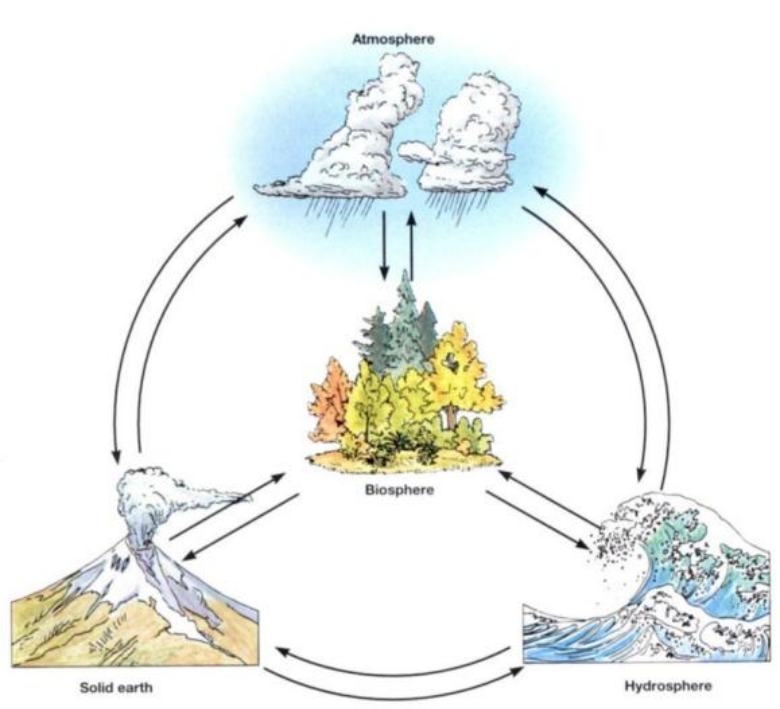
GB471
Stabilita a dynamika přírodních systémů

3

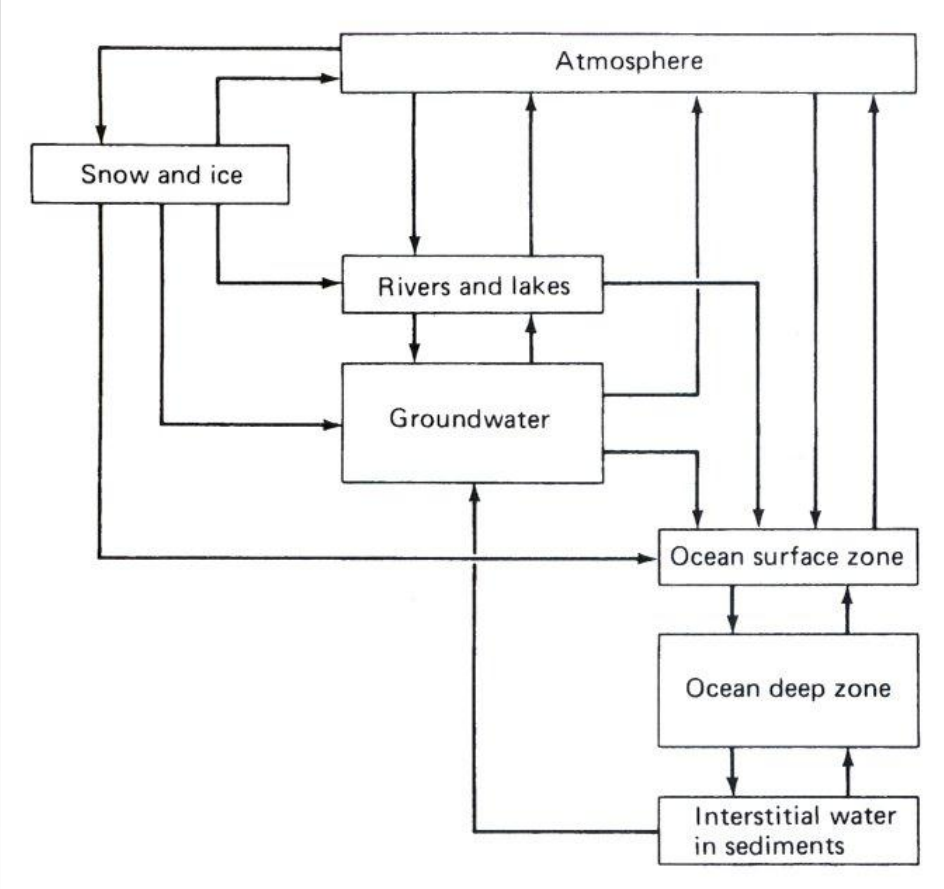
Josef Zeman

KONCEPCE REZERVOÁRŮ

Zemský systém

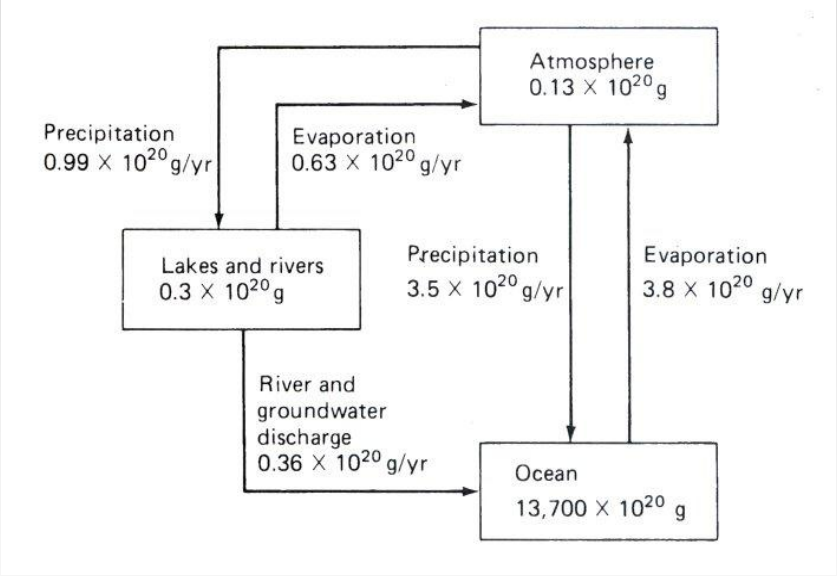


Hydrogeologický systém

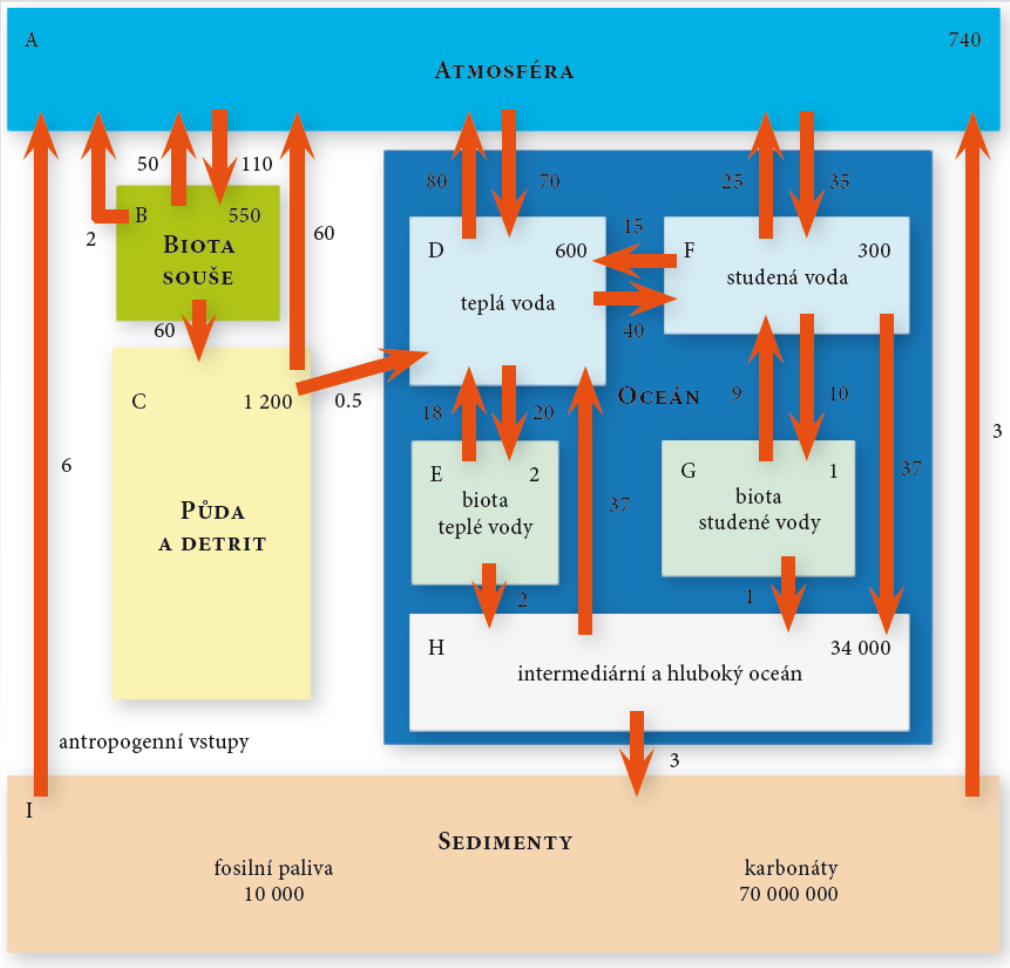


KONCEPCE REZERVOÁRŮ

Zemský systém – voda

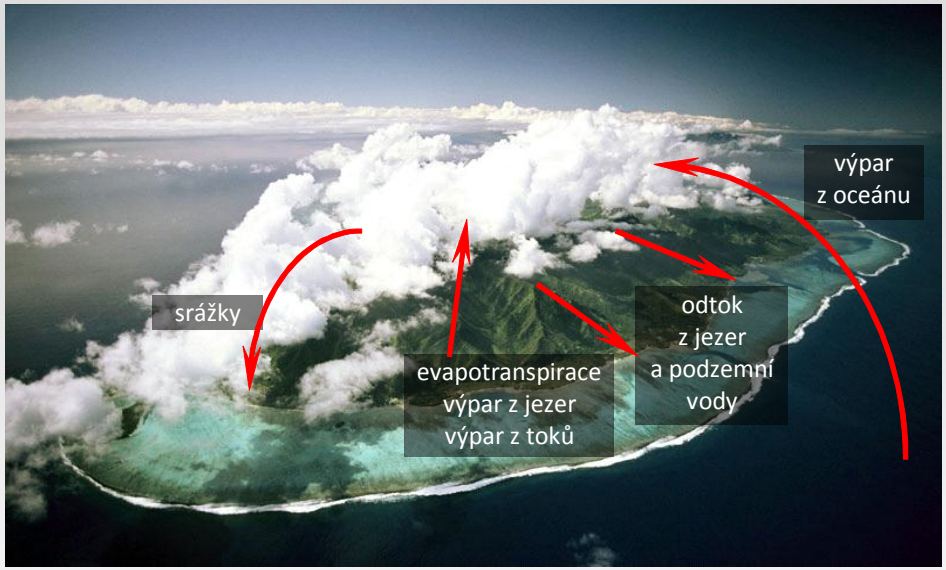


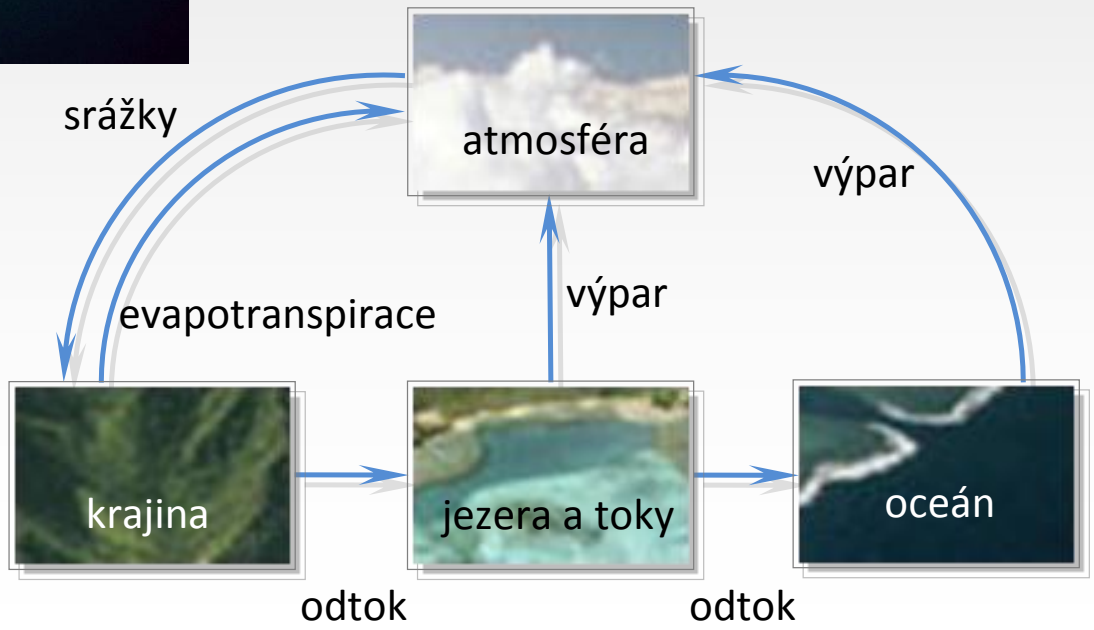
Globální cyklus uhlíku



KONCEPCE REZERVOÁRŮ

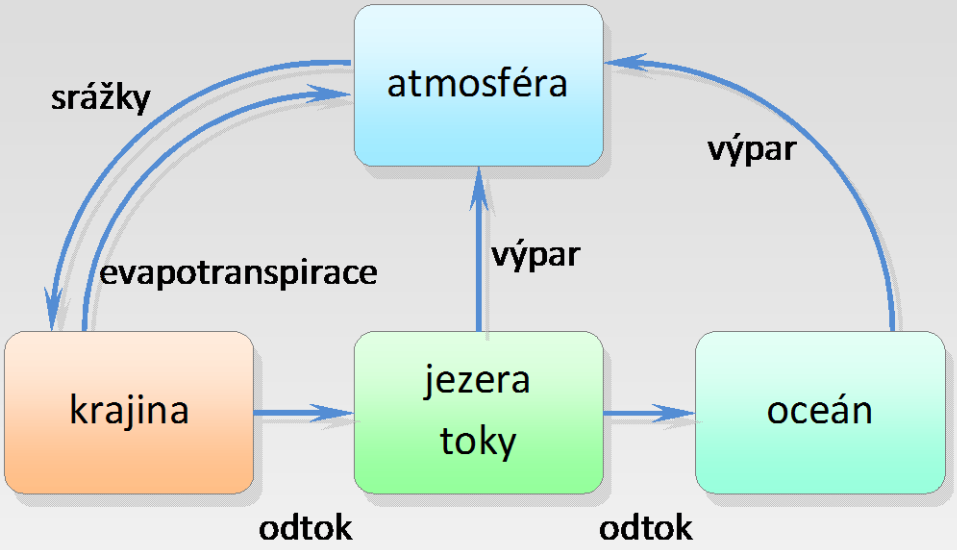
Ostrov

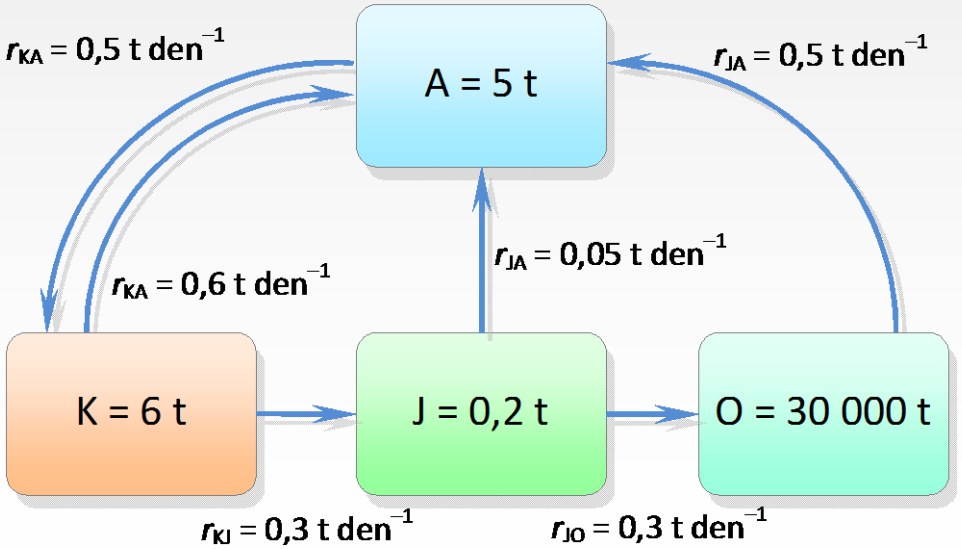




KONCEPCE REZERVOÁRŮ

Ostrov





KONCEPCE REZERVOÁRŮ

Hranice – fyzická, ale také pouze virtuální

$$CO_2(aq) + H_2O \underset{r_{-1}}{\overset{r_{+1}}{\rightleftharpoons}} H_2CO_3 \underset{r_{-2}}{\overset{r_{+2}}{\rightleftharpoons}} H^+ + HCO_3^- \underset{r_{-3}}{\overset{r_{+3}}{\rightleftharpoons}} H^+ + CO_3^{2-}$$

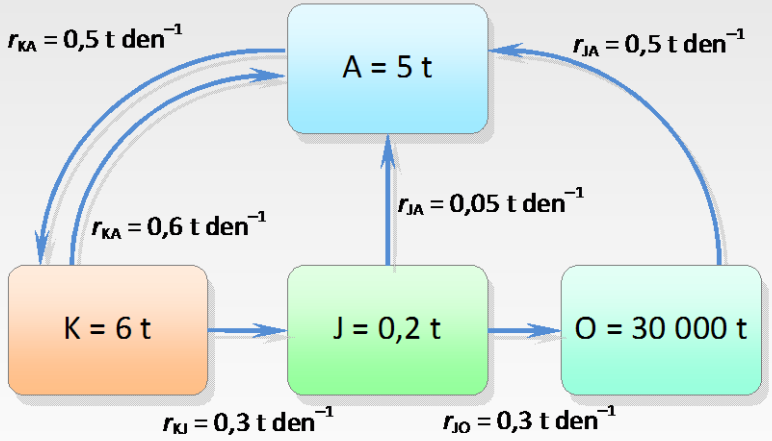
Rychlost $r_Y = kX$

$r_{jo} = k_{jo} m_j$

toky lineární $r_{+1} = k_{+1} a_{CO_2(aq)} a_{H_2O}$

toky nelineární $r_Y = kX^2$

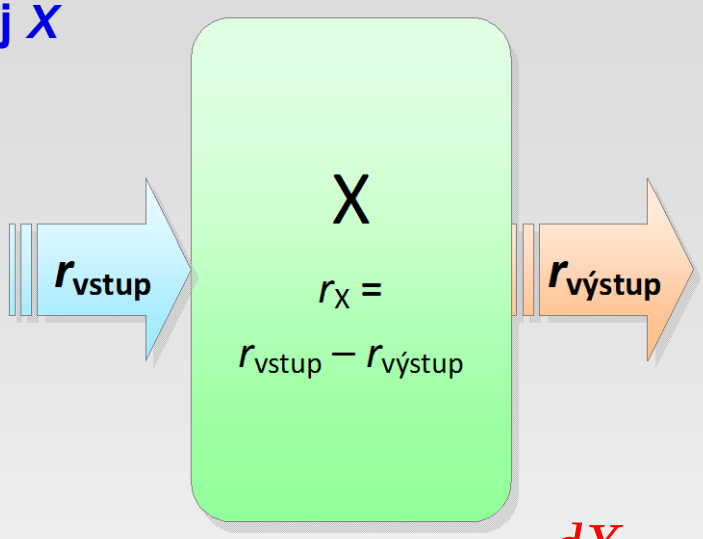
výsledný tok $r_{výsledný} = \sum_i r_i - \sum_j r_j$



časová změna obsahu rezervoáru $\frac{dm_C}{dt} = \sum_i r_i - \sum_j r_j$

JEDNOREZERVOÁROVÝ SYSTÉM

Časový vývoj *X*



$$r_{vstup} = A$$

$$r_{výstup} = kX$$

$$r_{výsledný} = r_{vstup} - r_{výstup}$$

$$\frac{dX}{dt} = r_{vstup} - r_{výstup}$$

$$\frac{dX}{dt} = A - kX$$

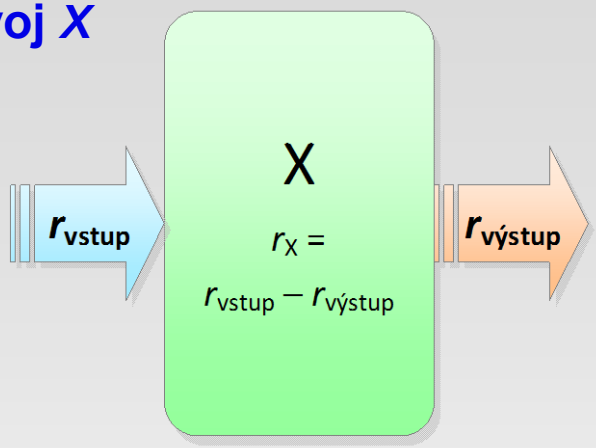
$$\frac{dX}{A - kX} = dt$$

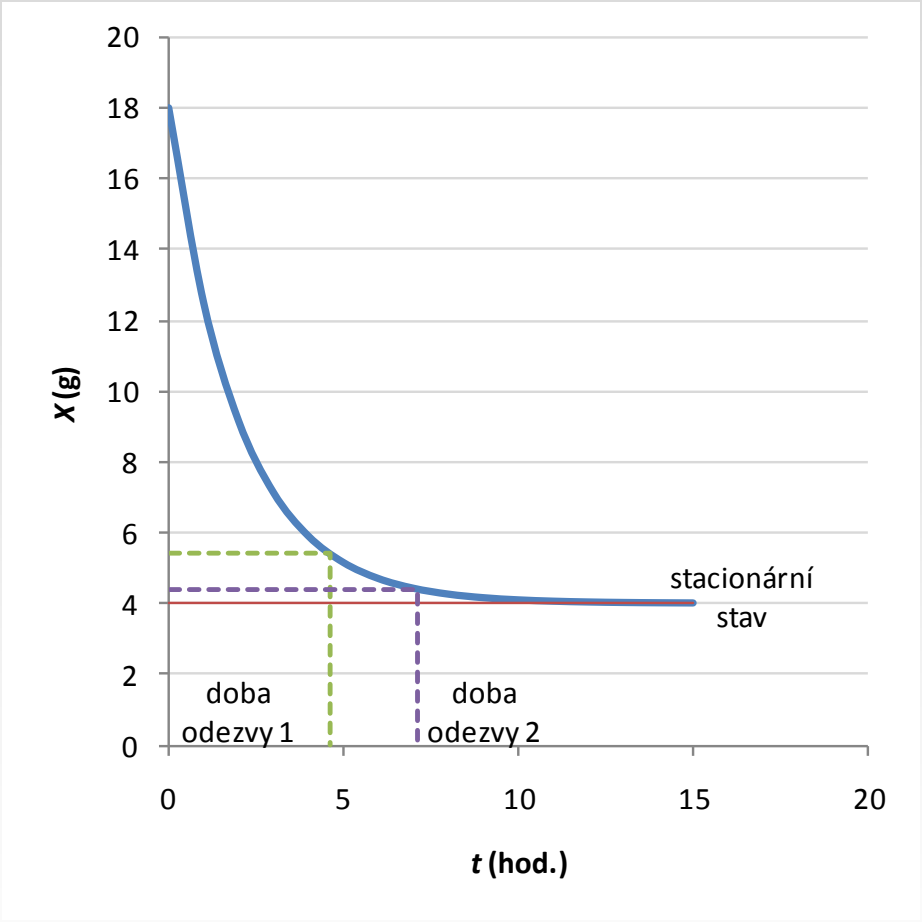
$$\int_{X_0}^{X} \frac{dX}{A - kX} = \int_0^t dt$$

$$\ln(A - kX) - \ln(A - kX_0) = -k(t - 0)$$

JEDNOREZERVOÁROVÝ SYSTÉM

Časový vývoj *X*





$$\ln\frac{A-kX}{A-kX_0}=-kt$$

$$\frac{A-kX}{A-kX_0}=e^{-kt}$$

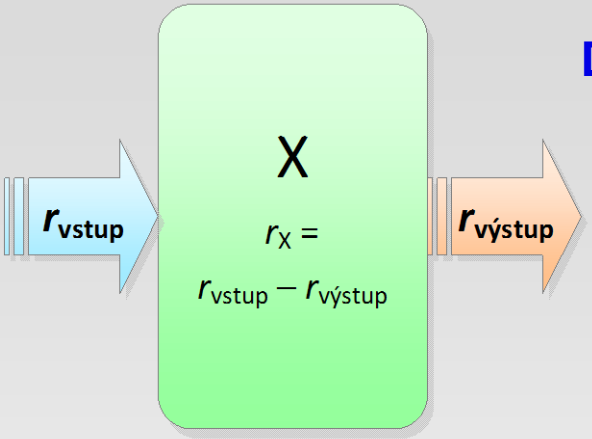
$$A-kX=\left(A-kX_0\right)e^{-kt}$$

$$-kX=\left(A-kX_0\right)e^{-kt}-A$$

$$X=\frac{A}{k}+\left(X_0-\frac{A}{k}\right)e^{-kt}$$

JEDNOREZERVOÁROVÝ SYSTÉM

Stacionární stav



$$r_{vstup} = r_{výstup}$$

$$r_{výsledný} = \frac{dX}{dt} = A - kX_S = 0$$

$$X_S = \frac{A}{k}$$

$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k} \right) e^{-kt}$$

$$X = X_S + \left(X_0 - X_S \right) e^{-kt}$$

Doba zdržení

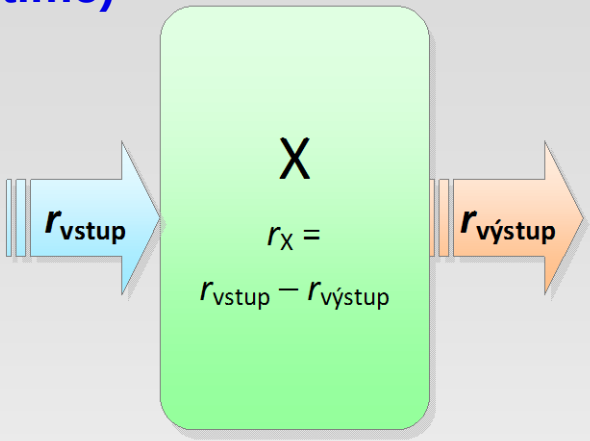
$$t_{zdržení} = \frac{X_S}{r_{vstup}} = \frac{X_S}{r_{výstup}}$$

$$t_{zdržení} = \frac{X_S}{A} = \frac{\frac{A}{k}}{A} = \frac{1}{k}$$

$$t_{zdržení} = \frac{X_S}{kX_S} = \frac{1}{k}$$

JEDNOREZERVOÁROVÝ SYSTÉM

Doba odezvy (response time)



$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k} \right) e^{-kt}$$

absolutní

$$\left(X_0 - \frac{A}{k} \right) e^{-kt} = \left(X_0 - X_S \right) e^{-kt}$$

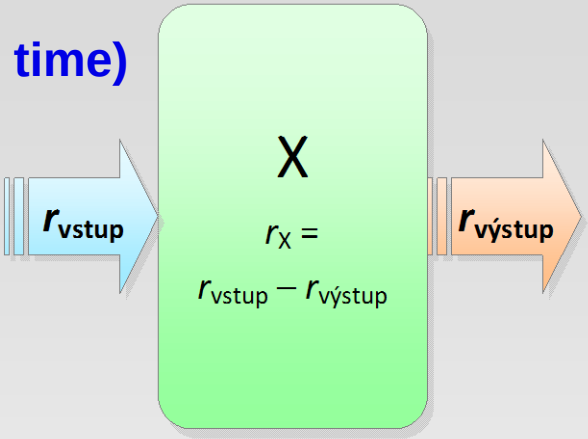
$$e^{-kt_{odezvy}} = 0,1$$

$$t_{odezvy} = -\frac{\ln 0,1}{k} = \frac{2,303}{k}$$

$$-kt_{odezvy} = \ln 0,1$$

JEDNOREZERVOÁROVÝ SYSTÉM

Doba odezvy (response time)



$$X = \frac{A}{k} + \left(X_0 - \frac{A}{k} \right) e^{-kt}$$

relativní

$$\Delta X_{rel} = \frac{velikost\ odchylky}{stacionární\ stav} = \frac{\left(X_0 - X_S \right) e^{-kt}}{X_S} = \left(\frac{X_0}{X_S} - 1 \right) e^{-kt}$$

$$0{,}1 = \left(\frac{X_0}{X_S} - 1 \right) e^{-kt_{odezvy}}$$

$$\frac{0{,}1X_S}{X_0 - X_S} = e^{-kt_{odezvy}}$$

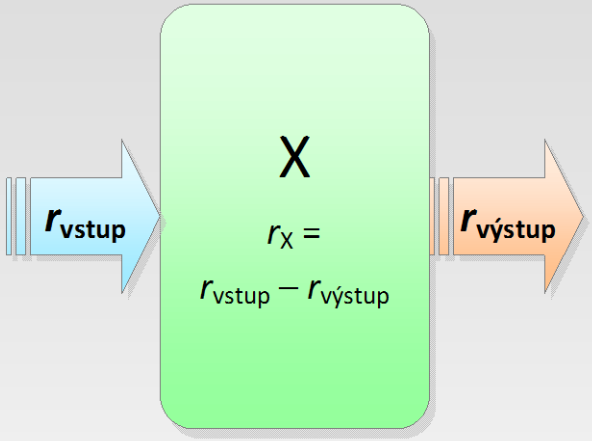
$$t_{odezvy} = \frac{1}{k} \ln \frac{X_0 - X_S}{0{,}1X_S}$$

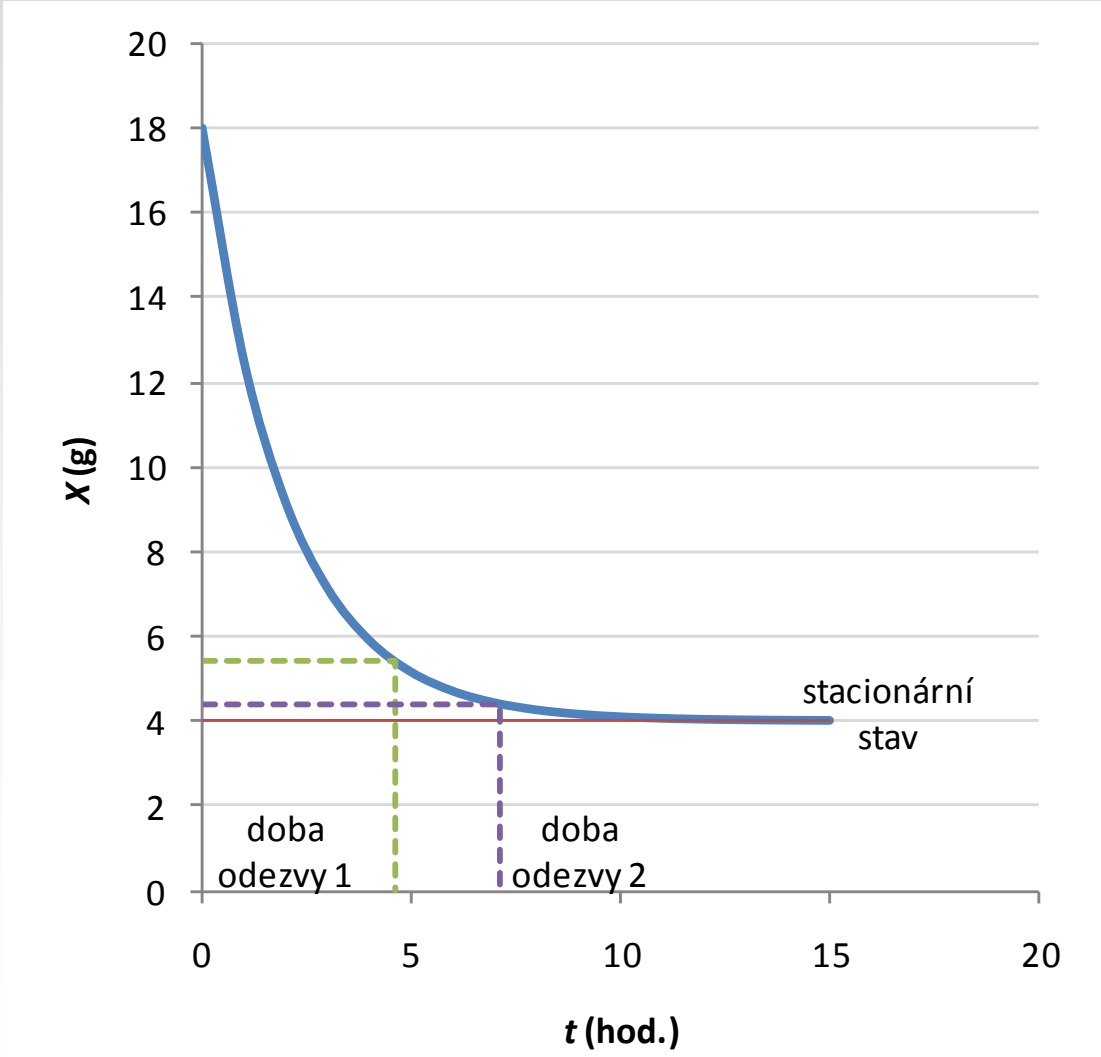
$$0{,}1 = \left(\frac{X_0 - X_S}{X_S} \right) e^{-kt_{odezvy}}$$

$$\ln \frac{0{,}1X_S}{X_0 - X_S} = -kt_{odezvy}$$

JEDNOREZERVOÁROVÝ SYSTÉM

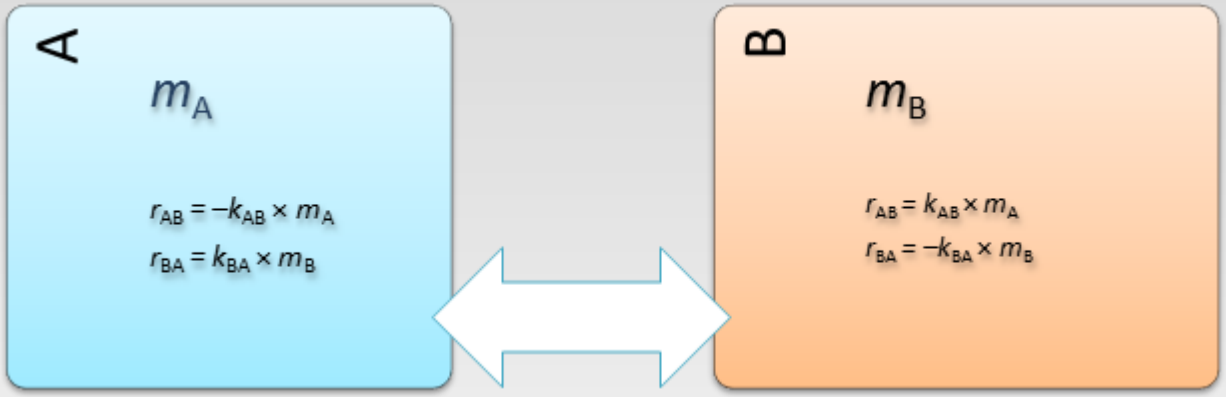
Doba odezvy (response time)





DVOUREZERVOÁROVÝ SYSTÉM

Časový vývoj *X*



$$r_{AB} = k_{AB} m_A$$

$$r_{BA} = k_{BA} m_B$$

$$r_A = -r_{AB} + r_{BA} = -k_{AB} m_A + k_{BA} m_B$$

$$r_B = -r_{BA} + r_{AB} = -k_{BA} m_B + k_{AB} m_A$$

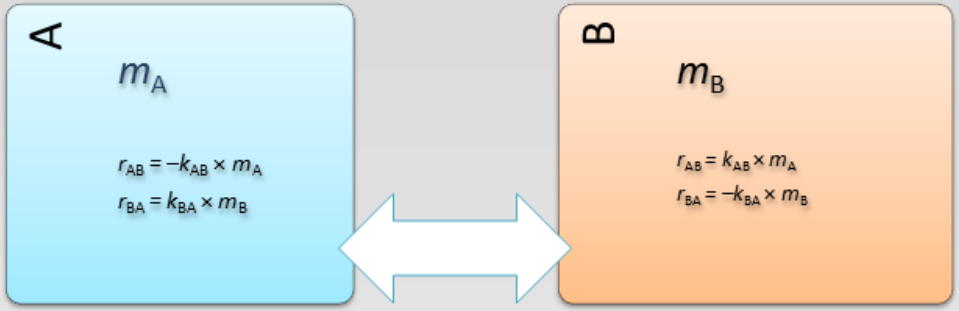
$$\frac{dm_A}{dt} = -k_{AB} m_A + k_{BA} m_B$$

$$\frac{dm_B}{dt} = k_{AB} m_A - k_{BA} m_B$$

$$m = m_{A0} + m_{B0} = m_A + m_B$$

DVOUREZERVOÁROVÝ SYSTÉM

Časový vývoj *X*



$$m_B = m - m_A$$

$$\frac{dm_A}{dt} = -k_{AB} m_A + k_{BA}\left(m - m_A\right)$$

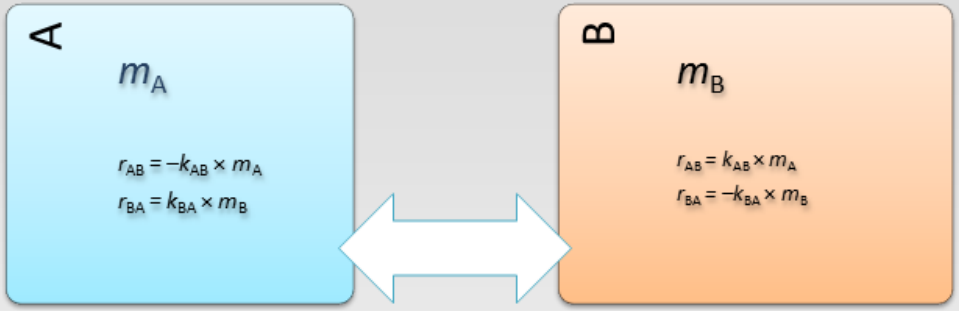
$$\frac{dm_A}{dt} = -\left(k_{AB} + k_{BA}\right) m_A + k_{BA} m$$

$$\int_{m_{A0}}^{m_A} \frac{dm_A}{-\left(k_{AB} + k_{BA}\right) m_A + k_{BA} m} = \int_0^t dt$$

$$-\frac{1}{k_{AB} + k_{BA}} \ln \frac{-\left(k_{AB} + k_{BA}\right) m_A + k_{BA} m}{-\left(k_{AB} + k_{BA}\right) m_{A0} + k_{BA} m} = t$$

DVOUREZERVOÁROVÝ SYSTÉM

Časový vývoj *X*

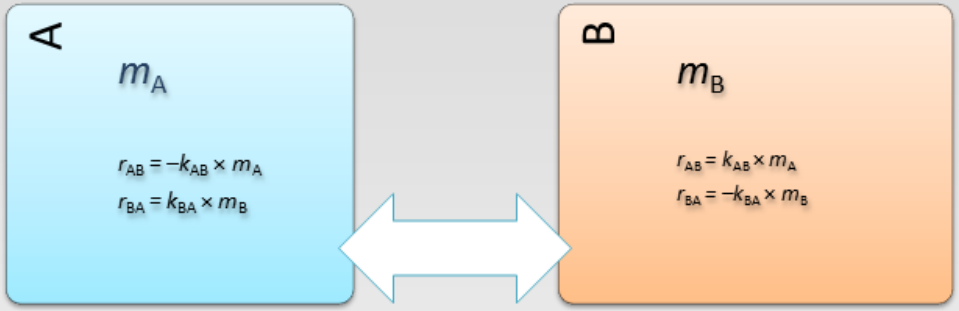


$$m_A = \frac{k_{BA}}{k_{AB}+k_{BA}} m + \left(m_{A0} - \frac{k_{BA}}{k_{AB}+k_{BA}} m \right) e^{-(k_{AB}+k_{BA})t}$$

$$m_B = \frac{k_{AB}}{k_{AB}+k_{BA}} m + \left(m_{B0} - \frac{k_{AB}}{k_{AB}+k_{BA}} m \right) e^{-(k_{AB}+k_{BA})t}$$

DVOUREZERVOÁROVÝ SYSTÉM

Stacionární stav



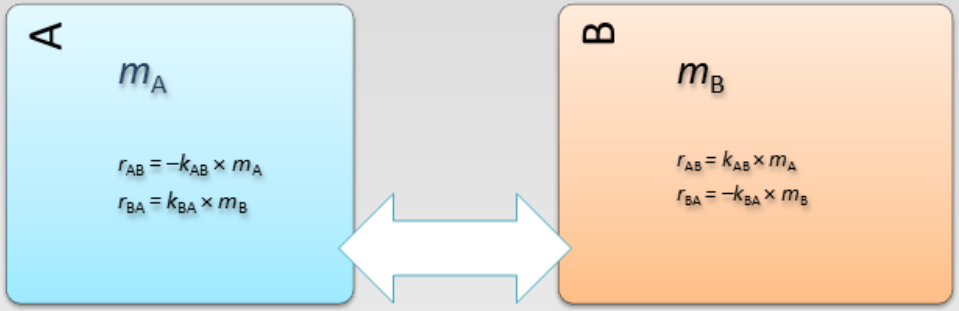
$$\frac{dm_A}{dt} = 0 = -\left(k_{AB} + k_{BA}\right) m_{AS} + k_{BA} m$$

$$m_{AS} = \frac{k_{BA}}{k_{AB} + k_{BA}} m$$

$$m_{BS} = \frac{k_{AB}}{k_{AB} + k_{BA}} m$$

DVOUREZERVOÁROVÝ SYSTÉM

Doba zdržení



$$t_{zdržení\ A} = \frac{m_{AS}}{r_A} = \frac{m_{AS}}{r_B}$$

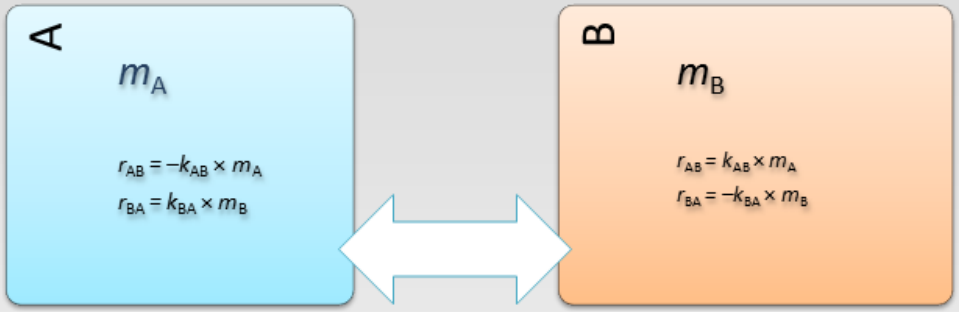
$$t_{zdržení\ B} = \frac{m_{BS}}{r_A} = \frac{m_{BS}}{r_B}$$

$$t_{zdržení\ A} = \frac{m_{AS}}{k_{AB} m_{AS}} = \frac{m_{AS}}{k_{BA} m_{BS}} = \frac{1}{k_{AB}}$$

$$t_{zdržení\ B} = \frac{m_{BS}}{k_{AB} m_{AS}} = \frac{m_{BS}}{k_{BA} m_{BS}} = \frac{1}{k_{BA}}$$

DVOUREZERVOÁROVÝ SYSTÉM

Doba odezvy



$$m_A = m_{AS} + (m_{A0} - m_{AS})e^{-(k_{AB}+k_{BA})t}$$

$$m_B = m_{BS} + (m_{B0} - m_{BS})e^{-(k_{AB}+k_{BA})t}$$

Odchylky od stacionárního stavu

$$(m_{A0} - m_{AS})e^{-(k_{AB}+k_{BA})t}$$

$$(m_{B0} - m_{BS})e^{-(k_{AB}+k_{BA})t}$$

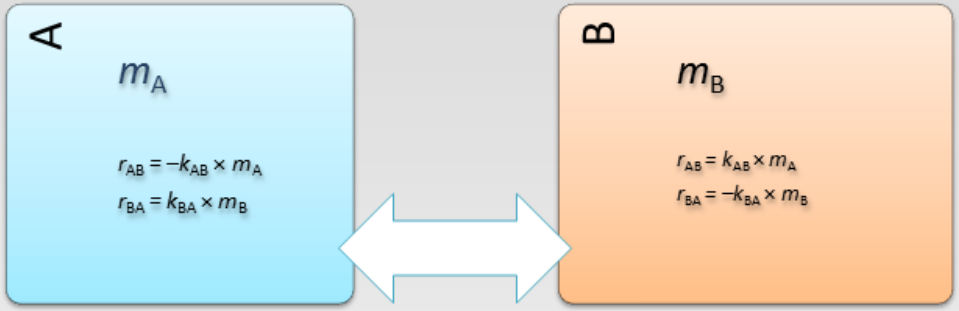
Pro 10 % původní hodnoty

$$e^{-(k_{AB}+k_{BA})t_{odezvy}} = 0,1$$

$$t_{odezvy} = -\frac{1}{k_{AB}+k_{BA}} \ln 0,1$$

DVOUREZERVOÁROVÝ SYSTÉM

Doba odezvy



Pro 10 % odchylky ze stacionárního stavu

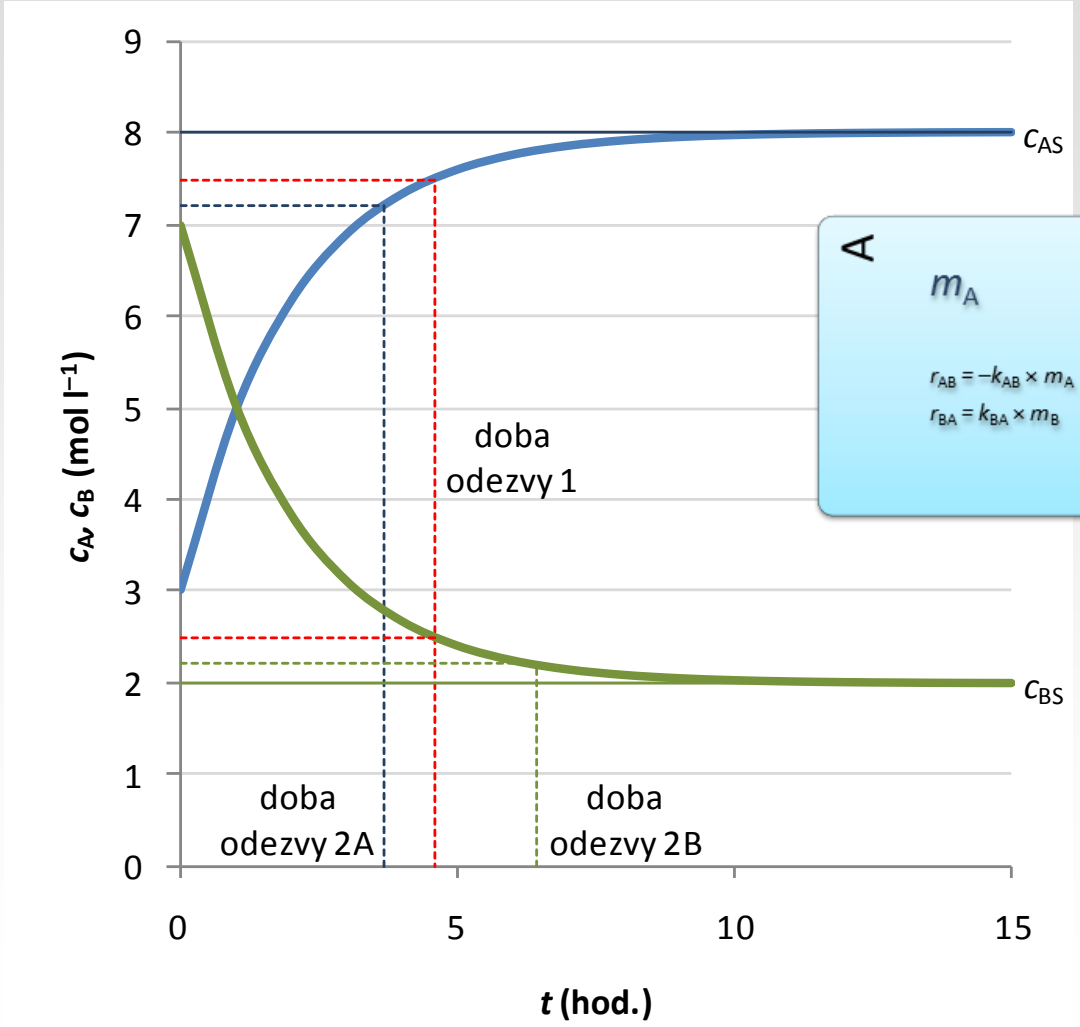
$$\frac{(m_{A0} - m_{AS})}{m_{AS}} e^{-(k_{AB}+k_{BA})t} = 0{,}1$$

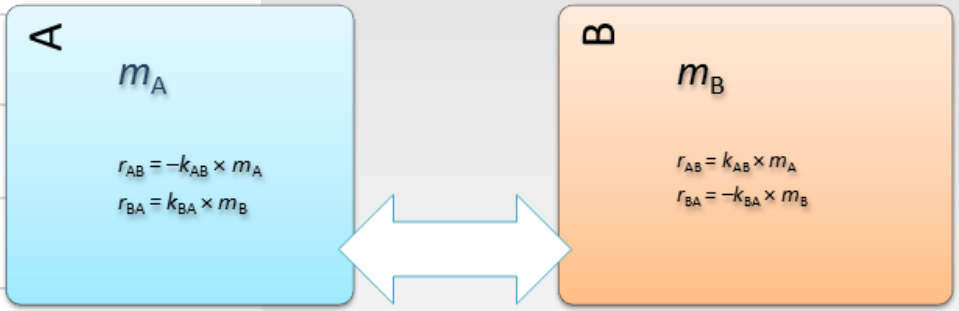
$$t_{odezvy\ A} = -\frac{1}{k_{AB} + k_{BA}} \ln \frac{0{,}1 \times m_{AS}}{|m_{A0} - m_{AS}|}$$

$$t_{odezvy\ B} = -\frac{1}{k_{AB} + k_{BA}} \ln \frac{0{,}1 \times m_{BS}}{|m_{B0} - m_{BS}|}$$

DVOUREZERVOÁROVÝ SYSTÉM

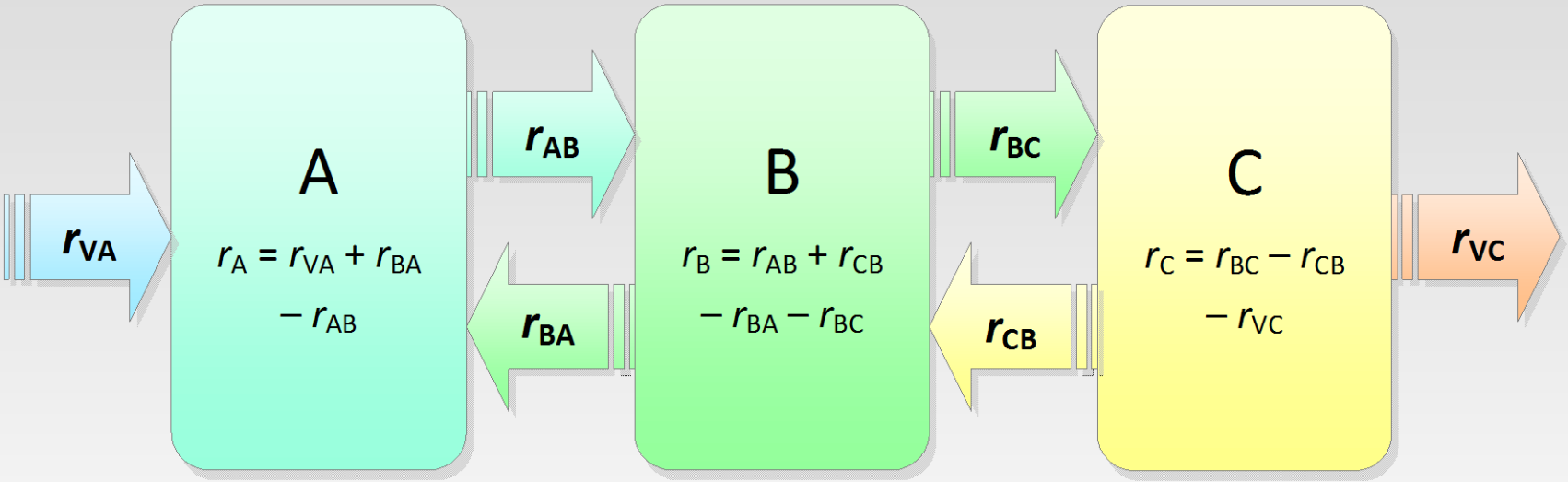
Doba odezvy





TŘÍREZERVOÁROVÝ SYSTÉM

Vývoj koncentrací (obsahů)

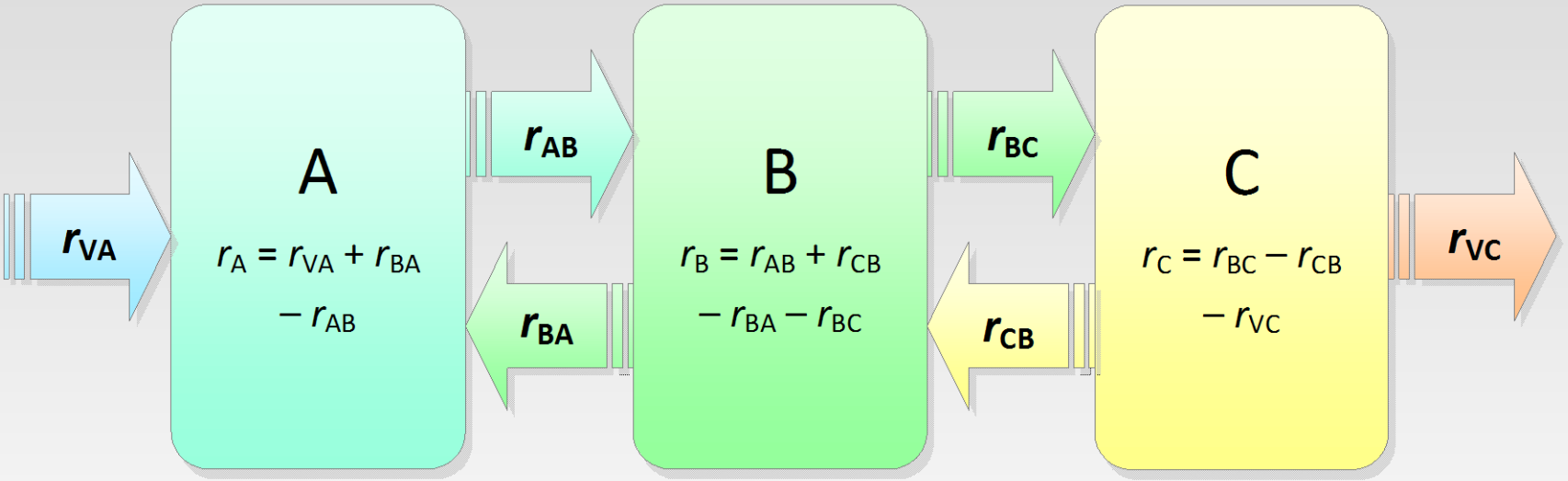


$$r_{VA} = A \qquad r_{BA} = k_{BA} m_B \qquad r_{CB} = k_{CB} m_C$$

$$r_{AB} = k_{AB} m_A \qquad r_{BC} = k_{BC} m_B \qquad r_{VC} = k_{VC} m_C$$

TŘÍREZERVOÁROVÝ SYSTÉM

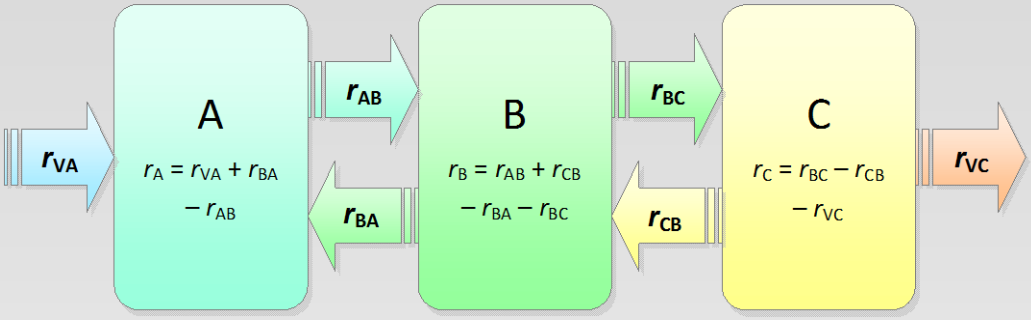
Vývoj koncentrací (obsahů)



$$\rightarrow CO_2(aq) + H_2O \underset{r_{-1}}{\overset{r_{+1}}{\rightleftharpoons}} H_2CO_3 \underset{r_{-2}}{\overset{r_{+2}}{\rightleftharpoons}} H^+ + HCO_3^- \underset{r_{-3}}{\overset{r_{+3}}{\rightleftharpoons}} H^+ + CO_3^{2-} \rightarrow$$

TŘÍREZERVOÁROVÝ SYSTÉM

Vývoj koncentrací (obsahů)



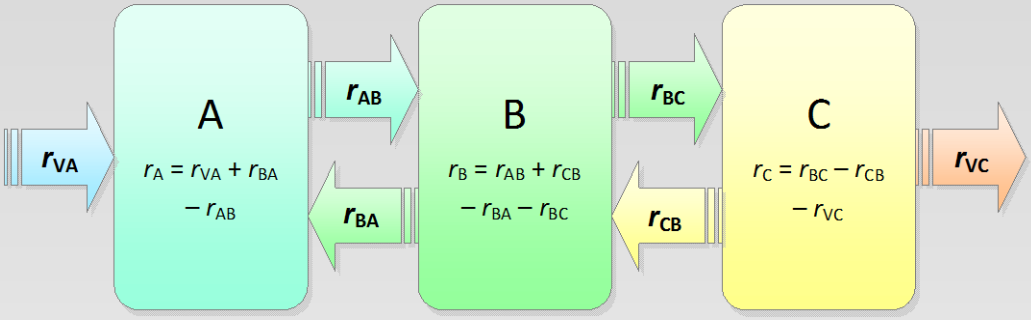
$$r_A = \frac{dm_A}{dt} = r_{VA} + r_{BA} - r_{AB} = A + k_{BA} m_B - k_{AB} m_A$$

$$r_B = \frac{dm_B}{dt} = r_{AB} - r_{BA} + r_{CB} - r_{BC} = k_{AB} m_A - k_{BA} m_B + k_{CB} m_C - k_{BC} m_B$$

$$r_C = \frac{dm_C}{dt} = r_{BC} - r_{CB} - r_{VC} = k_{BC} m_B - k_{CB} m_C - k_{VC} m_C$$

TŘÍREZERVOÁROVÝ SYSTÉM

Stacionární stav



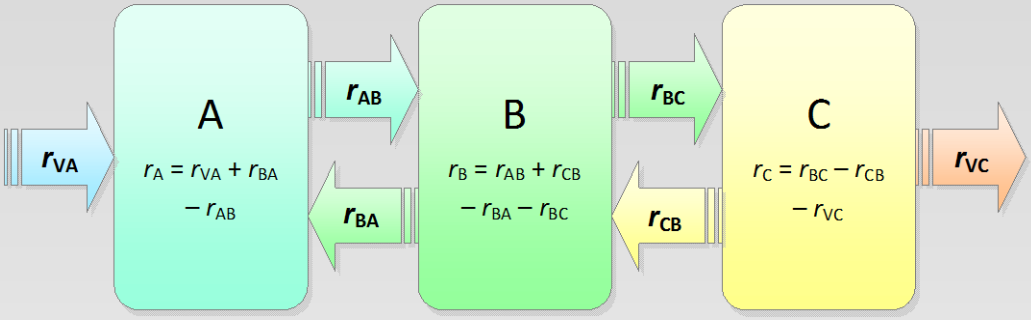
$$\frac{dm_A}{dt} = A + k_{BA} m_{BS} - k_{AB} m_{AS} = 0 \qquad m_{AS} = \frac{A k_{BA} \left(k_{CB} + k_{VC} \right) + A k_{BC} k_{VC}}{k_{AB} k_{BC} k_{VC}}$$

$$\frac{dm_B}{dt} = k_{AB} m_{AS} - k_{BA} m_{BS} + k_{CB} m_{CS} - k_{BC} m_{BS} = 0 \qquad m_{BS} = \frac{A\left(k_{CB} + k_{VC} \right)}{k_{BC} k_{VC}}$$

$$\frac{dm_C}{dt} = k_{BC} m_{BS} - k_{CB} m_{CS} - k_{VC} m_{CS} = 0 \qquad m_{CS} = \frac{A}{k_{VC}}$$

TŘÍREZERVOÁROVÝ SYSTÉM

Doba zdržení



$$t_{zdržení\ A} = \frac{m_{AS}}{k_{AB} m_{AS}} = \frac{1}{k_{AB}}$$

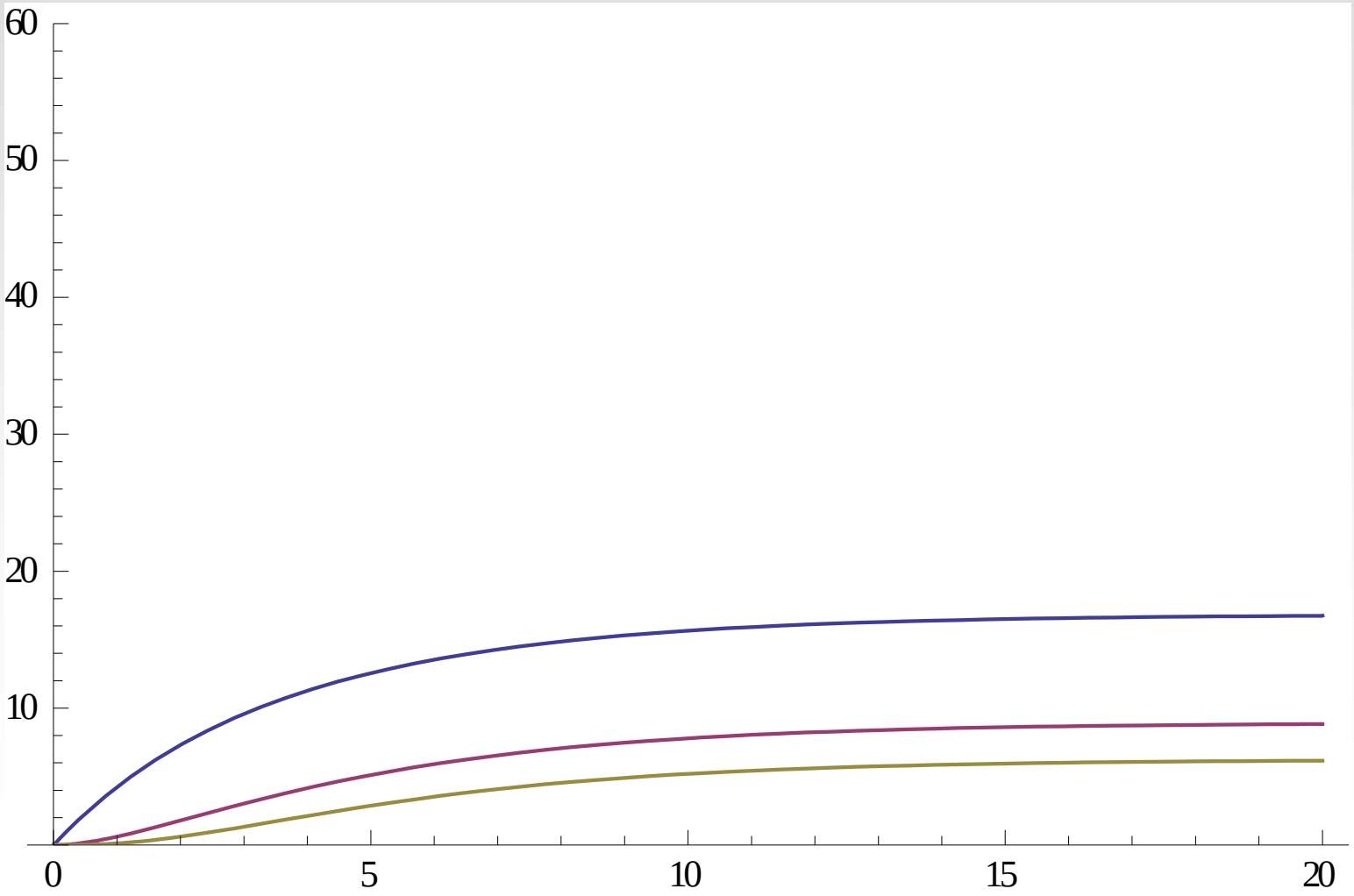
$$t_{zdržení\ B} = \frac{m_{BS}}{\left(k_{BA} + k_{BC}\right) m_{BS}} = \frac{1}{k_{BA} + k_{BC}}$$

$$t_{zdržení\ C} = \frac{m_{CS}}{\left(k_{CB} + k_{VC}\right) m_{CS}} = \frac{1}{k_{CB} + k_{VC}}$$

TŘÍREZERVOÁROVÝ SYSTÉM

A = 5,0; k_{AB} = 0,35; k_{AB} = 0,1; k_{BC} = 0,7; k_{CB} = 0,2; k_{VC} = 0,8

m_{AS} = 16,84; m_{BS} = 8,93; m_{CS} = 6,25 **m_{A0} = 0; m_{B0} = 0; m_{C0} = 0**

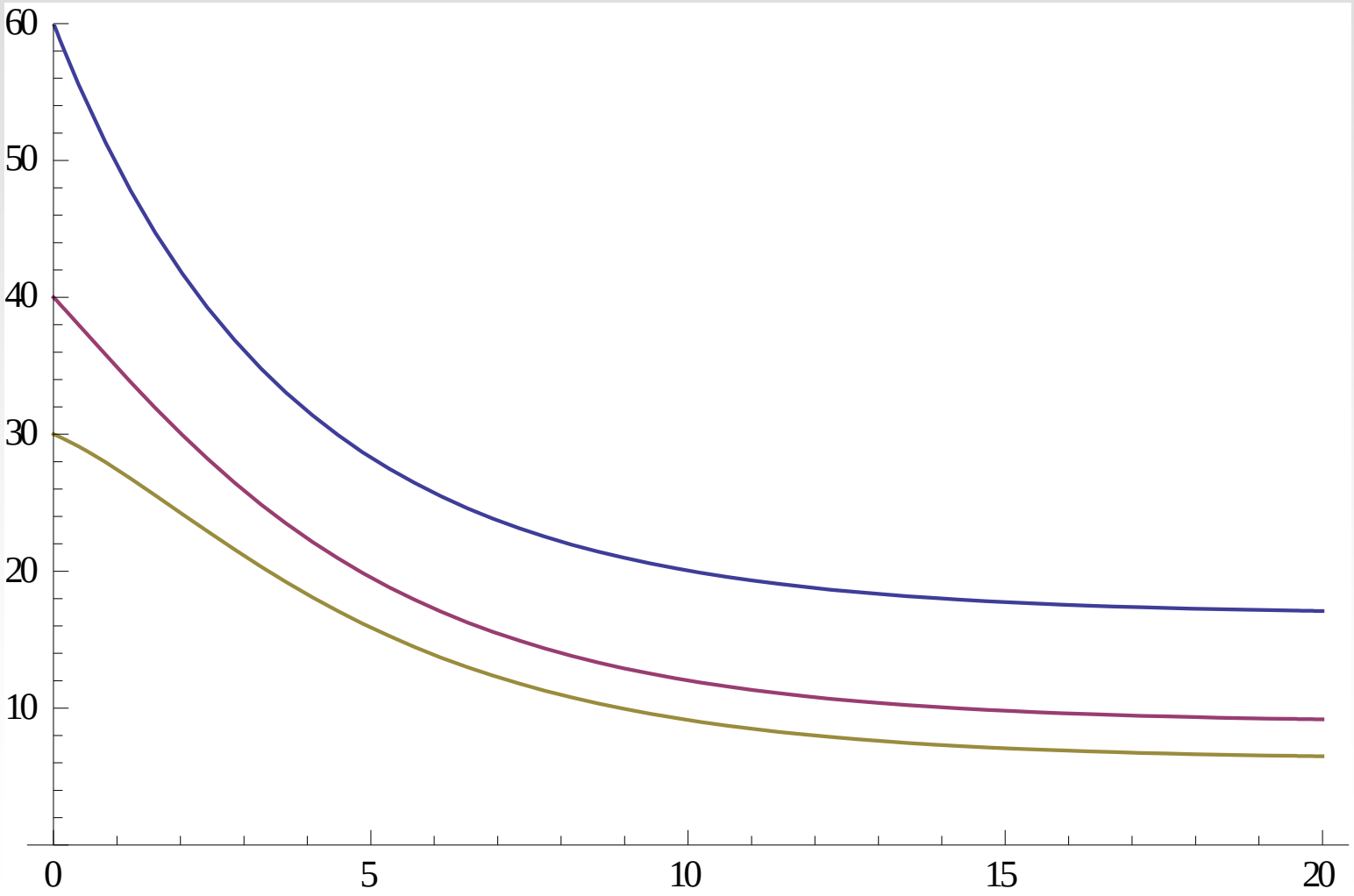


TŘÍREZERVOÁROVÝ SYSTÉM

A = 5,0; k_{AB} = 0,35; k_{AB} = 0,1; k_{BC} = 0,7; k_{CB} = 0,2; k_{VC} = 0,8

m_{AS} = 16,84; m_{BS} = 8,93; m_{CS} = 6,25

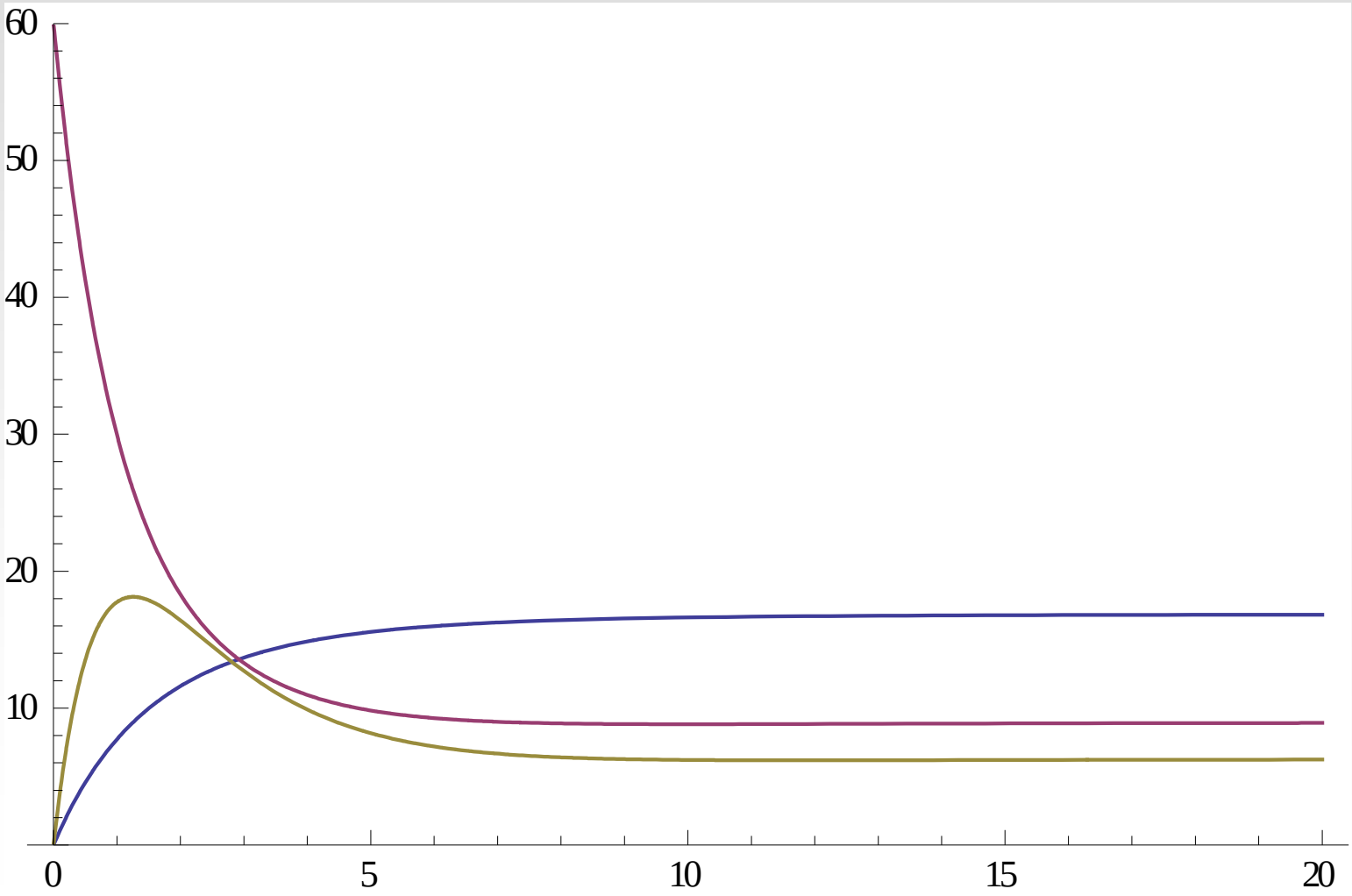
m_{A0} = 60; m_{B0} = 40; m_{C0} = 30



TŘÍREZERVOÁROVÝ SYSTÉM

A = 5,0; k_{AB} = 0,35; k_{AB} = 0,1; k_{BC} = 0,7; k_{CB} = 0,2; k_{VC} = 0,8

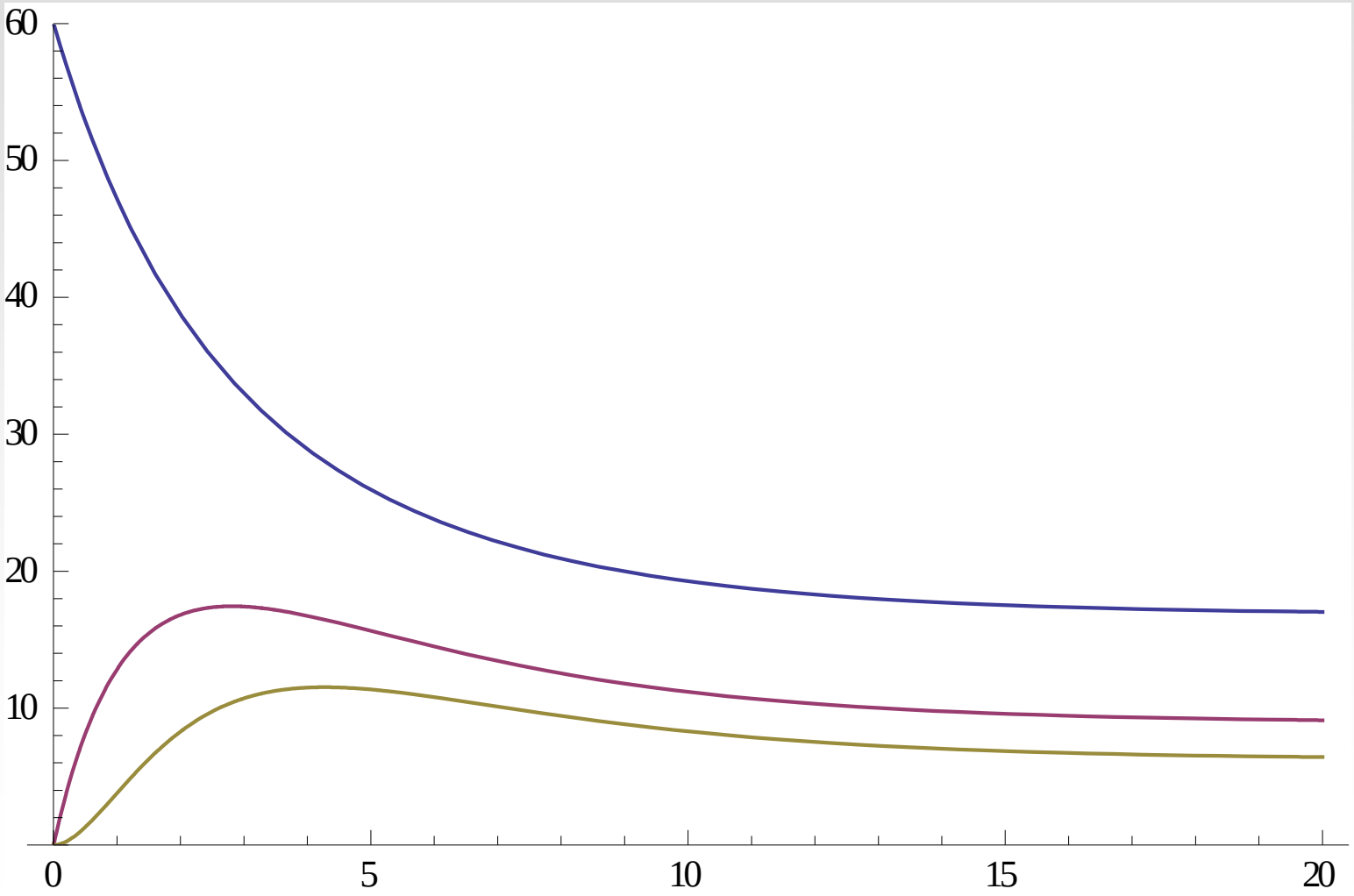
m_{AS} = 16,84; m_{BS} = 8,93; m_{CS} = 6,25 **m_{A0} = 0; m_{B0} = 60; m_{C0} = 0**



TŘÍREZERVOÁROVÝ SYSTÉM

A = 5,0; k_{AB} = 0,35; k_{AB} = 0,1; k_{BC} = 0,7; k_{CB} = 0,2; k_{VC} = 0,8

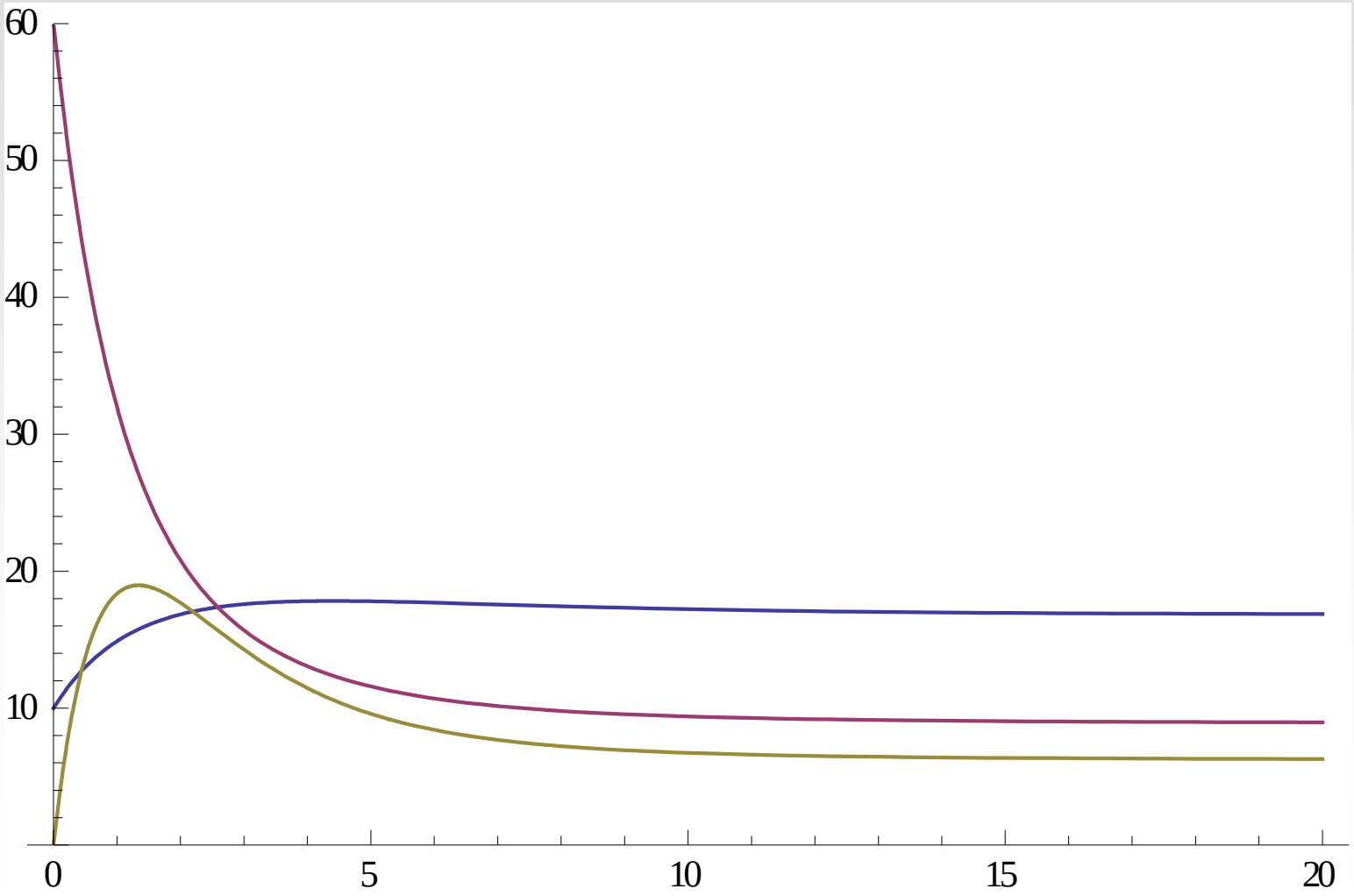
m_{AS} = 16,84; m_{BS} = 8,93; m_{CS} = 6,25 **m_{A0} = 60; m_{B0} = 0; m_{C0} = 0**



TŘÍREZERVOÁROVÝ SYSTÉM

A = 5,0; k_{AB} = 0,35; k_{AB} = 0,1; k_{BC} = 0,7; k_{CB} = 0,2; k_{VC} = 0,8

m_{AS} = 16,84; m_{BS} = 8,93; m_{CS} = 6,25 **m_{A0} = 10; m_{B0} = 60; m_{C0} = 0**



TŘÍREZERVOÁROVÝ SYSTÉM

A = 5,0; k_{AB} = 0,35; k_{AB} = 0,1; k_{BC} = 0,7; k_{CB} = 0,2; k_{VC} = 0,8

m_{AS} = 16,84; m_{BS} = 8,93; m_{CS} = 6,25 **m_{A0} = 60; m_{B0} = 0; m_{C0} = 13**