Calculate at home

- Derive continuity equation using fluid dynamics.
- Derive continuity equation using general transport equation (Bittencourt, page 194).
- Write the Jacobian matrix and the Jacobian matrix determinant for the transformation from spherical coordinates (r, θ, φ) to Cartesian coordinates (x, y, z).
- Sir Edward Victor Appleton was awarded the Nobel Prize in physics in 1947 for his investigation of radio wave propagation in the ionosphere. In his Nobel Lecture, he described the variations in the reception of radio waves when the direct waves (near ground) interferred with waves reflected from the ionosphere. Consider radio waves reflected from the so-called E layer. The E-layer has a plasma density of $10^5\,\mathrm{cm}^{-3}$ and is located at an altitude of about 100 km. What frequency radio waves can reflect from the E-layer?
- Suppose a particle of mass m is moving in the force field of a one-dimensional harmonic oscillator. The force $\mathbf{F}$ (in x-direction $\hat{\mathbf{x}}$) and the potential energy U are given by

  $$\mathbf{F} = \mathbf{F}(x) = -k\,x\,\hat{\mathbf{x}}, \qquad U = U(x) = \frac{1}{2}\,k\,x^2$$

  The parameter k denotes a constant.

  $$f_1(x,v) = C_1 \exp\left(-\frac{m\,v^2}{2} - kx\right), \quad f_2(x,v) = C_2 \exp\left(-\frac{m\,v^2}{2} + kx\right),$$

  $$f_3(x,v) = C_3 \exp\left(-\frac{m\,v^2}{2} - \frac{kx^2}{2}\right), \quad f_4(x,v) = C_4 \exp\left(-\frac{m\,v^2}{2} + \frac{kx^2}{2}\right).$$

  a) Only one of the following four functions solves the collisionless Boltzmann kinetic equation for this case. Which one? Give a simple argument or calculation.

  b) From part (a) choose one of the three remaining functions and verify that it does not solve the collisionless Boltzmann kinetic equation (a few calculations are required).