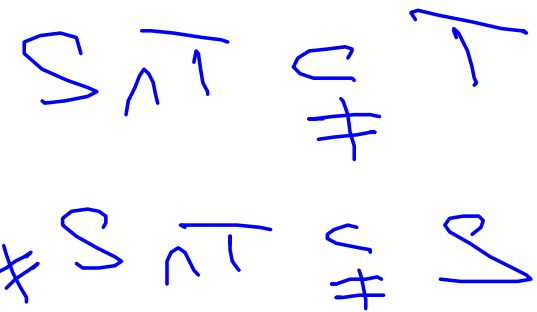
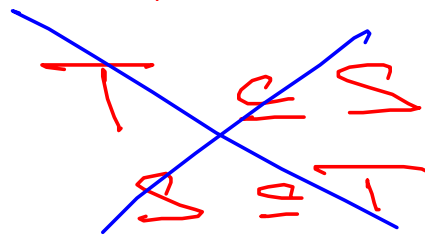
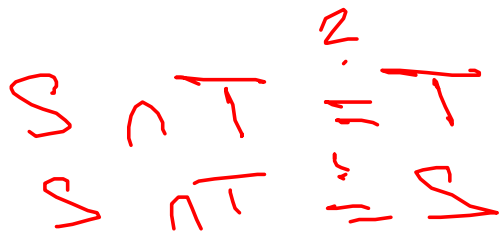


M a N párhuzamos, nem lezárt, $? \dim V =$

$M \neq N$

$M \cap N = \emptyset$

$\dim(M) \cap \dim(N) \neq \emptyset$



$\dim(S \cap T) \geq 1$

$\dim(T) \geq 2, \dim(S) \geq 2$

$$\dim(S+T) = \dim S + \dim T - \dim(S \cap T)$$

$$\dim(S \cap T) < \min(\dim(S), \dim(T))$$

$$\dim(S \cap T) + 1 \leq \min(\dim(S), \dim(T))$$

$$\rightarrow \quad \quad \quad \rightarrow \quad \quad \text{--- min}$$

$$\dim(S+T) \geq \underbrace{\min(\dim(S), \dim(T)) + 1}$$

$$S+T \not\subseteq V$$

$$\dim(V) \geq 4$$

$$\begin{aligned}
 & f(x) = \varphi(x) + u && \varphi \text{ LZ} \\
 & \leftarrow && u \in \mathbb{V} \Rightarrow \\
 & \varphi \text{ is } \underline{\text{AZ}} && \forall \alpha \in \mathbb{K} \\
 & f(\alpha x_1 + (1-\alpha)x_2) \stackrel{?}{=} \alpha f(x_1) + (1-\alpha)f(x_2) \\
 & \varphi(\alpha x_1 + (1-\alpha)x_2) + u = \alpha \varphi(x_1) + (1-\alpha)\varphi(x_2) \\
 & + \alpha u + (1-\alpha)u = \alpha (\underbrace{\varphi(x_1) + u}_{f(x_1)}) + (1-\alpha) (\underbrace{\varphi(x_2) + u}_{f(x_2)})
 \end{aligned}$$

$$\Rightarrow f: X \rightarrow W \quad \varphi: W$$

$$f = \varphi + u$$

$$\underline{\underline{\varphi(0) = 0}}$$

$$f(0) = \varphi(0) + u = u$$

$$\varphi = f - u$$

$$= f(\lambda x_1 + \sigma x_2 + (1-\lambda-\sigma)0)$$

$$- u = \lambda f(x_1) + \sigma f(x_2)$$

$$\varphi(\lambda x_1 + \sigma x_2) = \lambda f(x_1) + \sigma f(x_2) - u$$

$$= \lambda (f(x_1) - u) + \sigma (f(x_2) - u)$$

$$\begin{aligned} \varphi &= \psi + u, & \psi &= \varphi - u \\ (\varphi \circ \varphi)(R) &= \varphi(\psi + u) = \\ &= \varphi(\varphi(R) + u) + u = \\ &= \varphi(\varphi(R)) + \varphi(u) + u \end{aligned}$$

$f(M) \stackrel{?}{=} AP$

$P_1, P_2 \in f(M)$

$M = Q + S \quad \checkmark \Rightarrow \checkmark$

$f(M) = f(Q) + f(S)$ \boxed{AP}

$f^{-1}(M) = \emptyset$ (VP) g.m.:

$$f^{-1}(N) \neq \emptyset$$

$$\exists q \in f^{-1}(N)$$

$$f(q) \in N$$

$$N = f(q) + T$$

$$T = N - f(q)$$

$$f^{-1}(N) = \underbrace{q}_{\text{is UP}} + f^{-1}(T) \Rightarrow \underline{\underline{AP}}$$

$$\dim V = \dim \ker \varphi + \dim \operatorname{Im} \varphi$$

$$\operatorname{Im} f = \operatorname{Im} \varphi + \mathbb{P} \quad f = \varphi + \alpha$$

$$? \quad \dim f^{-1}(y) = \dim \ker \varphi$$

$$\forall y \in \operatorname{Im} f \quad y = f(x), \quad x \in V$$

$$x \in f^{-1}(y) \quad \underline{f^{-1}(y)} = \underbrace{x}_{\ker \varphi} + \underbrace{f^{-1}(0)}_{\ker \varphi}$$

$$f = \varphi + \psi$$

$$? f^{-1} = \varphi + \psi^{-1}$$

$$(\varphi + \psi) \circ (\varphi + \psi)(z) = \varphi(\varphi(z)) + \varphi(\psi(z)) + \psi(z) \stackrel{!}{=} id(z) = z.$$

$$\psi = -\varphi(\psi) = -\varphi^{-1}(\psi)$$

f is bi.

\Leftrightarrow

φ is bi.

$$\varphi = \varphi^{-1}$$

$\Rightarrow \varphi = \varphi^{-1}$

$$\varphi(x) = Ax$$

$$R(\varphi) = \dim(\text{Im}(\varphi)) \stackrel{\text{def}}{=} \boxed{R(A)} \quad | \quad \cdot \quad \boxed{=}$$

$$= R(A)$$

$$\varphi: \mathbb{K}^m \rightarrow \mathbb{K}^m$$

$$m = \dim \text{Ker } \varphi + \dim \begin{array}{c} \text{Im } \varphi \\ \parallel \\ R(A) \end{array}$$

$$B = \{2, 5, 4\}$$

$$x_2 = 1, \quad x_5 = 0, \quad x_1 = -2, \quad x_3 = x_4 = 0$$

$$(-2, 1, 0, 0, 0)$$

$$x_2 = 0, \quad x_5 = 1$$

$$x_4 = 2, \quad x_3 = -\frac{1}{2}, \quad x_1 = \frac{1}{3}$$

$$\left(\frac{1}{3}, 0, -\frac{1}{2}, 2, 1\right)$$

$$Ax = b$$

$$\Rightarrow \exists \pi \in \mathcal{R}(A|b) \Rightarrow \exists \pi \in \mathcal{R}(A)$$

$$Ay = b$$

$$A\pi = b$$

$$A(y - \pi) = 0$$

$$A(x + \pi) = b$$

$$\pi \in \mathcal{R}(A|b)$$

$$x \in \mathcal{R}(A)$$

$$x + \pi \in \mathcal{R}(A|b)$$

$$A\pi = b$$

$$Ax = 0$$

$$\begin{aligned}
 & Ax = b \text{ has solution} \iff \exists R(A) = R(A|b) \\
 & (c_1, \dots, c_m) \in \mathbb{K}^m \\
 & \sum_{j=1}^m c_j D_j(A) = b \iff \\
 & b \in [D_1(A), \dots, D_m(A)] \\
 & \iff [D_1(A), \dots, D_m(A)] = [D_1(A), \dots, D_m(A), b]
 \end{aligned}$$

$$\begin{array}{cccc|c}
 J_1 & J_1' & R & R = |J_1'| & \\
 \hline
 1 & 0 & 3 & 1 & 2 \\
 0 & 1 & 4 & 2 & 1 \\
 0 & 0 & 1 & 5 & 2 \\
 0 & 0 & 0 & 0 & 0
 \end{array}$$

$$x_4 = x_5 = x_6 = 0$$

$$x_1 = 2, x_2 = -1, x_3 = -\frac{2}{7}$$

$$(2, -1, -\frac{2}{7}, 0, 0, 0)$$

PART. RES.

$$x_4 = 1, \quad x_5 = 0, \quad x_6 = 0$$

$$x_1 = -3, \quad x_2 = -4, \quad x_3 = -1$$

$$\left(-3, -4, -1, 1, 0, 0 \right)$$

$$\left(-\frac{1}{4}, -2, 5, 0, 1, 0 \right)$$

$$\left(0, 1, -6, 0, 0, 1 \right)$$