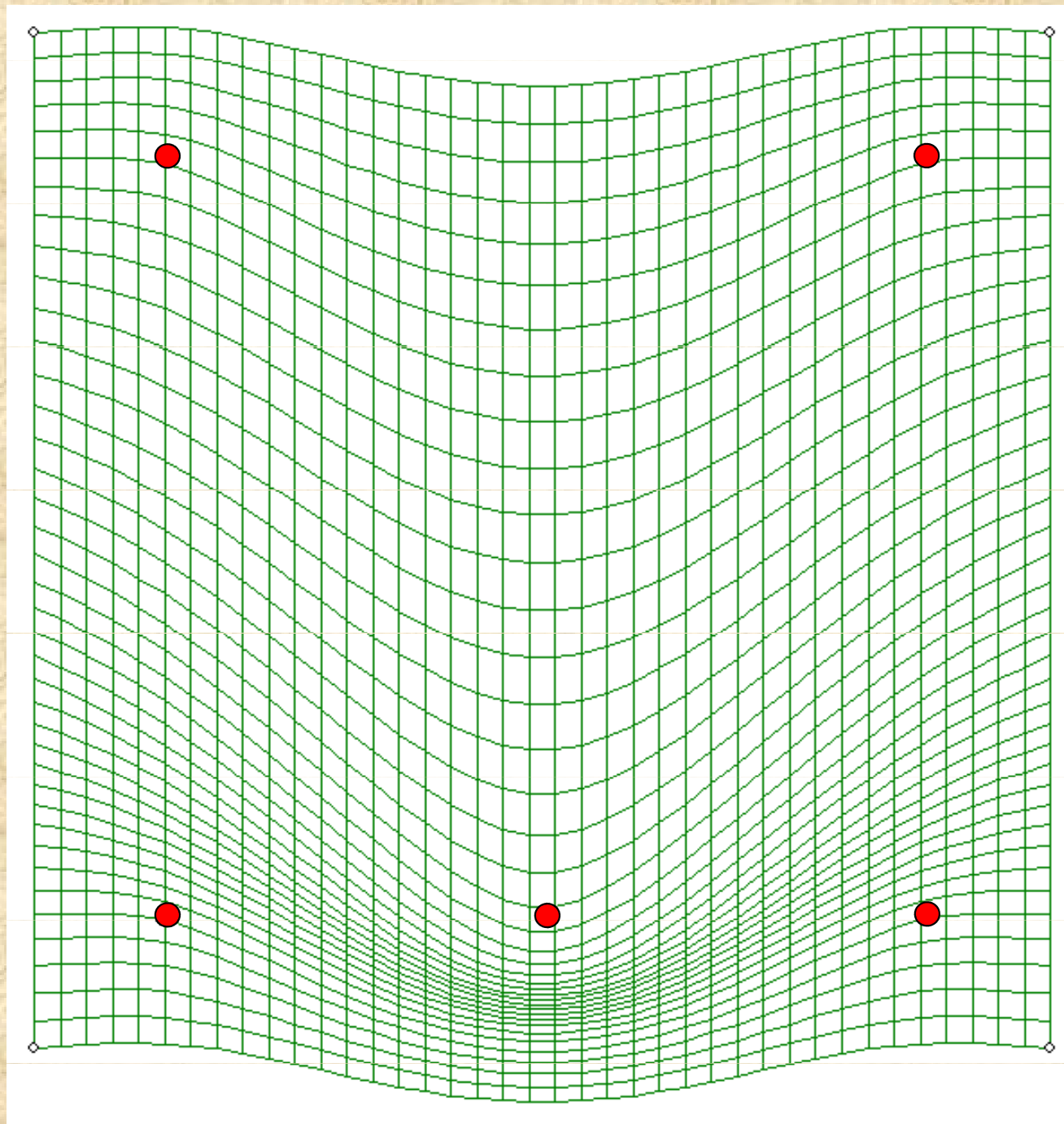
- **metafora nekonečně velkého, nekonečně tenkého kovového plátu**











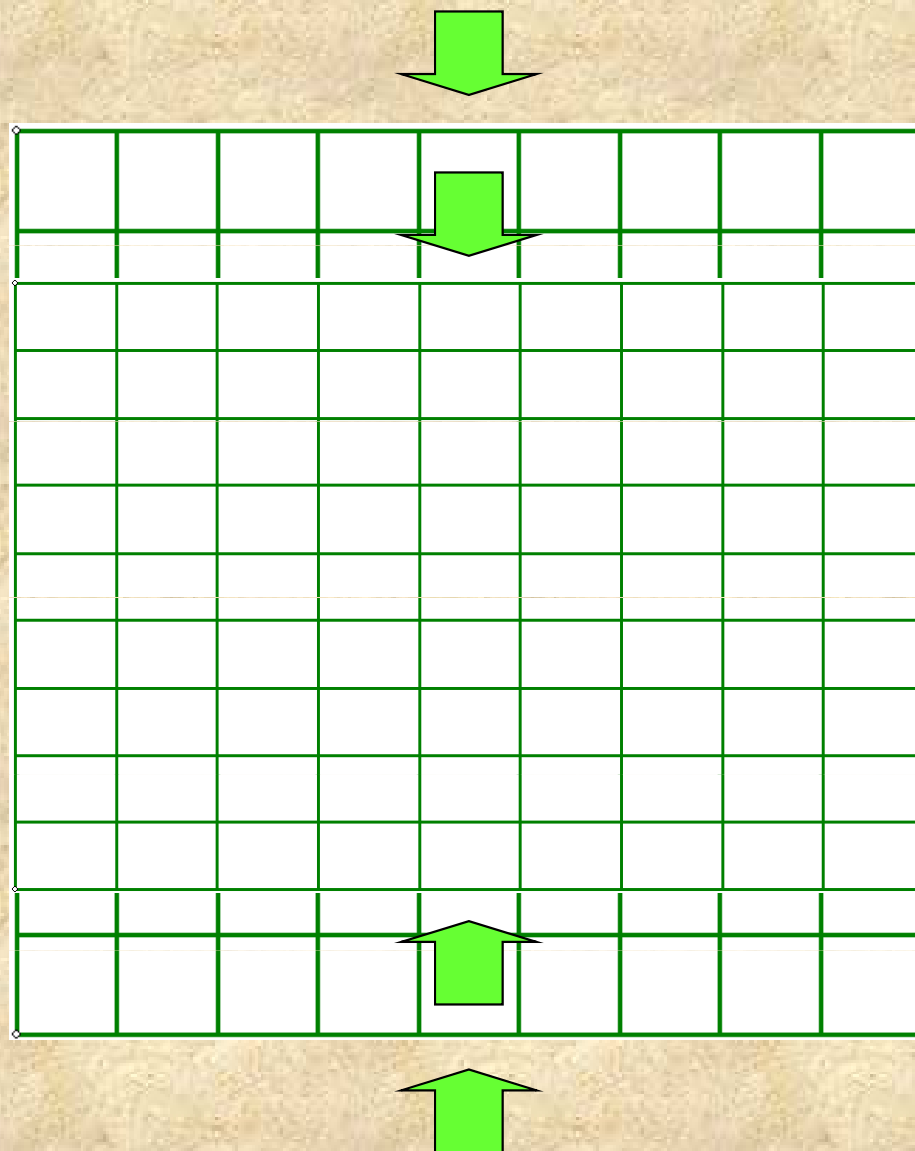
# Geometrická morfometrie III.

## Deformace souřadnicových sítí

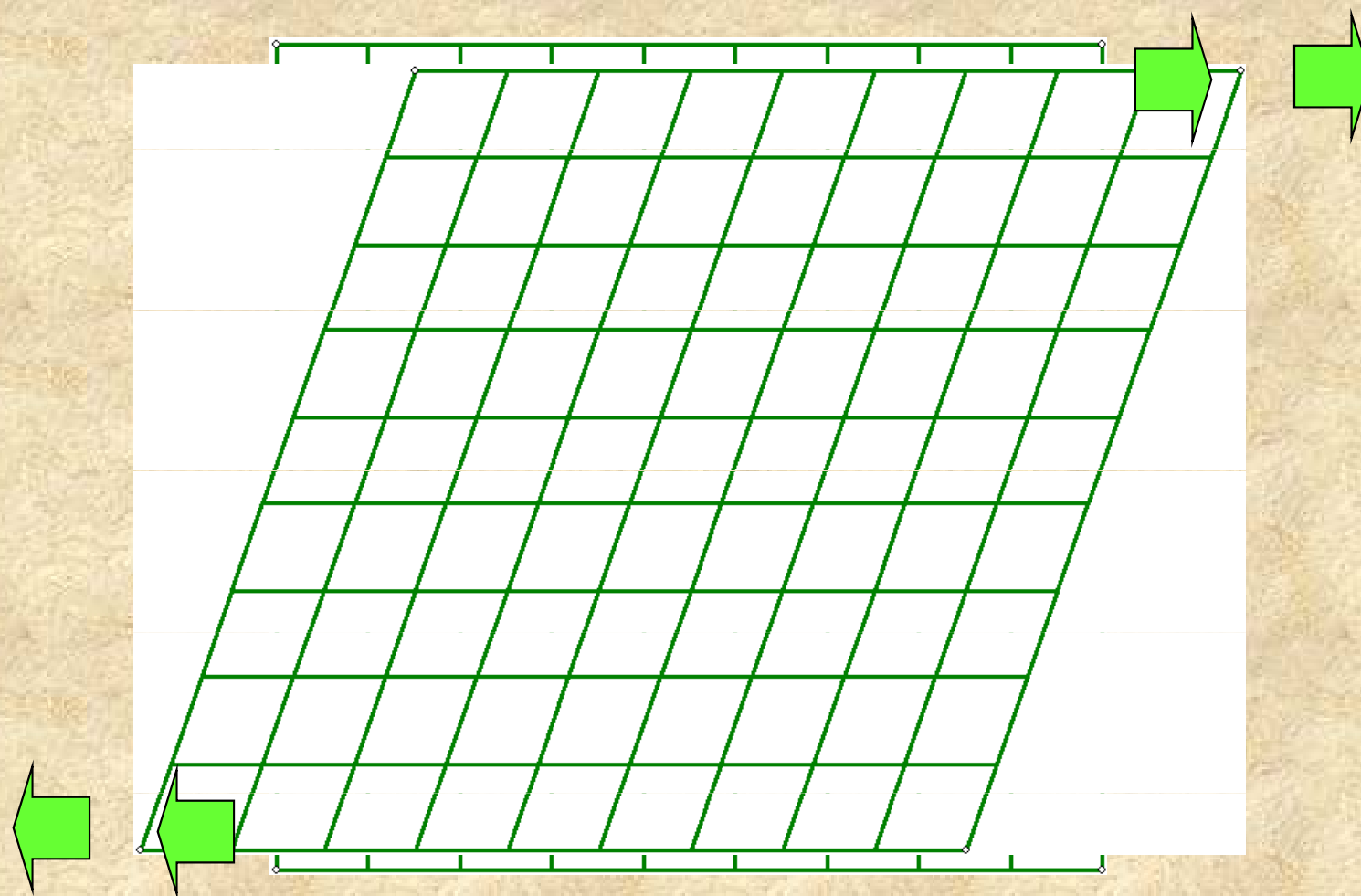
### Metoda ohebných pásků, TPS (Thin-Plate Spline)

- metafora nekonečně velkého, nekonečně tenkého kovového plátu
- energie nutná k deformaci plátu = deformační energie (bending energy)
- odlišení afinní a neafinní změny tvaru
- projekce latentních kořenů deformační energie do jednotlivých os = parciální deformace (partial warps)
- parciální deformace 0 ~ uniformní složce

# Afinní (uniformní) změna tvaru

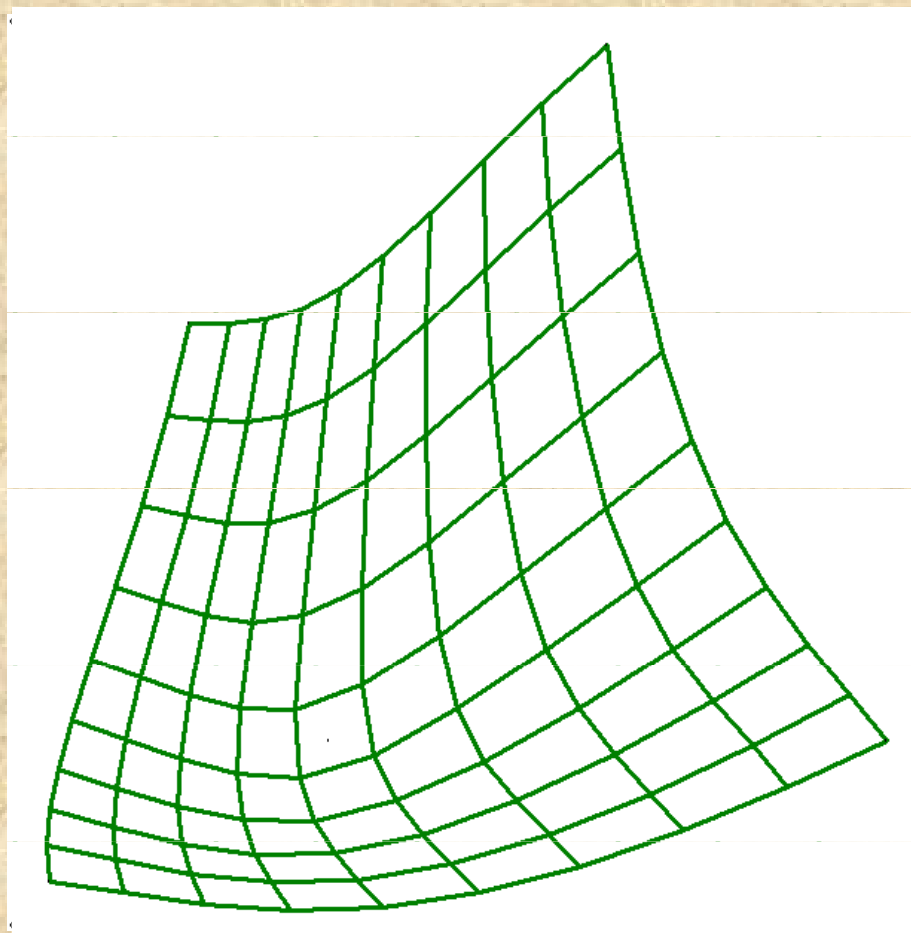


## Afinní (uniformní) změna tvaru



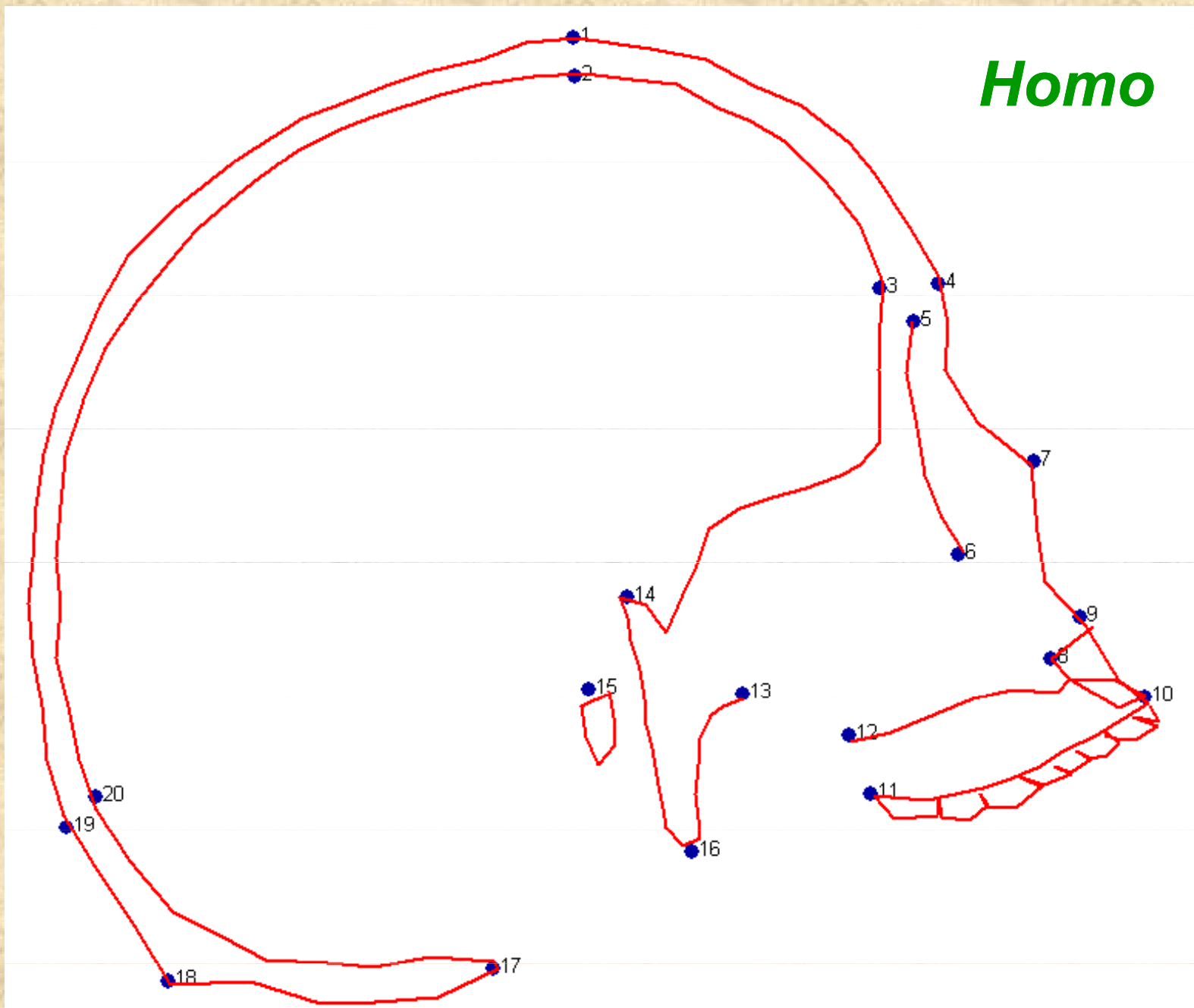
- rovnoběžky zůstávají rovnoběžné

## Neafinní (neuniformní) změna tvaru

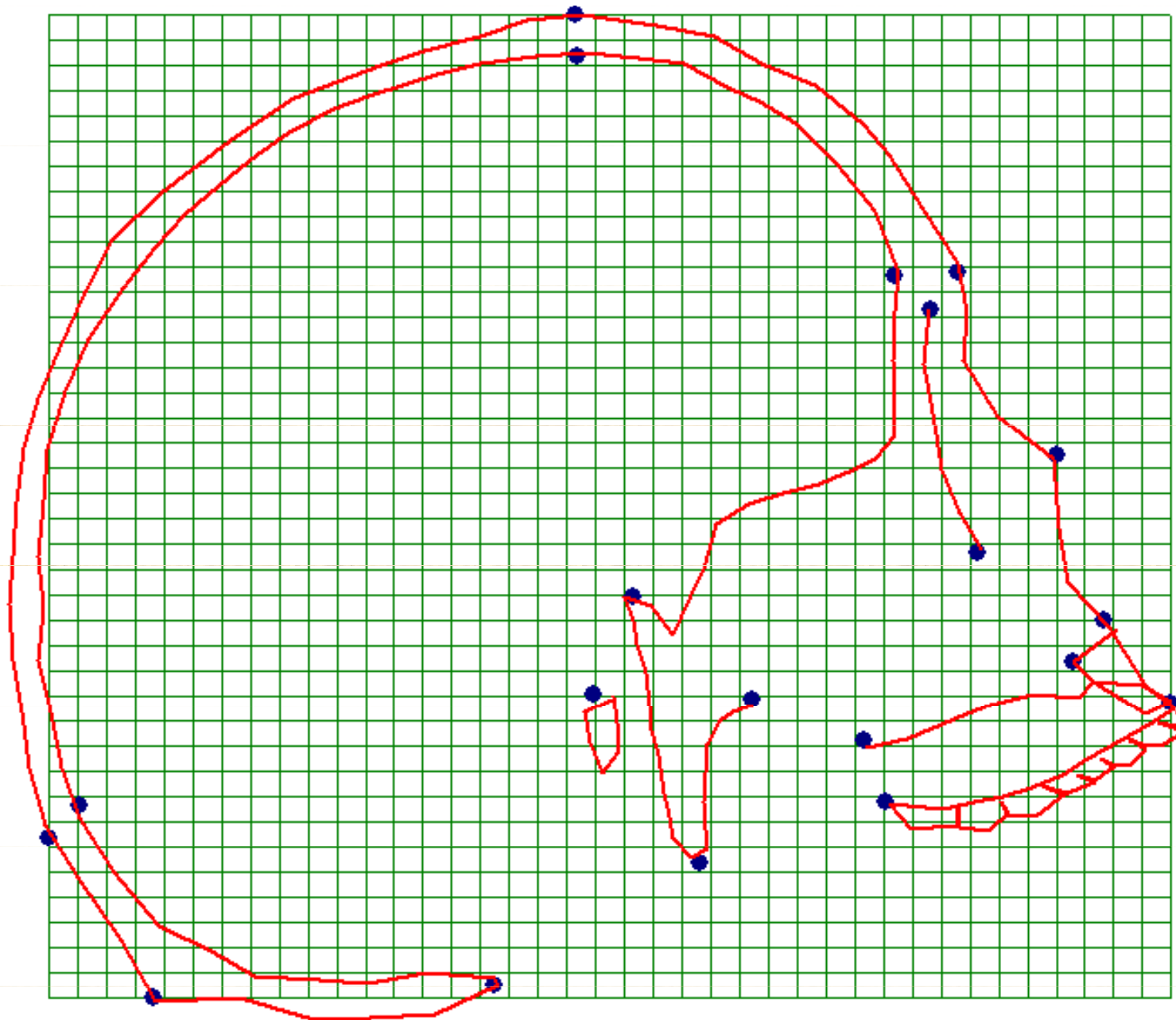


- změna tvaru sítě lokální

# *Homo*

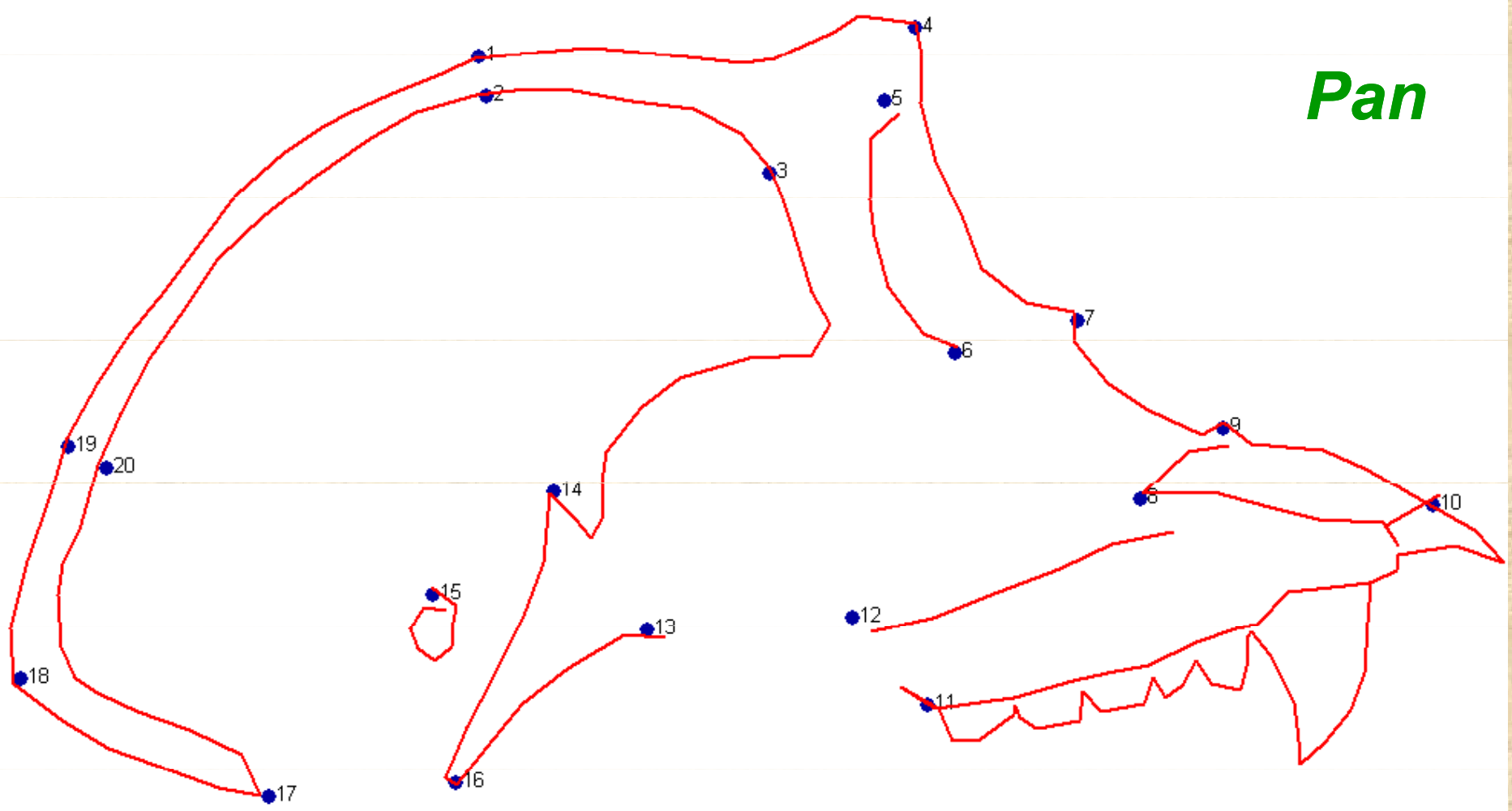


## referenční objekt

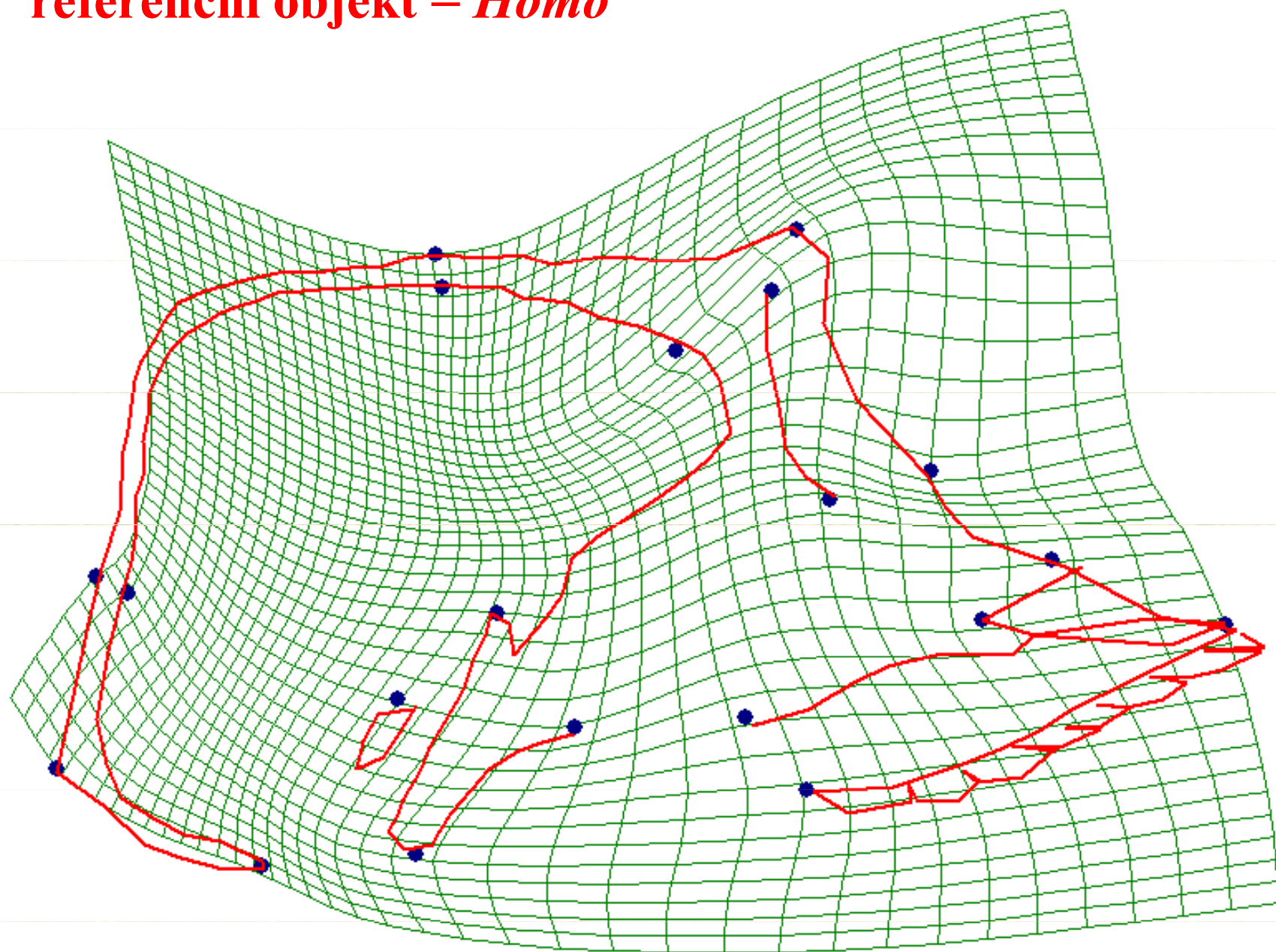


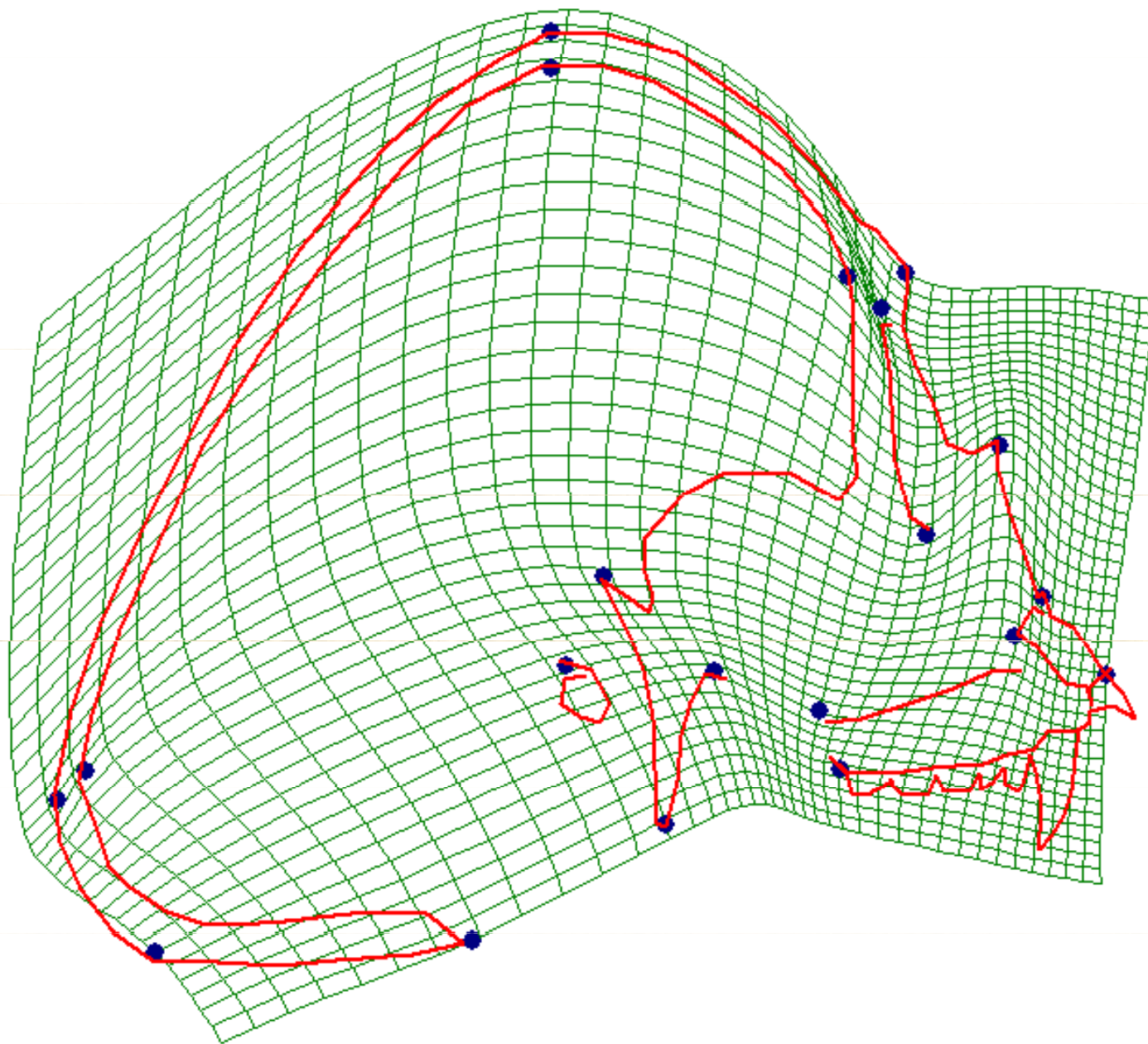


*Pan*



referenční objekt = *Homo*

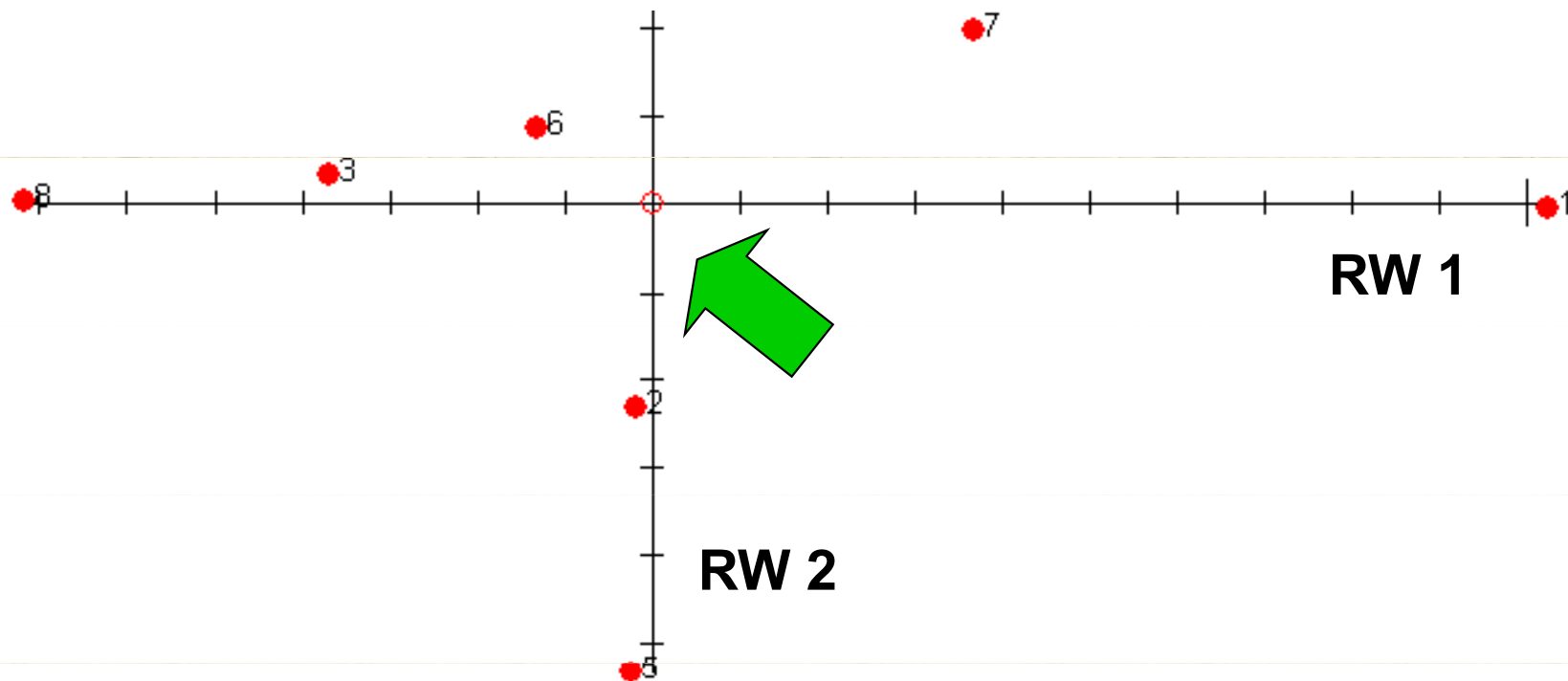


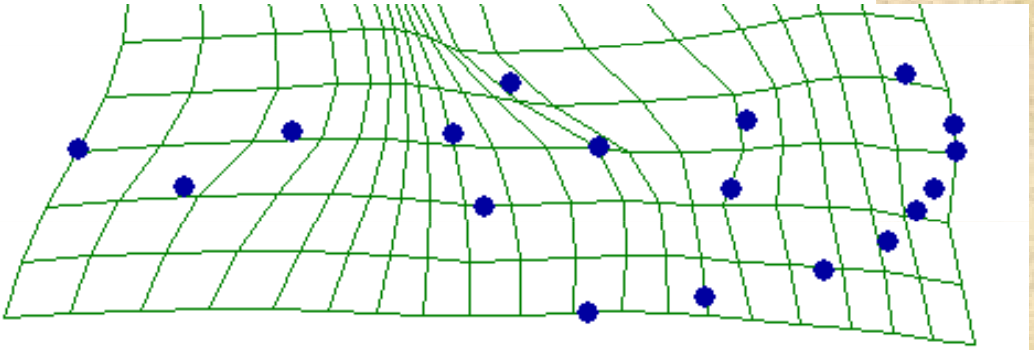
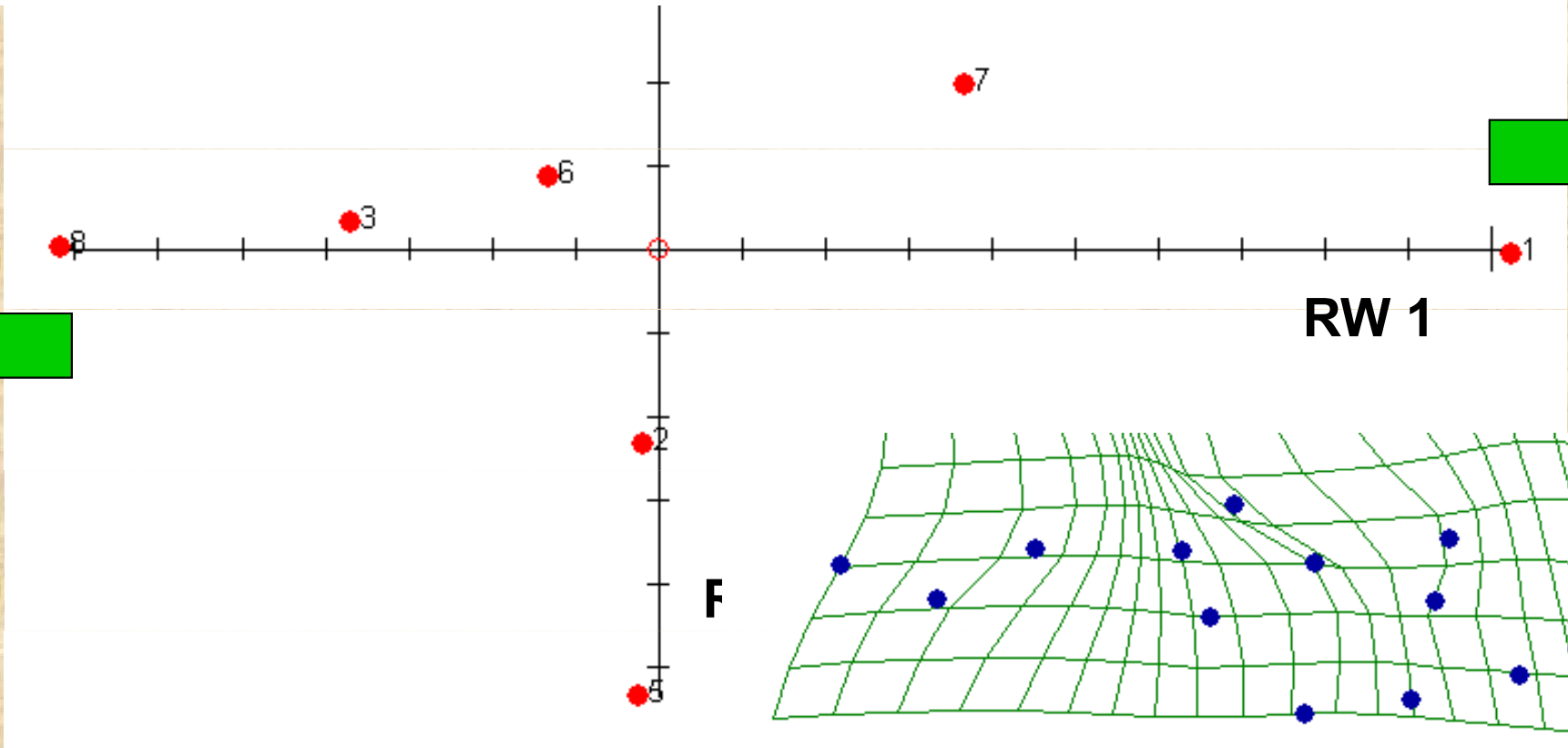
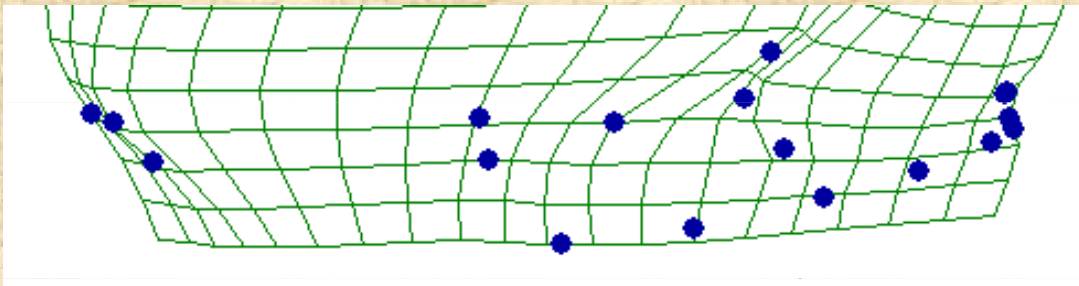


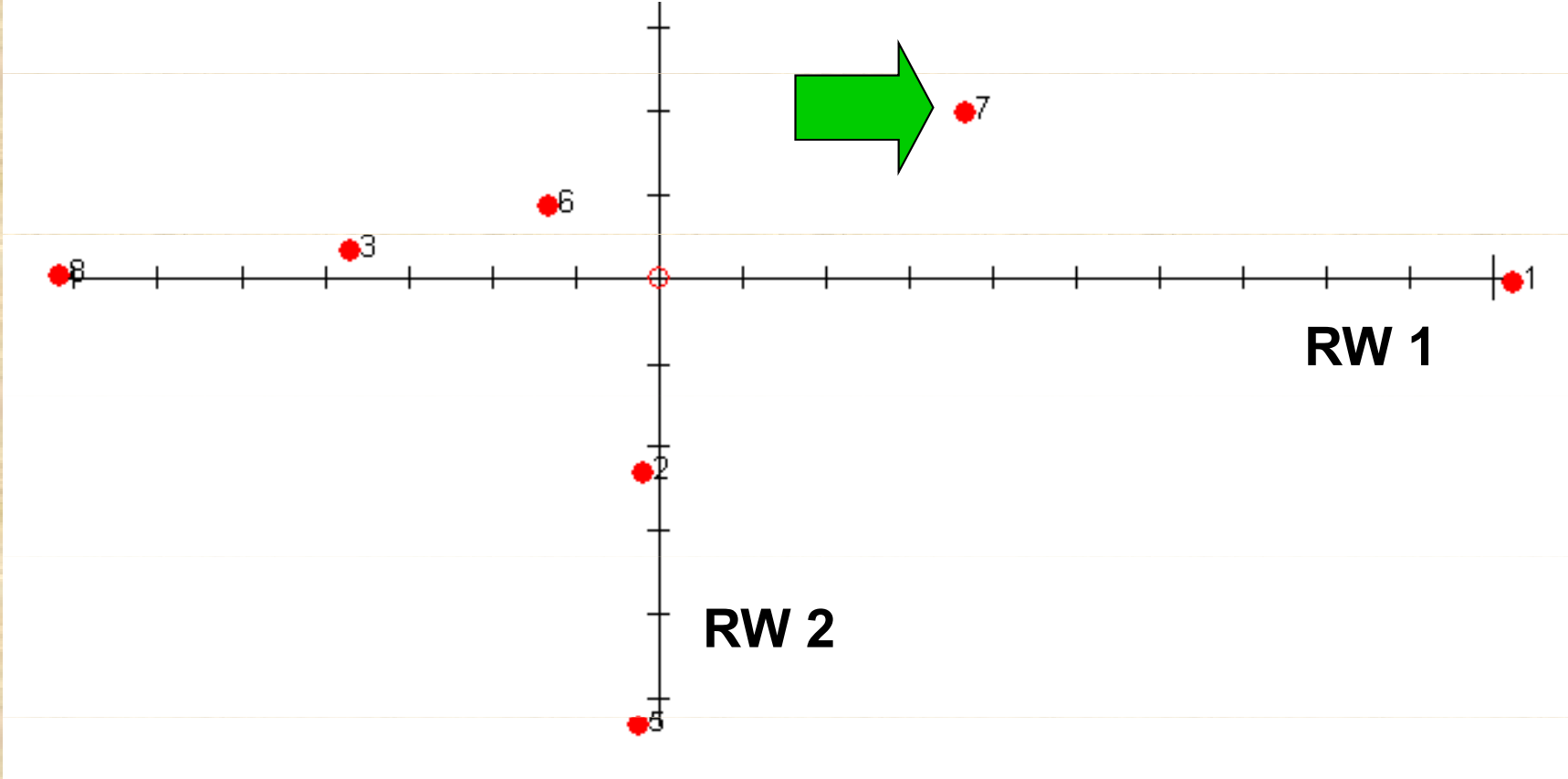
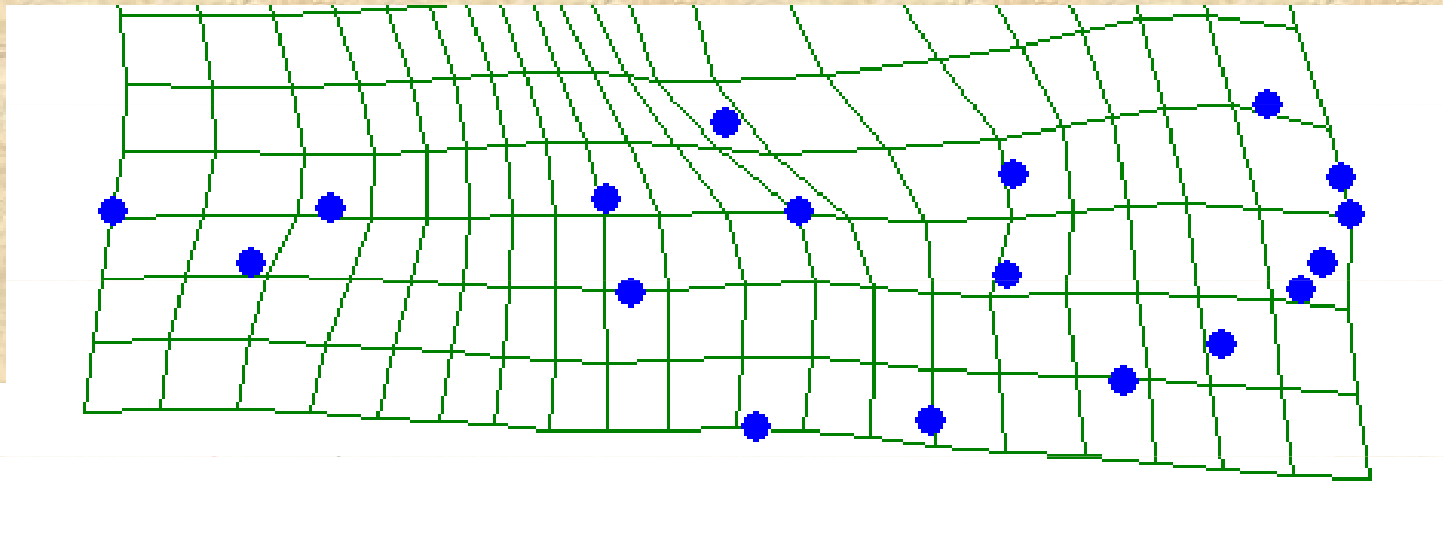
**referenční objekt = *Pan***

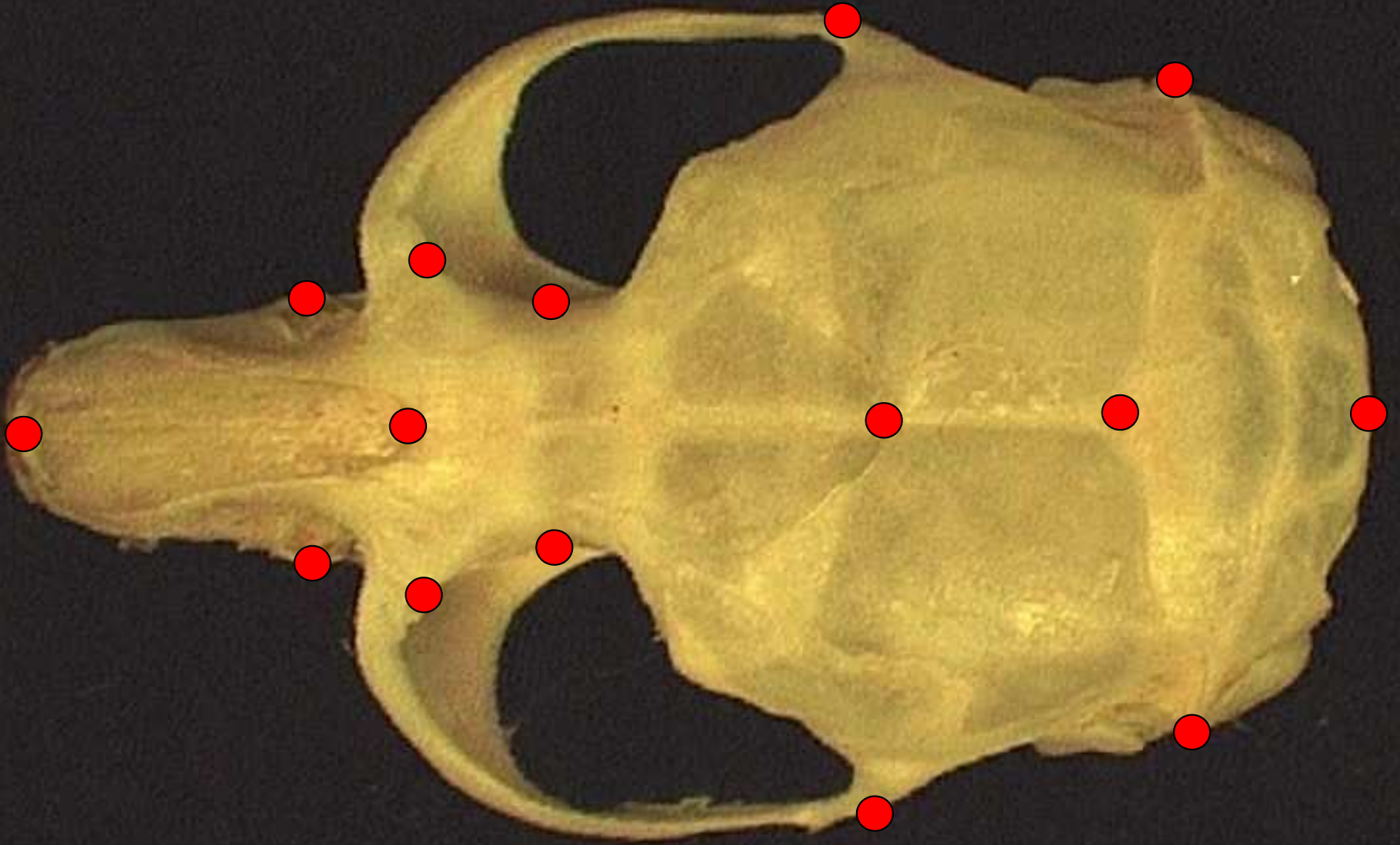
## Více než 2 objekty:

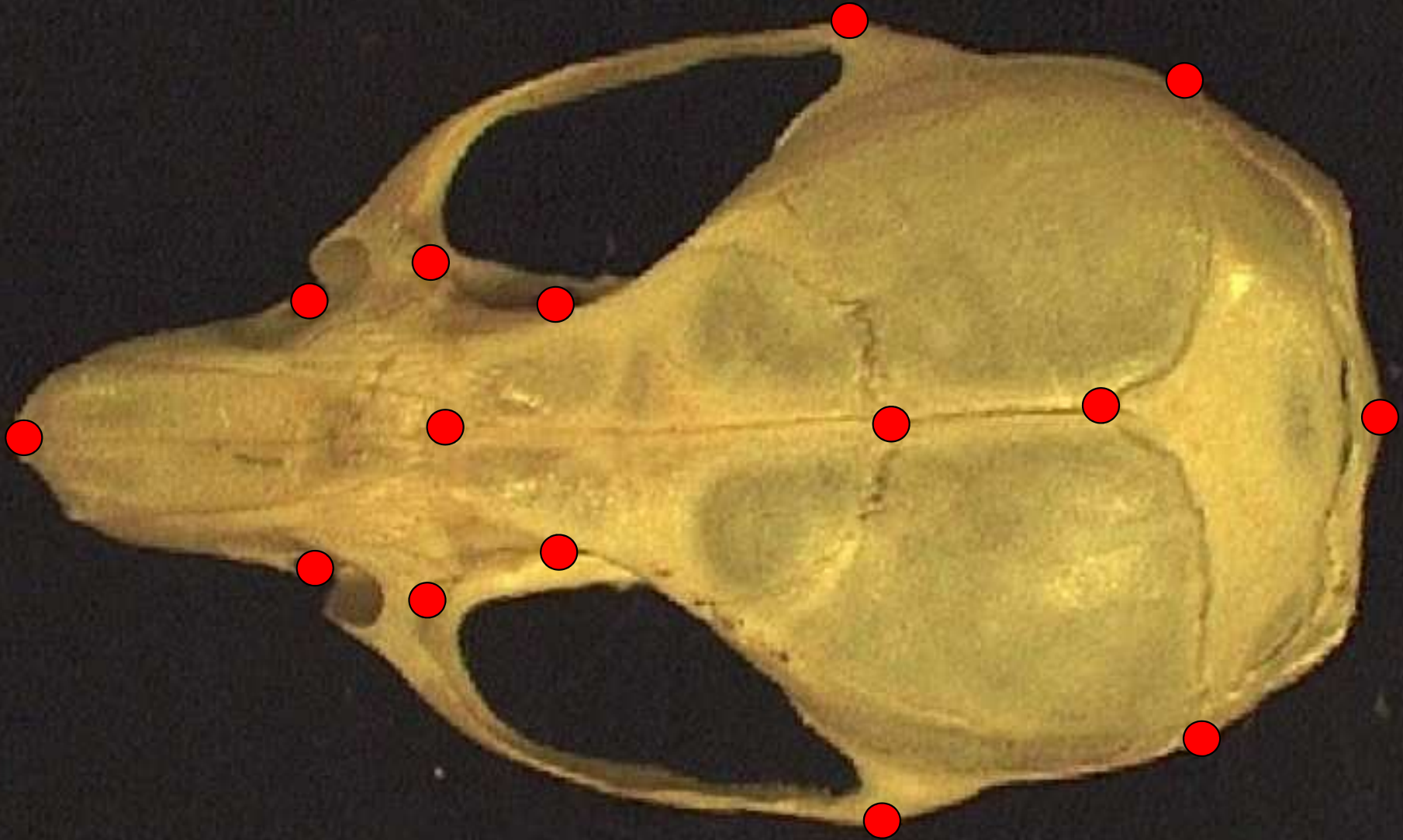
Relativní deformace, TPSRW (Thin-Plate Spline  
Relative Warps)













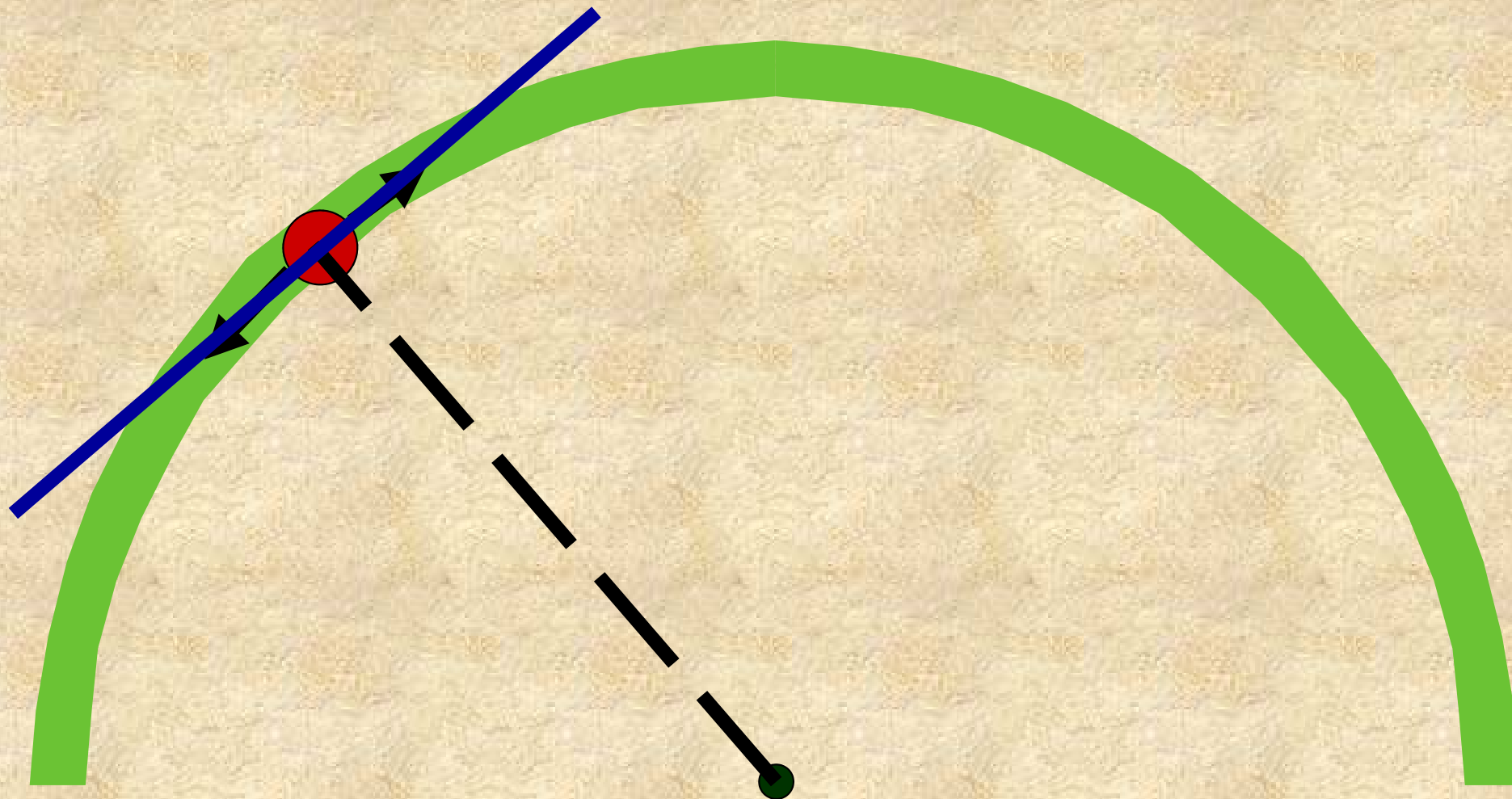


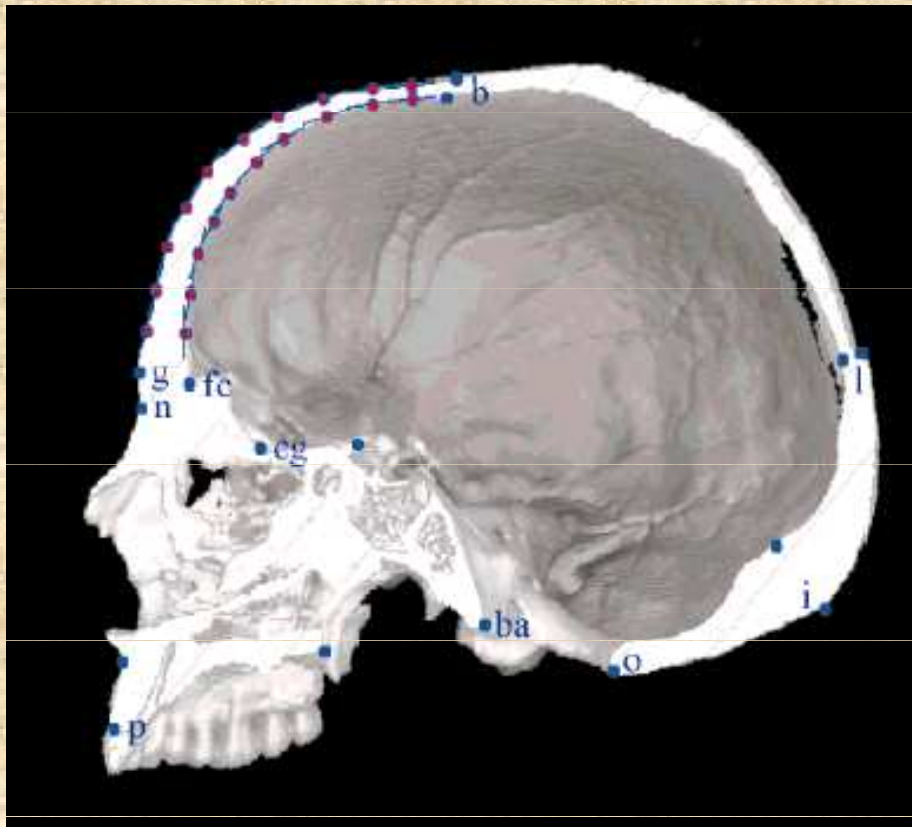
## **Software:**

- **tpsDig:** digitalizace bodů, měření rozměrů
- **tpsSpln:** Thin-Plate Spline
- **tpsRelw:** TPS Relative Warps
- **tpsRegr:** regrese na nezávislou proměnnou
- **tpsPLS:** parciální nejmenší čtverce (např. korelace 2 sad bodů)
- **tpsSuper:** deformace obrázků („unwarping“)
- **tpsTree:** analýza tvarových změn podél fylogenetického stromu

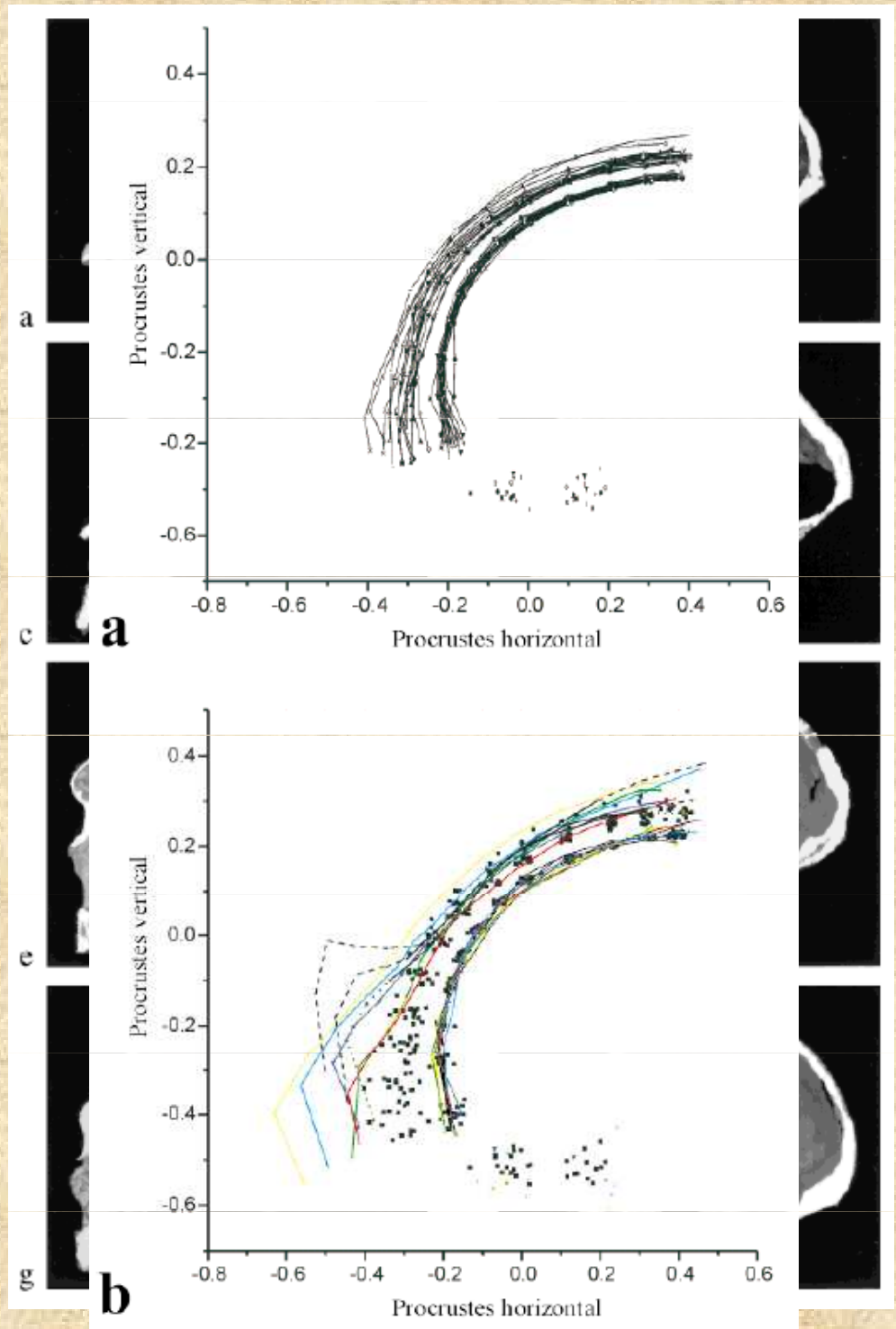
**<http://life.bio.sunysb.edu/morph/>**

# Metody založené na význačných bodech bez landmarků – „sliding semilandmarks“

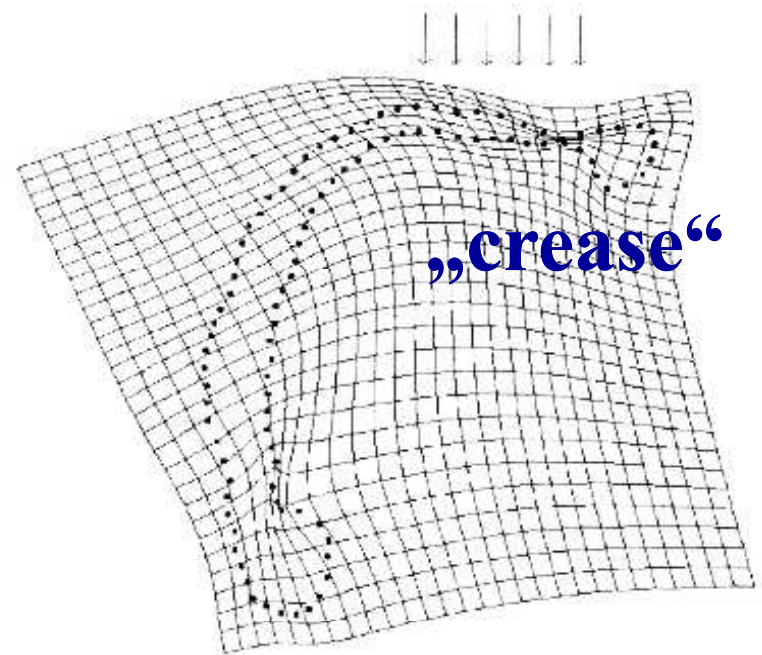
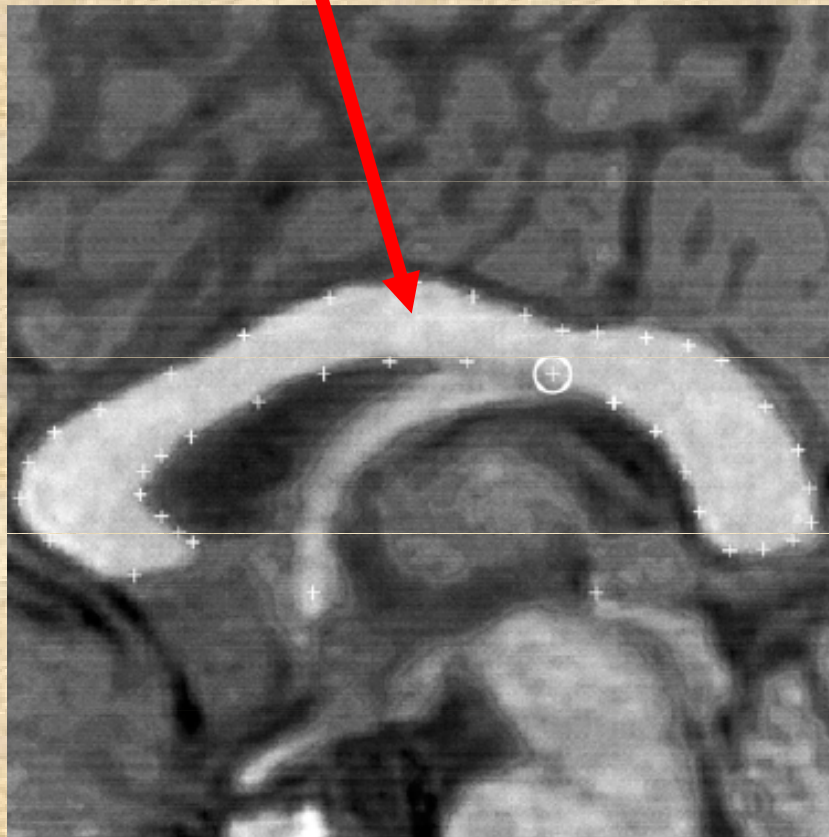




**Bookstein et al.,  
Anat. Record (1999)**

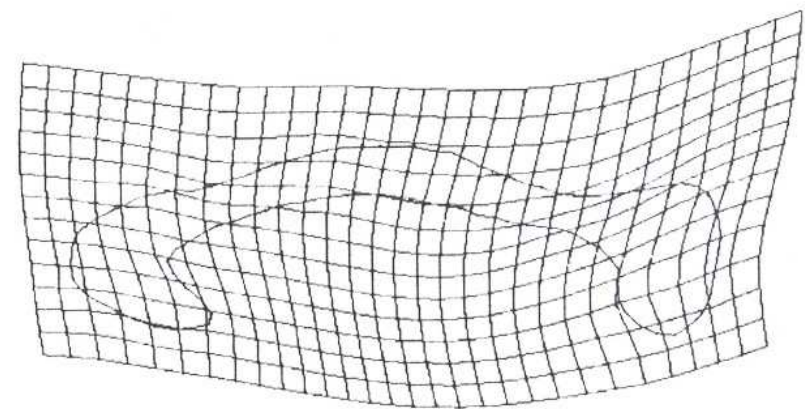


*corpus callosum*

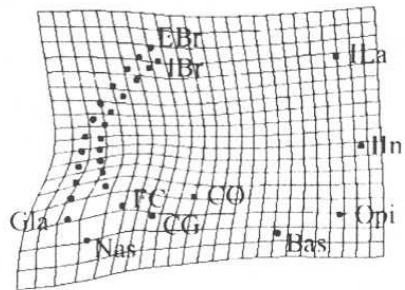


„crease“

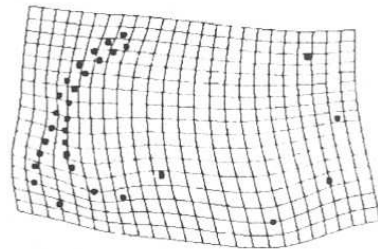
Extremum of Deformation Tensor:  
Male → Female  
(Extrapolated -2.42-Fold to a Crease)



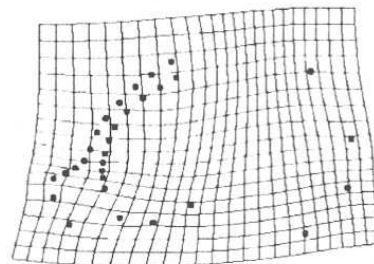
Deformation: Male → Female (x2)



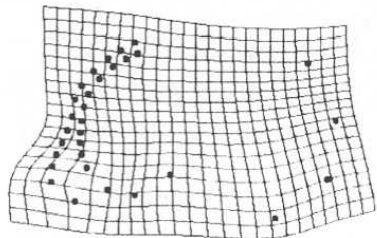
Modern *Homo* → Archaic *Homo*



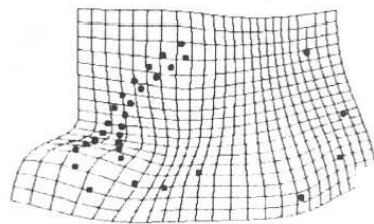
Archaic *Homo* → STS5



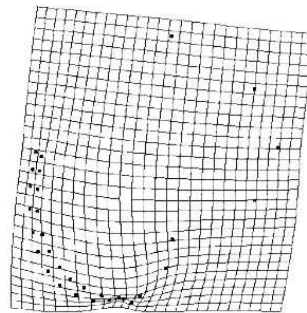
STS5 → *Pan*



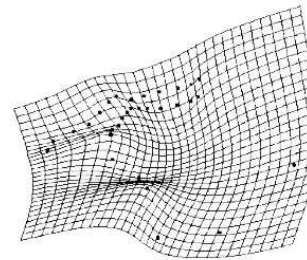
Modern *Homo* → STS5



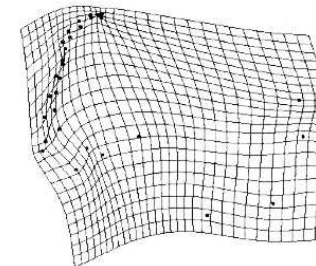
Modern *Homo* → *Pan*



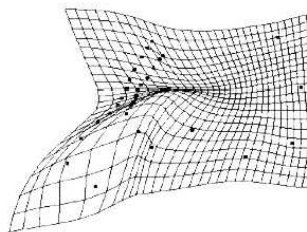
Modern *Homo* → Archaic *Homo*  
(Positive Crease, Strain = 2.362)



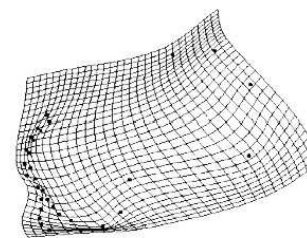
Archaic *Homo* → STS5  
(Positive Crease, Strain = 1.377)



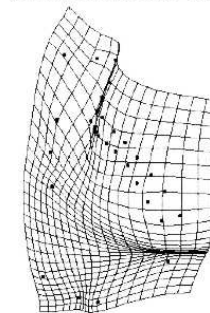
STS5 → *Pan*  
(Negative Crease, Strain = 0.693)



Modern *Homo* → Archaic *Homo*  
(Negative Crease, Strain = 0.701)



Archaic *Homo* → STS5  
(Positive Crease, Strain = 0.553)



STS5 → *Pan*  
(Positive Crease, Strain = 2.362)

**<http://life.bio.sunysb.edu/morph/>**